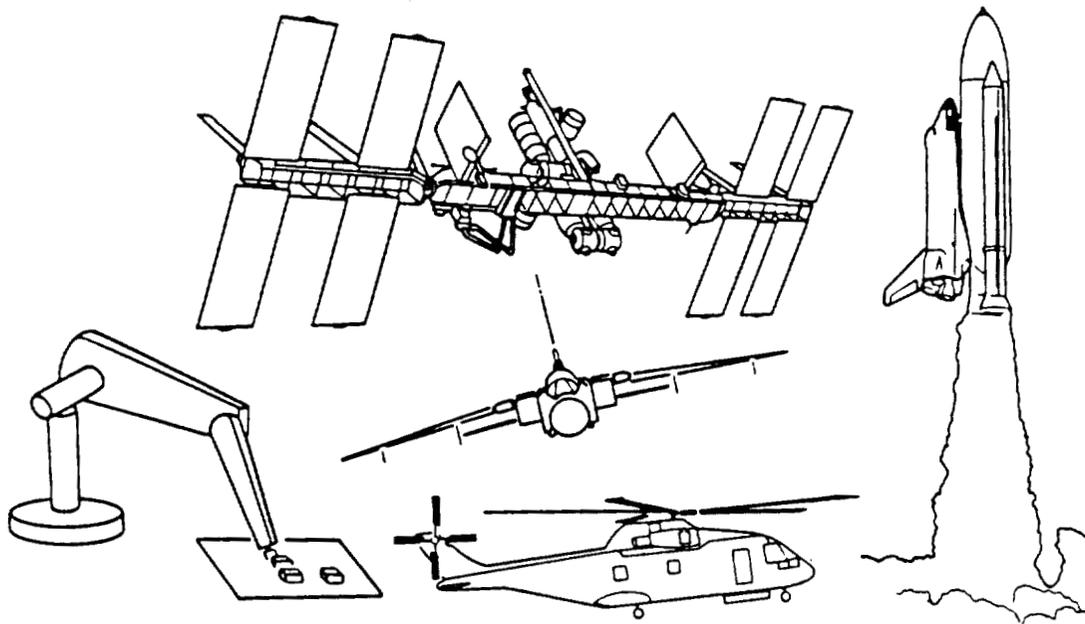


NASA Technical Memorandum 101578, Part One

WORKSHOP on COMPUTATIONAL ASPECTS  
in the CONTROL of FLEXIBLE SYSTEMS

*Held at the Royce Hotel in Williamsburg, Virginia*

*July 12-14, 1988*



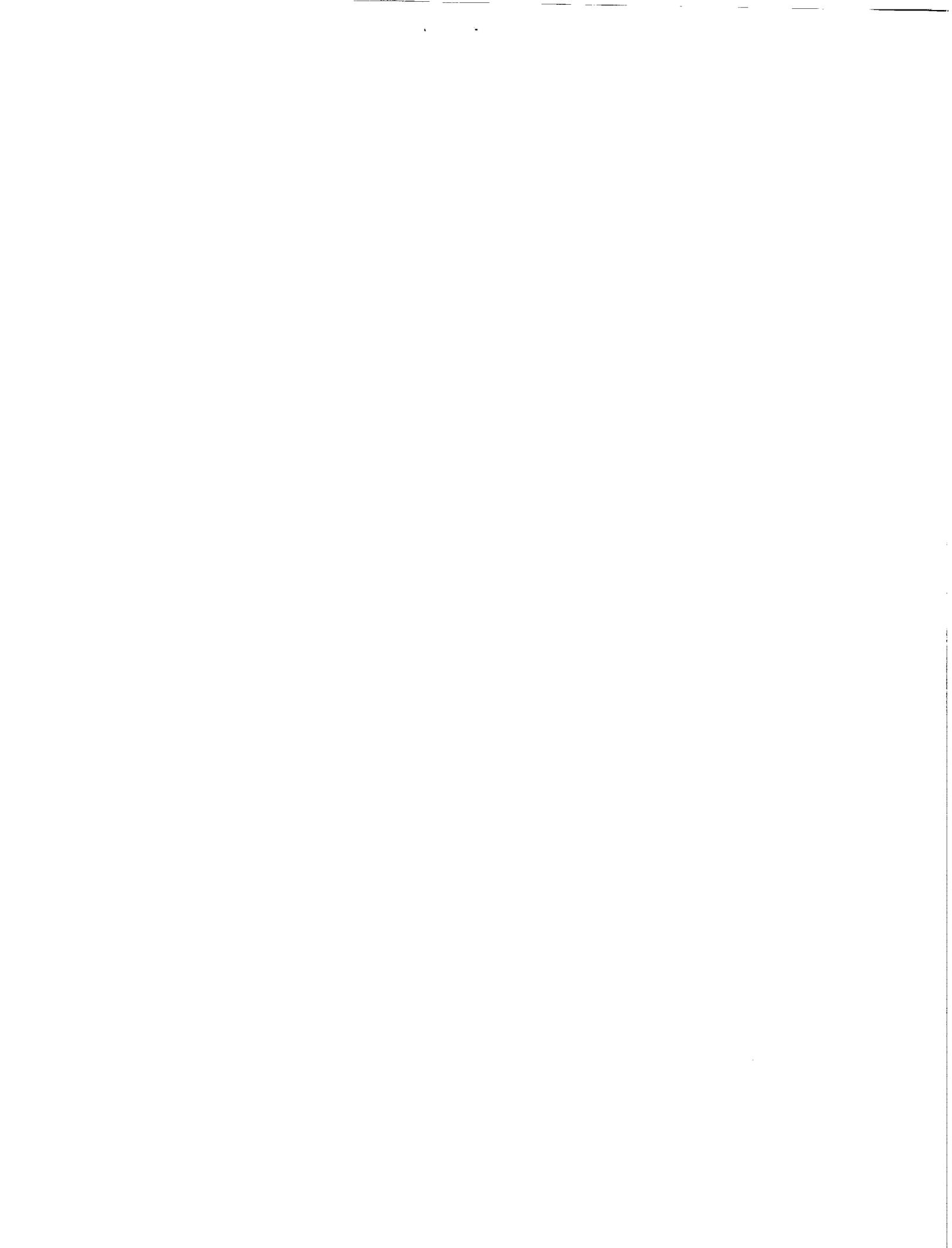
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**Proceedings Compiled by Larry Taylor**



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Reply to Attn of

479

April 12, 1988

**TO:** Invited Workshop Participants  
**FROM:** Lawrence W. Taylor, Jr., Chairman  
**SUBJECT:** Workshop on Computational Aspects in the Control of Flexible Systems

**BACKGROUND:** As aerospace vehicles and robotic systems become larger and more flexible, the attendant complexity results in an increased demand for high fidelity dynamic models. This in turn can cause an excessive computational burden for model development, systems analysis and real-time simulation.

A number of software packages are available for modeling flexible structures and for control analysis, but there remain unsatisfied needs for more efficient and more comprehensive software which is easy to use for modeling, analysis, synthesis and simulation. Low cost parallel processing promises significant increases in computational speed.

**GOAL:** To assess the state of the technology in software tools for simulation analysis and synthesis for the control of flexible aerospace systems; establish capabilities and performance of these tools when applied to specific example problems; and to identify gaps and shortcomings of software tools.

**APPROACH:** A workshop will be held at the Royce Hotel in Williamsburg, Virginia, July 12-14, 1988. This workshop is being organized under the auspices of the Office of Aeronautics and Space Technology (OAST) in NASA Headquarters, and is being chaired by the following people:

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NASA Langley Research Center  
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Hampton, VA 23665-5225  
(804)865-3716

Virginia B. Marks  
NASA Langley Research Center  
M/S 479  
Hampton, VA 23665-5225  
(804)865-2077

Presentations and demonstrations will be given as requested in the Call for Papers (below). In addition, a panel of experts will be assembled to summarize workshop presentations, lead a discussion on the state of the technology concerning computational aspects in the control of flexible structures and recommend future direction of research and areas of concentration. A proceedings will be published and distributed following the workshop which will include presentation materials with brief explanations and an address list of workshop attendees.

**CALL FOR PAPERS:** Presentations will be scheduled for 30 minute time slots including a period for questions and answers. We encourage submissions of one-page abstracts of proposed presentations for the following categories:

**MODELING SOFTWARE FOR FLEXIBLE STRUCTURES:** Special model formulations, model building, model reduction, modeling articulated, flexible structures.

**SYSTEMS ANALYSIS AND SYNTHESIS SOFTWARE:** Control synthesis, time-varying system analysis, and nonlinear system analysis

**ROBOTIC SYSTEMS APPLICATIONS:** Modeling experiences, needed software advances, comparison simulation and actual experience.

**SPACECRAFT APPLICATIONS:** Comparisons of actual and expected system stability and performance, integrated design techniques.

**AIRCRAFT APPLICATIONS:** Techniques by which flexibility is treated, capabilities of high fidelity simulation, comparison between actual and expected flight system stability and performance.

**SIMULATION COMPUTERS:** Workstations for analysis and synthesis, parallel processing for simulation.

**SOFTWARE/COMPUTER DEMONSTRATIONS:** Software developers and vendors are encouraged to demonstrate the capabilities of their wares. Sun and Microvax workstations will be available and space will be provided for additional equipment and displays. For details call Larry Taylor, (804)865-3716.

**PARTICIPATION:** Attendance at this conference by nonpresenters is also encouraged to facilitate thorough discussions in all areas. Special arrangements are not required for non-U.S. citizens attending the workshop in Williamsburg. However, if a non-U.S. citizen desires access to the Langley Research Center for any reason, a letter of endorsement from your Embassy in Washington, DC, must be forwarded to NASA Headquarters before plans can be arranged. The address is:

National Aeronautics and Space Administration  
International Affairs Division  
Code XIC  
Washington, DC 10546-0001

**RESERVATIONS:** A block of rooms has been reserved at the Royce Hotel for attendees of the Workshop on Computational Aspects in the Control of Flexible Systems. The special room rates for this workshop are:

<u>Types of Room</u>	<u>Rate</u>
Single	\$60.00 + 6.5% tax
Double	\$70.00 + 6.5% tax

The cut-off date for these rooms is June 11, 1988. Reservations requested beyond the cut-off date are subject to availability. Rooms may be available after this date, but not necessarily at the same rate. To make reservations, call (804)229-4020 and request a room reserved for the Workshop on Computational Aspects in the Control of Flexible Systems or return the

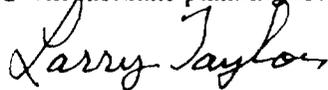
enclosed reservation form before the cut-off date. Reservations must be arranged by the individual attendees, keeping in mind that during the summer Colonial Williamsburg is a popular vacation site.

**REGISTRATION:** A registration form for the workshop is enclosed. Please complete this form and return it by June 11, 1988.

**DEADLINES:** All submissions must be accompanied by the title, full name, affiliation, complete address and telephone number of each co-author in regular sessions and each participant in panel sessions.

- MAY 1, 1988:** Indicate interest in demonstrating software packages or computers by calling Larry Taylor, Chairman.
- JUNE 1, 1988:** Submit one-page abstract for proposed technical presentations to Gin Marks, Vice Chairman.
- JUNE 11, 1988:** Registration forms due to Gin Marks.  
Cut-off date for reservations at the Royce Hotel at the special group rate.
- JUNE 22, 1988:** Authors are notified of presentation acceptance. Demonstrators are notified on demonstration acceptance.
- JULY 1, 1988:** Finalized agenda is sent to all registered attendees.
- JULY 12, 1988:** First day of workshop. Authors are asked to provide camera ready copies of presentation material with brief explanations included for workshop proceedings.

Early response to the Call for Papers and submission of your registration forms is appreciated and will facilitate planning for a successful workshop.



Lawrence W. Taylor, Jr.  
Chairman

Enclosures:  
Preliminary Agenda  
Registration Form  
Reservation Request  
Local Attractions  
Golf Facilities  
"Williamsburg Great Entertainer"

## COMPUTATIONAL ASPECTS WORKSHOP ORGANIZATION

---

<b>Chairman</b>	<b>Larry Taylor</b>
<b>Administrator</b>	<b>Trish Johnson</b>
<b>Aero Program Chairman</b>	<b>Jerry Elliott</b>
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<b>Proceedings Compiler</b>	<b>Larry Taylor</b>
<b>Consultant</b>	<b>Gin Marks</b>

**Attendance List for the  
WORKSHOP ON  
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IN THE CONTROL OF FLEXIBLE SYSTEMS**

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**SESSION I - NEEDS FOR ADVANCED CSI SOFTWARE**

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**NASA'S CONTROLS-STRUCTURES INTERACTION PROGRAM**

By

Brantley R. Hanks  
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**ABSTRACT**

Spacecraft design is conducted conventionally by estimating sizes and masses of mission-related components, designing a structure to maintain desired component relationships during operations, and then designing a control system to orient, guide and/or move the spacecraft to obtain required performance. This approach works well in cases where a relatively high stiffness structural bus is attainable and where nonstructural components are massive relative to the structure.

Occasionally, very flexible, distributed-mass, structural components, such as solar arrays and antennas are attached to the structural bus. In these, the primary purpose is to maintain geometric relationships rather than support masses which are large relative to the structural mass. Because of their flexibility, potential interactions of such components with the spacecraft control system can reduce performance or restrict operations. This interaction, referred in this document as controls-structures interaction (CSI), also occurs in small components if precision pointing and/or surface shapes/orientations are critical performance factors and in very large systems where attaining a high structural stiffness is detrimental to launch and operations requirements. The degree of success in handling these situations in past designs is uncertain. Reduced performance and unexpected dynamic motions have been observed in operational spacecraft; but, in most cases, the spacecraft were not sufficiently instrumented to determine the cause.

Designing to avoid CSI generally requires either stiffening the structure (costly in mass, inertia and fuel consumption) or slowing down the control system response (costly in performance capability). Using the power available in the control system to reduce the interactive motions is theoretically possible; a great number of approaches to do so have been advanced in the literature. However, reduction of these approaches to practice on hardware has not been accomplished on any meaningful scale. The techniques generally require analytical representations of the system within the control loop. The fidelity, size, accuracy and computational speed of these analyses are integrally related to, and affect the performance of, the combined structure-control system. The structural hardware, the control hardware, and the analytical models cannot be separated in the process of verifying that the system performs as required. Furthermore, if improperly designed, the closed-loop system is subject not only to inadequate performance, but also to destructive dynamic instability.

Future NASA missions are likely to increase the likelihood of CSI because of increased size of distributed-mass components, greater requirements for surface and pointing precision, increased use of articulated moving components, and increased use of multi-mission science platforms (with multiple control systems on board). An SSTAC

develop the technology to solve the CSI problem. More recently, a NASA CSI Requirements Committee reviewed potential future NASA missions and found the need for CSI technology to be widespread.

Control system Interf. for (CSI)

A NASA program is about to start which has the objective to advance ~~CSI~~ technology to a point where it can be used in spacecraft design for future missions. Because of the close interrelationships between the structure, the control hardware, and the analysis/design, a highly interdisciplinary activity is defined in which structures, dynamics, controls, computer and electronics engineers work together on a daily basis and are co-located to a large extent. Methods will be developed which allow the controls and structures analysis and design functions to use the same mathematical models. Hardware tests and applications are emphasized and will require development of concepts and test methods to carry out.

Because of a variety of mission application problem classes, several time-phased, focus ground test articles are planned. They will be located at the Langley Research Center (LaRC), the Marshall Space Flight Center (MSFC) and at the Jet Propulsion Laboratory (JPL). It is anticipated that the ground tests will be subject to gravity and other environmental effects to the extent that orbital flights tests will be needed for verification of some technology items. The need for orbital flight experiments will be quantified based on ground test results and mission needs. Candidate on-orbit experiments will be defined and preliminary design/definition and cost studies will be carried out for one or more high-priority experiments.

# A BRIEF OVERVIEW OF THE CONTROLS-STRUCTURES INTERACTION (CSI) PROGRAM

PRESENTED BY  
BRANTLEY R. HANKS

23

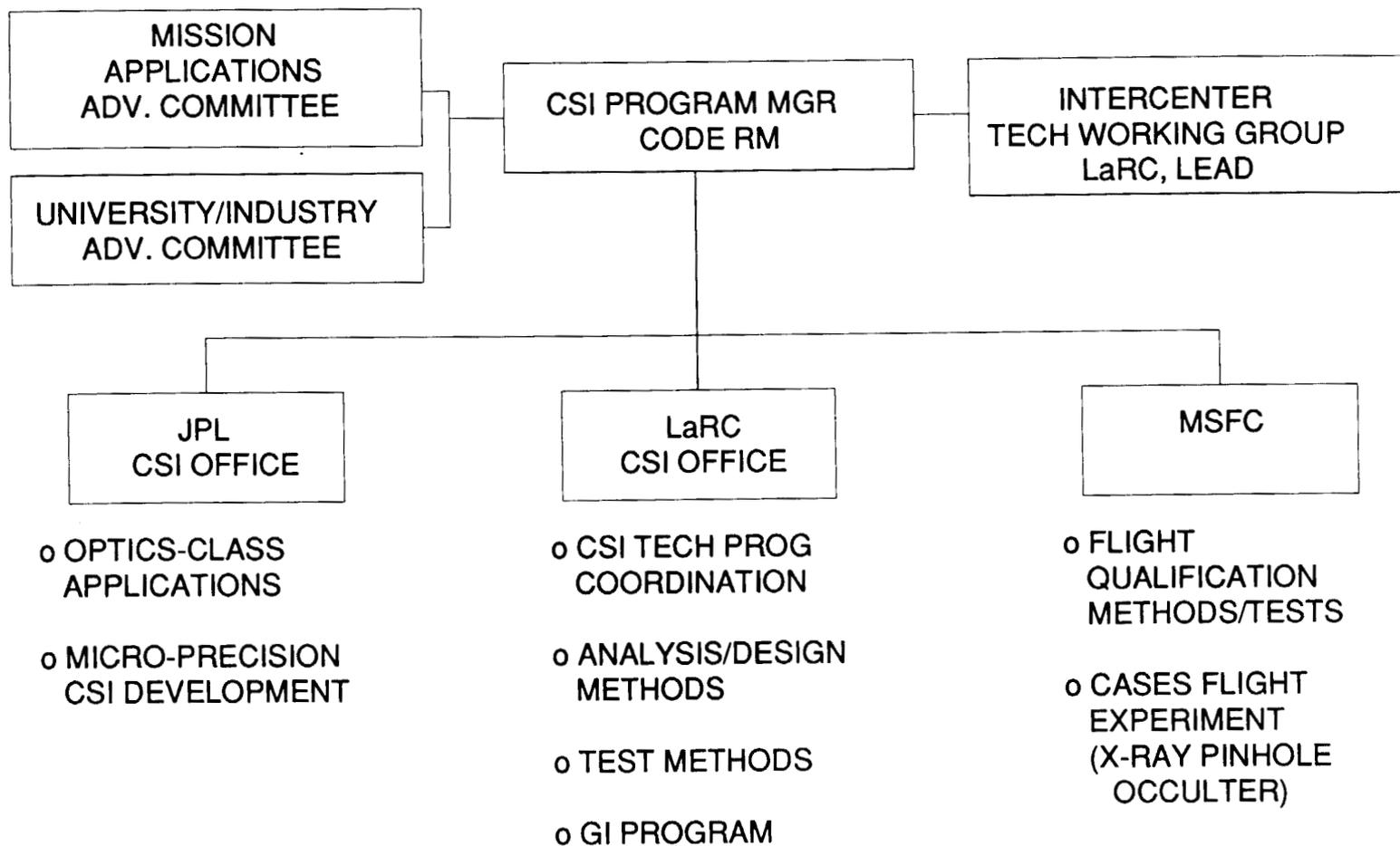
THE NASA LANGLEY RESEARCH CENTER

# THE NASA CONTROLS-STRUCTURES INTERACTION (CSI) PROGRAM

- o A RESTRUCTURING OF THE COFS PROGRAM
- o EMPHASIZES INCREASED GROUND TESTING AND ANALYSIS WITH A CONSERVATIVE FLIGHT EXPERIMENT SCHEDULE
- o MISSION APPLICATIONS WEIGHTED TOWARD EARTH OBSERVATION SPACECRAFT FOR 2000+
- o JOINT EFFORT OF NASA HEADQUARTERS AND THREE FIELD ORGANIZATIONS, LANGLEY, MARSHALL AND JPL
- o MANAGED BY HEADQUARTERS CODE RM, SPECIFIC ROLES FOR EACH FIELD ORGANIZATION, OVERALL TECHNICAL COORDINATION BY LANGLEY

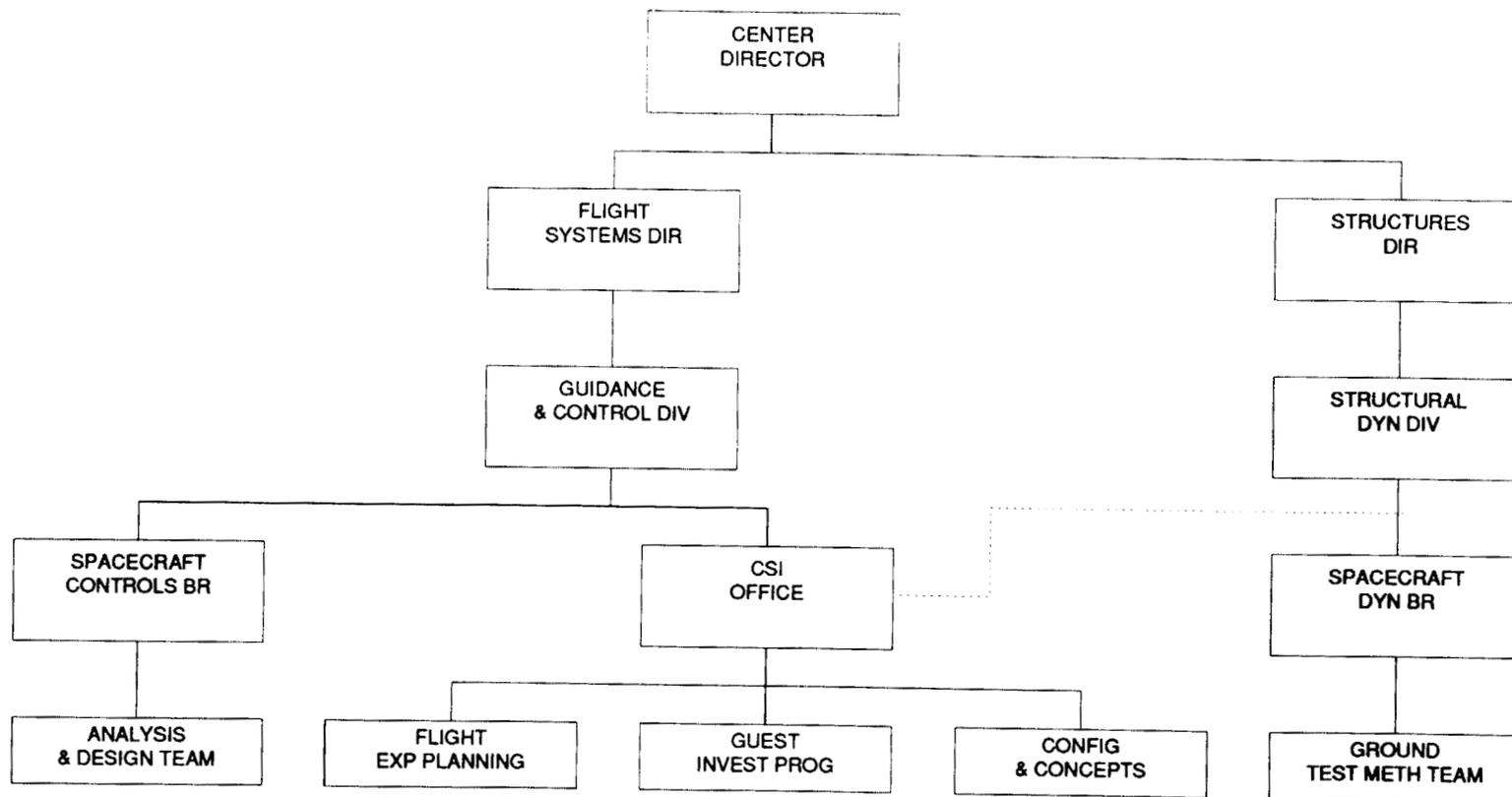
# NASA CSI PROGRAM ORGANIZATION

25



# LaRC CSI ORGANIZATION

26



# CSI PROGRAM GENERAL OBJECTIVES

- o REDUCE DYNAMIC RESPONSE FOR GIVEN MANEUVERS/LOADS WITHOUT INCREASING MASS OR CONTROL ENERGY
- o DEVELOP ACCURATE METHODS FOR PREDICTION OF ON-ORBIT RESPONSE BASED ON ANALYSIS TUNED BY GROUND TESTS
- o DEVELOP UNIFIED MODELING, ANALYSIS AND DESIGN METHODS WHICH PROVIDE BETTER AND FASTER RESULTS THAN CURRENT METHODS
- o VERIFY THE CAPABILITY TO VALIDATE ON-ORBIT CSI PERFORMANCE BY GROUND-BASED METHODS

# CSI PROGRAM ELEMENTS

## CONFIGURATIONS & CONCEPTS

- QUANTIFY MISSION REQUIREMENTS & BENEFIT TRADE-OFFS
- EXPAND CONFIGURATION AND TECHNOLOGY OPTIONS

## INTEGRATED ANALYSIS & DESIGN

- DEVELOP UNIFIED MODELING & ANALYSIS TECHNIQUES
- DEVELOP IMPROVED CSI SYSTEM DESIGN APPROACHES

## GROUND TEST METHODOLOGY

- DEVELOP TEST METHODS FOR VERIFYING CSI DESIGNS
- VALIDATE THEORETICAL CSI TECHNICAL APPROACHES

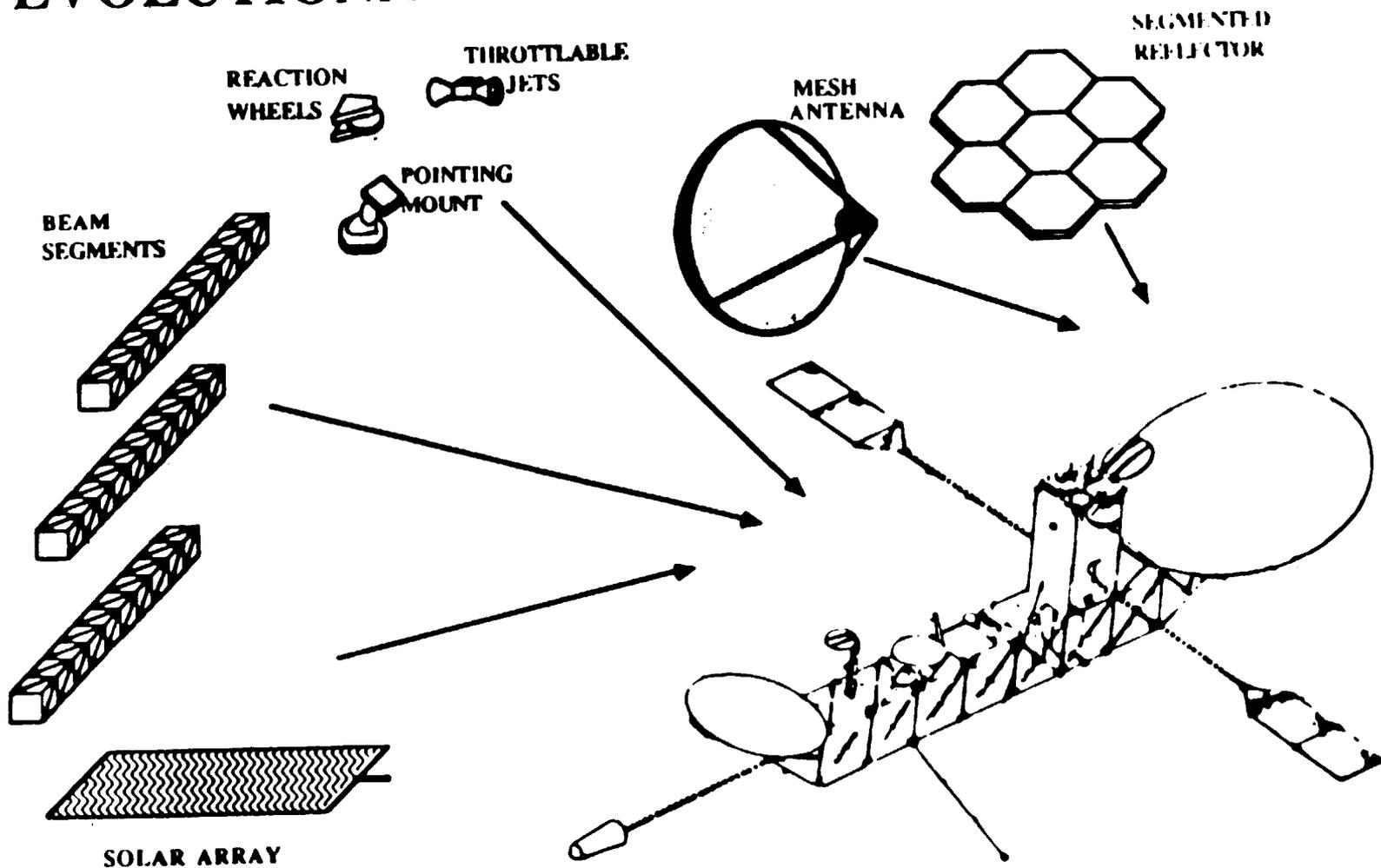
## IN-SPACE FLIGHT EXPERIMENTS

- INVESTIGATE PHENOMENA MASKED IN GROUND TESTS
- CALIBRATE PROPOSED VERIFICATION TEST & ANALYSIS METHODS

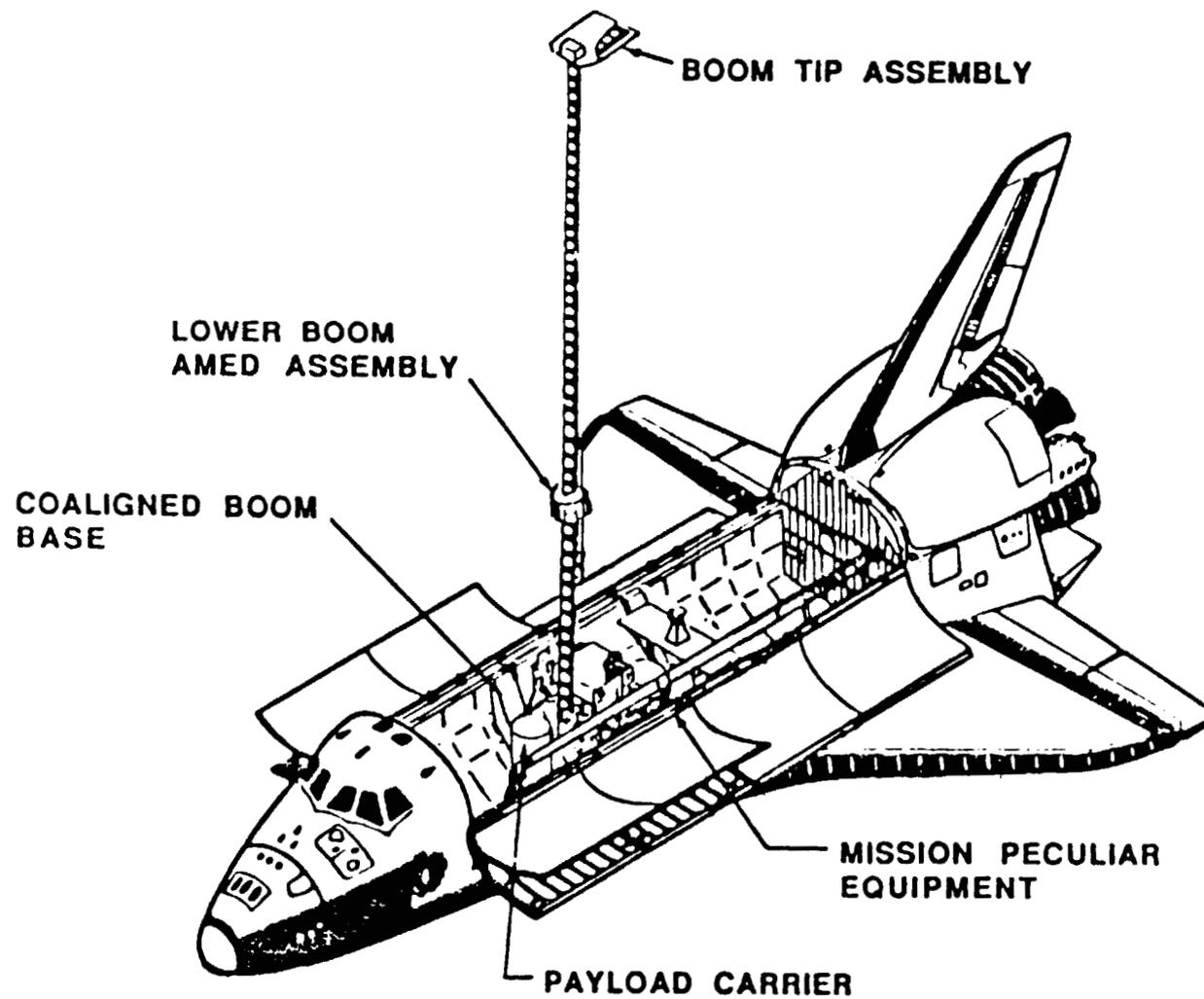
## GUEST INVESTIGATOR PROGRAM

- PROVIDE MECHANISM/FUNDS FOR INCORPORATING IDEAS & CAPABILITIES OF NON-NASA RESEARCHERS

# EVOLUTIONARY GROUND TEST SYSTEM



# FLIGHT STRUCTURES CONTROL EXPERIMENT



# USEFUL WORKSHOP OUTPUT

CASES WHERE PROBLEMS WERE CAUSED BY THE FOLLOWING:

- INACCURATE MATH MODELS
- INACCURATE COMPUTATIONAL ALGORITHMS
- INABILITY TO TEST SYSTEM
- SLOW DESIGN ITERATION TURNAROUND
- FLEXIBLE STRUCTURE INTERACTION WITH CONTROLS

EXAMPLES OF SIGNIFICANT DESIGN IMPACT TO AVOID CSI PROBLEMS:

- BY LIMITING CAPABILITY
- BY REDUCING REQUIREMENTS
- BY "BEEFING-UP" DESIGN

QUANTIFIED EXAMPLES OF THE COMPUTATIONAL BURDEN

- ITERATION TIMES
- COMPUTER "HORSEPOWER" REQUIREMENTS

PRIORITIZED AREAS OF EXPECTED BENEFIT FROM RESEARCH

# UPCOMING CSI PROGRAM EVENTS

- o FIRST GI CONTRACTS TO BE ANNOUNCED - AUGUST
- o GI/UNIVERSITY ENGR RESEARCH CENTERS/OUTREACH COORD MEETING - OCTOBER
- o THIRD NASA/DOD CSI CONFERENCE, JANUARY 89
- o NEXT GI PROPOSAL SOLICITATION - 1st QUARTER 89

COMPUTATIONAL CONTROLS FOR AEROSPACE SYSTEMS

**JPL**

GUY K. MAN

ROBERT A. LASKIN

A. FERNANDO TOLIVAR

12 JULY 1988

33

N90-10082

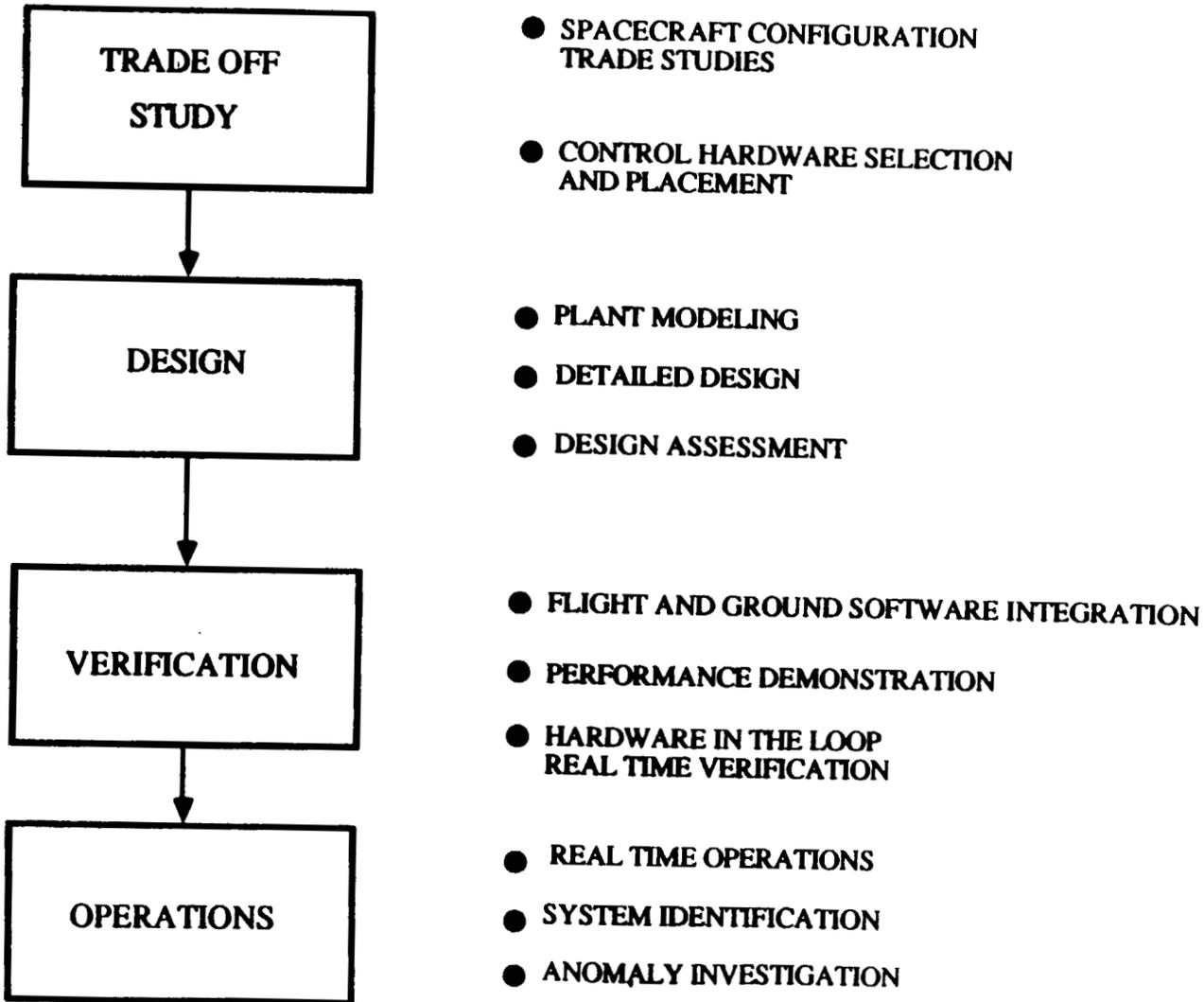
10

# COMPUTATIONAL CONTROLS OBJECTIVE

*The goal is to*

**DEVELOP THE NEXT GENERATION GUIDANCE AND CONTROL  
ANALYSIS AND DESIGN TOOLS TO ENABLE FUTURE MISSIONS  
AND TO IMPROVE PRODUCTIVITY AND RELIABILITY.**

# TOOLS FOR CONTROL SYSTEM DEVELOPMENT



TOOLS ARE INDISPENSIBLE FOR CONTROL SYSTEM DEVELOPMENT

## GOALS FOR NASA COMPUTATIONAL CONTROL

- IMPROVE QUICK-DESIGN TURN AROUND TIME BY A FACTOR OF 16  
(4 MONTHS → 1 WEEK)
- IMPROVE EVALUATION TURN AROUND TIME BY A FACTOR OF 40  
( 10 MONTHS→ 1 WEEK)
- ENABLE REAL TIME HARDWARE-IN-THE-LOOP SIMULATION OF  
COMPLEX SPACECRAFT
- ENABLE REAL TIME ANOMALY INVESTIGATION FOR OPERATIONS
- ENABLE TOOLS TO HANDLE 300 STATES BY 1992 AND 1000 STATES BY 1996

## **RATIONALE**

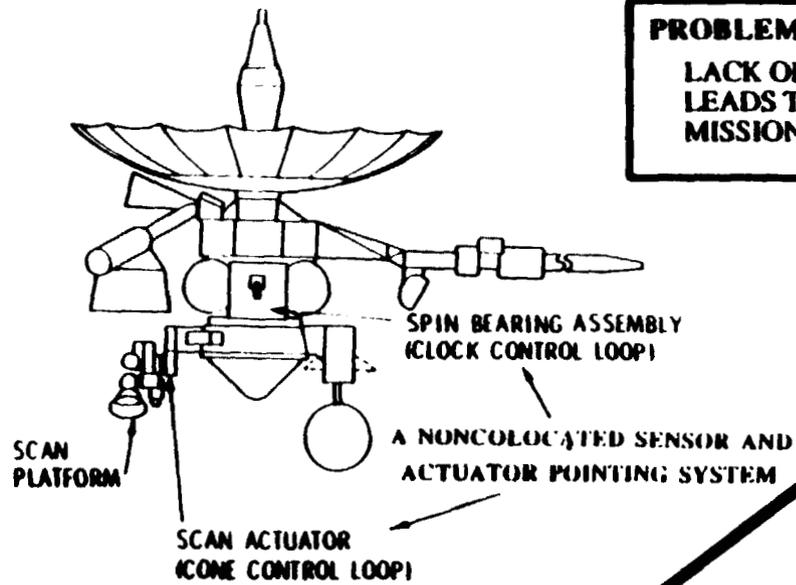
- **LACK OF QUICK-DESIGN TOOLS TO IMPACT SPACECRAFT DESIGN**
- **LACK OF EFFECTIVE EVALUATION TOOLS TO CHECK DESIGN MARGIN & PERFORMANCE**
- **LACK OF REAL TIME SIMULATION TOOL OF REALISTIC SPACECRAFT TO CERTIFY DESIGN**
- **LACK OF QUICK DIAGNOSTIC TOOLS FOR MISSION OPERATIONS**

**LACK OF PROPER TOOL CREATES  
INTOLERABLE RISK FOR FUTURE  
SPACECRAFT SYSTEMS**

## THE GALILEO CONTROL DESIGN PROBLEM

**PROBLEM:**

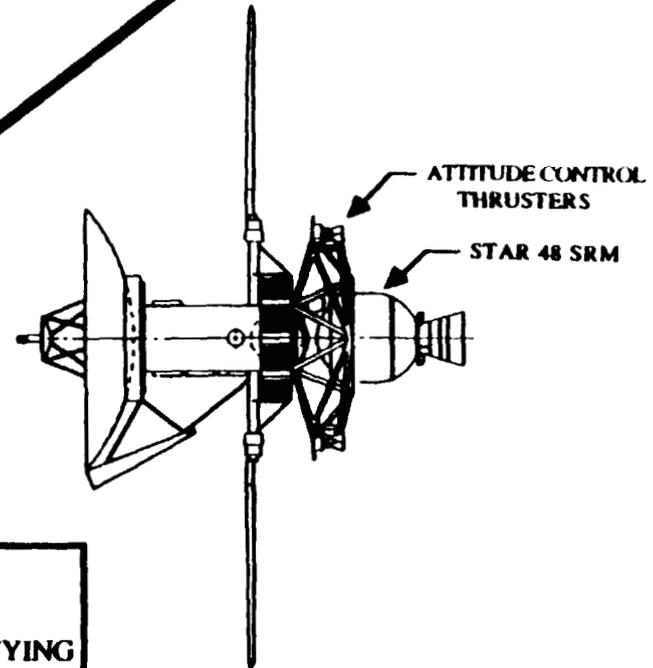
LACK OF QUICK-LOOK TOOL  
LEADS TO FAILURE TO MEET  
MISSION REQUIREMENTS



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**PROBLEM:**

LACK OF EFFECTIVE EVALUATION  
TOOL PROHIBITS US FROM IDENTIFYING  
A MISSION CATASTROPHIC FAILURE  
DURING VENUS ORBIT INSERTION



## MAGELLAN SPACECRAFT VENUS ORBIT INSERTION PROBLEM

# GALILEO CONTROL SYSTEM REAL TIME TESTING

Galileo AACs  
Test Area

SECTION 341

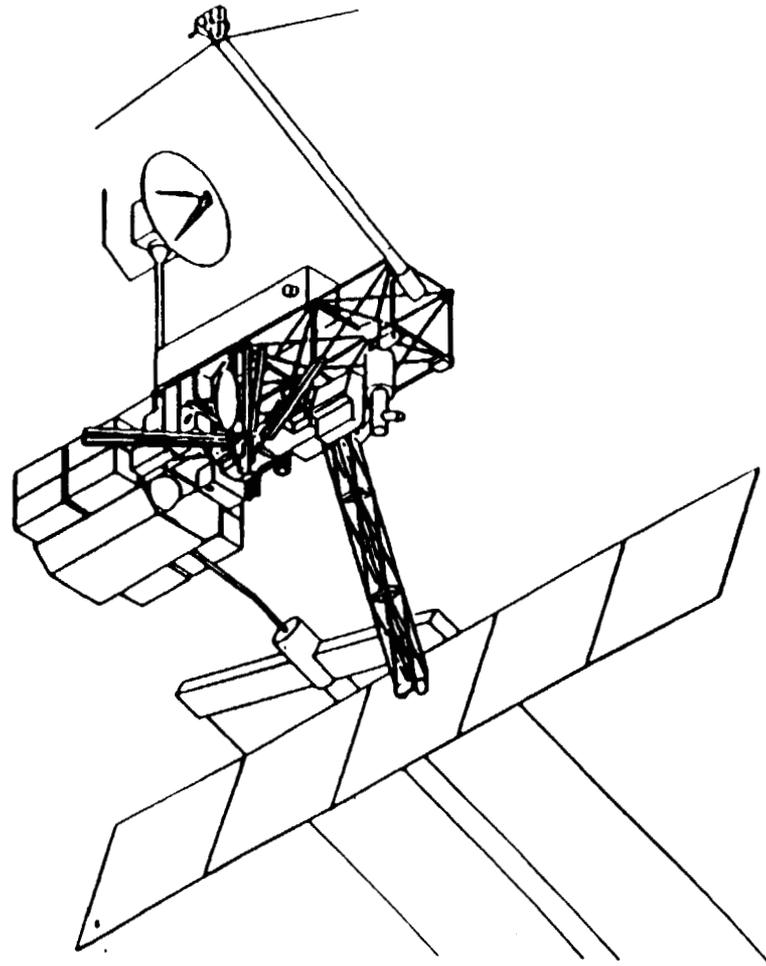
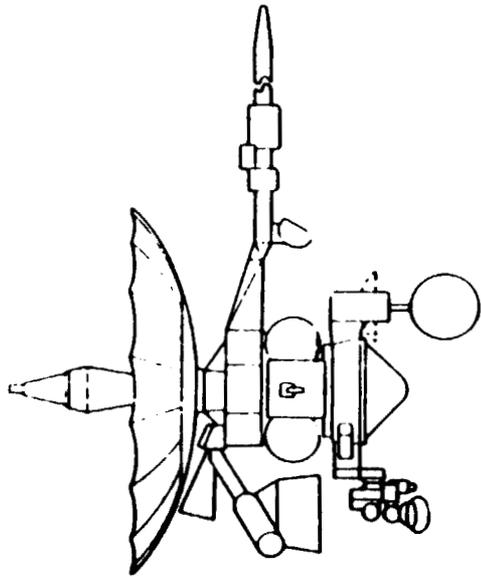
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BLACK AND WHITE PHOTOGRAPH

## PROBLEMS:

- THE ATTITUDE AND ARTICULATION CONTROL SYSTEM IS THE ONLY SPACERATE SUBSYSTEM WHICH CANNOT BE TESTED ON THE GROUND
- THERE IS A LACK OF REAL TIME SIMULATION TOOL TO CHECK CONTROL SYSTEM ROBUSTNESS FOR TODAY'S CONTROL PROBLEM

## MISSION OPERATIONS SUPPORT IS INADEQUATE

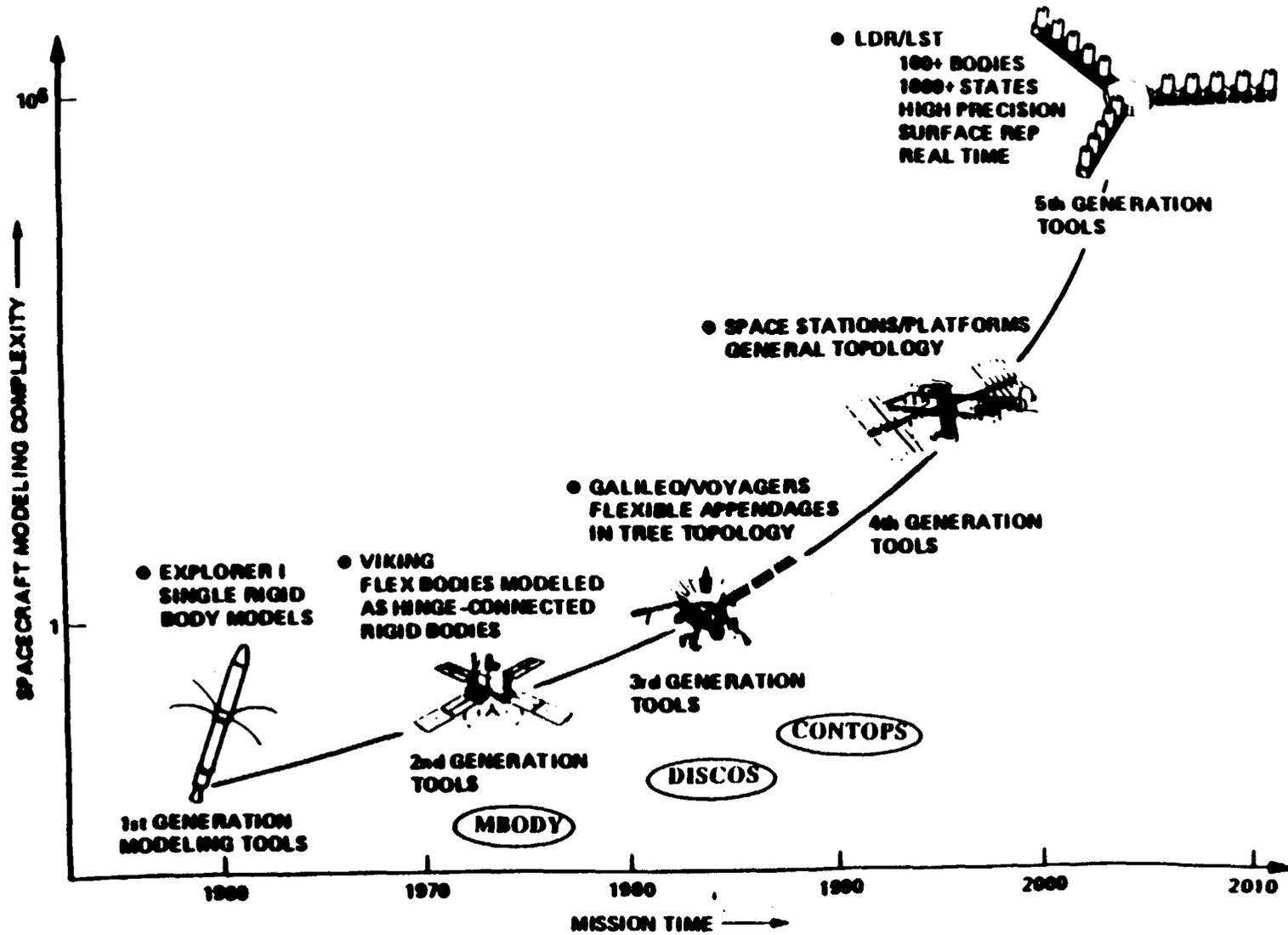
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**PROBLEM:**

LACK OF QUICK DIAGNOSTIC TOOL  
FOR ANOMALY INVESTIGATION  
LEAD TO CONCERNS IN TURN  
AROUND TIME FOR OPERATIONS

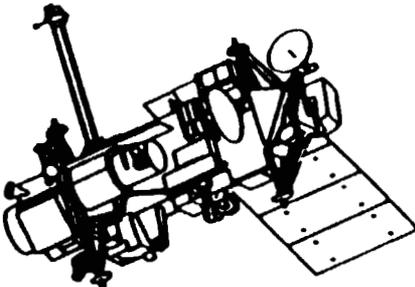
# GROWTH IN SPACECRAFT MODELING COMPLEXITY



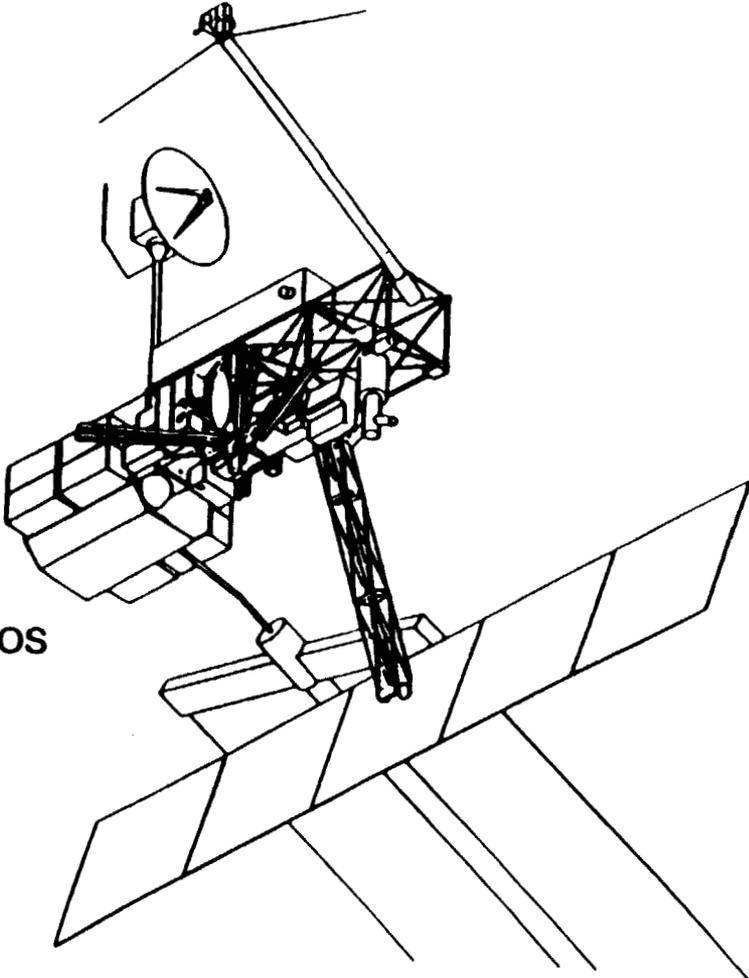
# EVOLUTION OF EARTH OBSERVING PLATFORMS



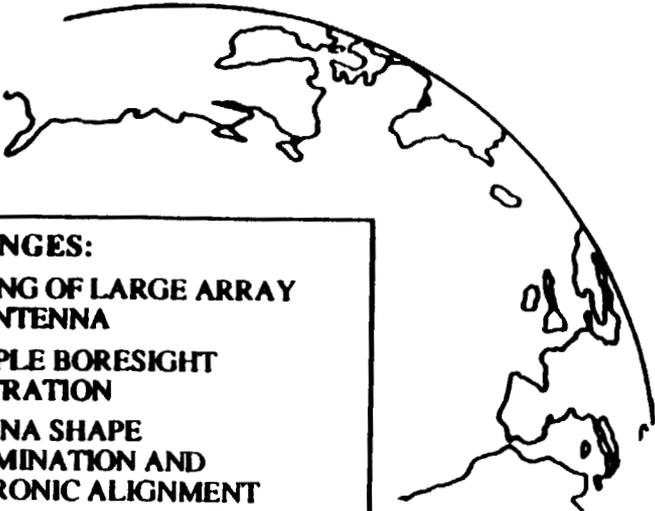
LANDSAT



UARS

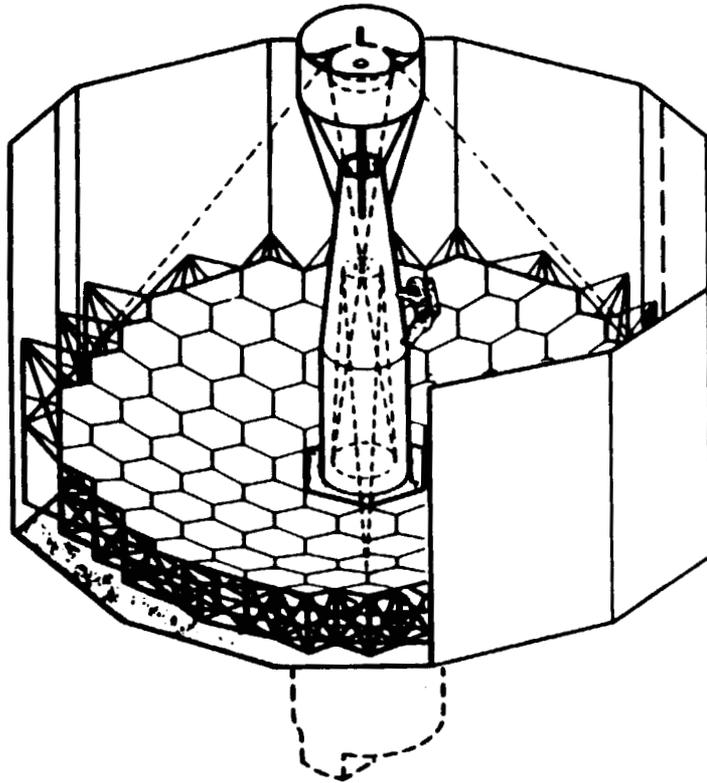


EOS

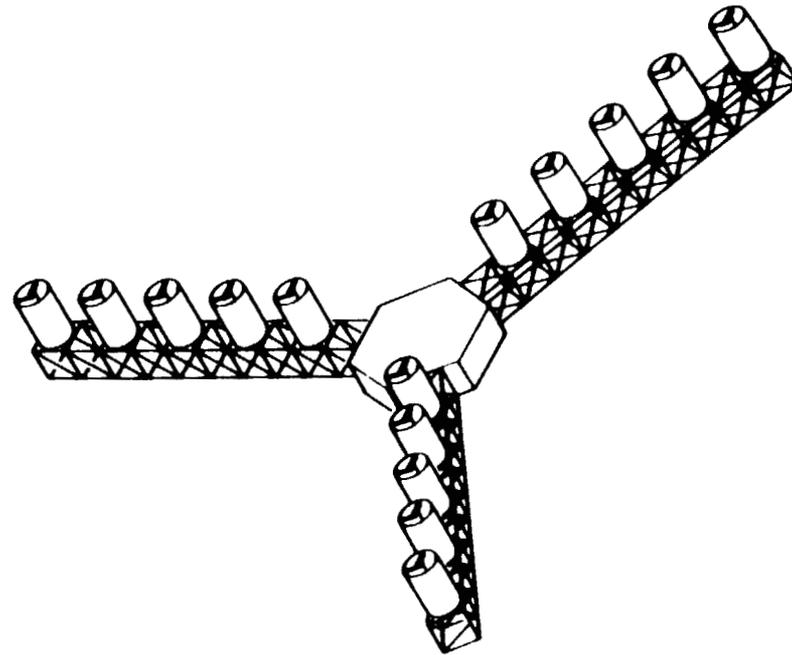


- CHALLENGES:**
- POINTING OF LARGE ARRAY AND ANTENNA
  - MULTIPLE BORESIGHT REGISTRATION
  - ANTENNA SHAPE DETERMINATION AND ELECTRONIC ALIGNMENT

## ADVANCED ASTROPHYSICAL INSTRUMENTS



LDR

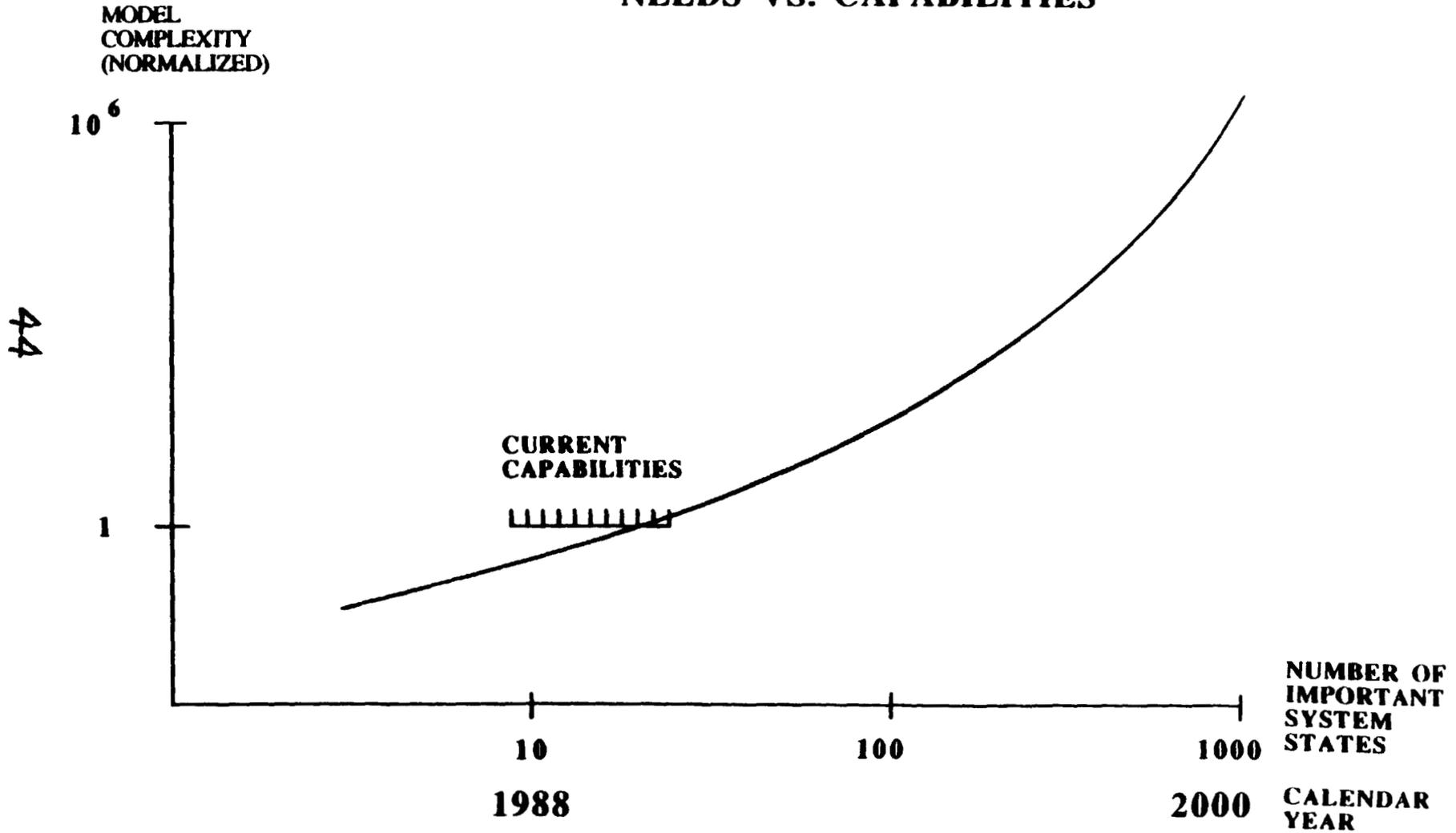


ASTROPHYSICAL INTERFEROMETER

### CHALLENGES:

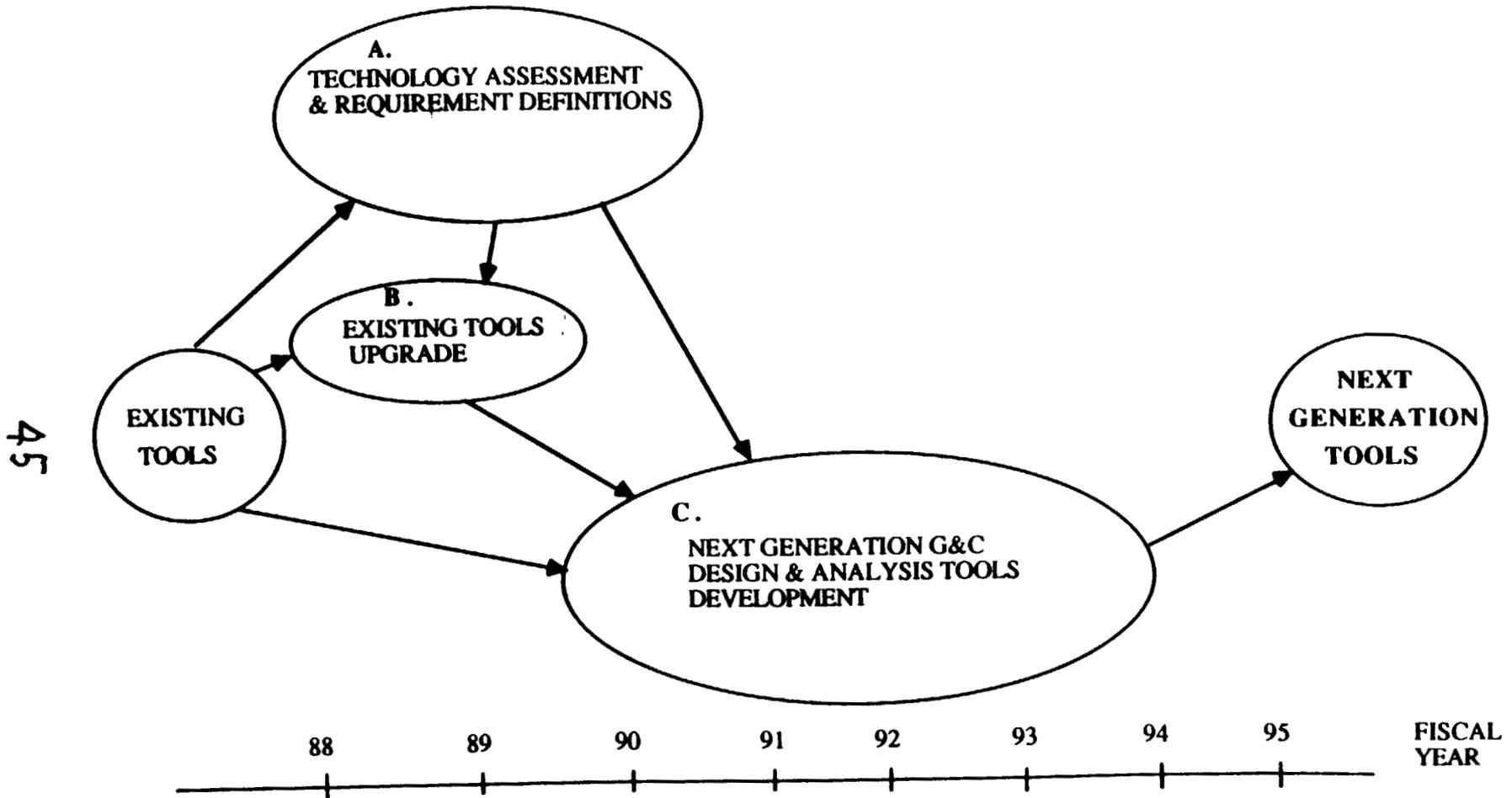
- SHAPE DETERMINATION AND ACTIVE CONTROL
- SUBWAVELENGTH PHASING OF OPTICAL PATHS
- DISTRIBUTED SENSING AND ACTUATION

# CONTROL DESIGN AND ANALYSIS NEEDS VS. CAPABILITIES



**EXISTING TOOLS ARE A LIMITING FACTOR IN TODAY'S CONTROL DESIGN  
AND VERIFICATION, AND ARE INADEQUATE FOR FUTURE NEEDS**

# COMPUTATIONAL CONTROLS APPROACH



## **COMPUTATIONAL CONTROLS APPROACH CONT.**

### **A. TECHNOLOGY ASSESSMENT & REQUIREMENT DEFINITIONS**

- MULTIBODY SIMULATION TECHNOLOGY VERIFICATION
- CONTROL SYSTEM DESIGN/ANALYSIS TOOL ASSESSMENT
- REQUIREMENT DEFINITION AND ANALYSIS

### **B. EXISTING TOOLS UPGRADE**

- UPDATE TOOLS WITH KNOWN DEFICIENCIES
- UPGRADE TOOLS TO MEET NEAR TERM NEEDS

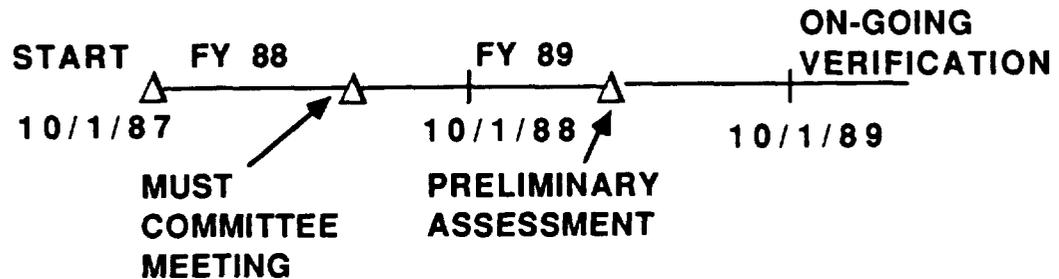
### **C. NEXT GENERATION TOOLS DEVELOPMENTS**

- MULTIBODY SIMULATION TOOLS
- CONTROL SYSTEM OPTIMIZATION
- TOOLS FOR MODERN COMPUTING ENVIRONMENT
- ACCURATE SURFACE MODELING & REPRESENTATION TOOLS
- INTEGRATED CONTROL DESIGN ENVIRONMENT

# MULTIBODY SIMULATION ASSESSMENT & VERIFICATION PLAN

## PLAN SUMMARY:

### ● ESTIMATED DURATION:



### SCHEDULE:

#### 1ST YEAR

- REQUIREMENT DEFINITION AND ANALYSIS
- ESTABLISH VERIFICATION LIBRARY
- TEST CASE DEVELOPMENT

#### 2ND YEAR

- TEST CASE EXECUTION AND EVALUATION
- EXPERIMENT EXECUTION AND EVALUATION
- TEST REPORT GENERATION

#### FUTURE YEARS:

- CONTINUE TO BUILD VERIFICATION LIBRARY
- VERIFY NEW TOOLS AS THEY ARE DEVELOPED

#### DELIVERABLES:

- QUESTIONNAIRES
- REQUIREMENTS MATRIX
- TEST PLAN
- TEST CASE REPORT
- FINAL REPORT
- TWO WORKSHOPS
  
- COMPUTATIONAL ASPECTS OF FLEXIBLE BODY SYSTEMS
  
- FINAL REPORT TO THE COMMUNITY



**ADDITIONAL SOFTWARE DEVELOPMENTS WANTED FOR MODELING AND CONTROL OF  
FLEXIBLE SPACE SYSTEMS**

By

Dr. Jiguan Gene Lin  
Control Research Corporation  
Lexington, Massachusetts

**ABSTRACT**

Existing modeling and control software packages are either inadequate or inefficient for applications to flexible space structures. Some additional software developments are wanted for effective design and evaluation of the control systems. The following will be discussed in this presentation.

1. Linear-quadratic optimal regulators as usual can be designed using various "modern control" design software packages. To design for active augmentation of (approximately) the specified amount of active damping to each "controlled modes," the common practice is to adjust repeatedly the state and control weights (i.e., the Q and R matrices) by mostly endless trial and error. The time consumed and effort spent in the trial-and-error repetition can be saved by using an analytical procedure for closely estimating the corresponding state and control weights. ~~Various numerical examples have shown that this is possible.~~ No software has been developed for automating such a time-saving analytical assignment procedure yet.
2. "Modal dashpots" are very effective output-feedback vibration controllers for flexible structures, not only effective for augmenting a small amount of active damping to a large number of vibration modes (like the so-called low-authority structural controllers), but also effective for quick suppression of large vibrations (like high-authority structural controllers). Recent numerical results on orbital SCOLE configuration have shown so. No software has been developed for facilitating the design process yet.
3. The actual performance of any control design needs to be evaluated against a faithful model of the flexible structure to be controlled. The potential of destabilization or serious performance degradation needs to be detected by numerical simulation of the structure with the control loops being closed. Except for some trivial cases, reduced-order normal-mode models are generally not appropriate: if they are computationally feasible to simulate the closed-loop system, then they are likely not accurate enough to represent the dynamics of the flexible structure; if they are satisfactorily accurate, then they are mostly too large for effective dynamic simulation even by a state-of-the-art mainframe computer. ~~Besides, computing a very large number of normal modes is very expensive, and the accumulated computational errors in the natural frequencies and mode shapes grow very rapidly.~~ The popular Guyan reduction technique is often used to reduce the large finite-element mass-stiffness model first. Such a reduction technique, unfortunately, introduces large additional errors which are proportional to the square of the natural frequency of the modes computed thereafter.

There is a trend towards some innovative use of non-normal modes (such as Ritz or Lanczos vectors) for representing the structures by a much smaller number of such modes. Available results are interesting and promising. Additional development effort is needed and will be very worthwhile.

ADDITIONAL SOFTWARE DEVELOPMENTS WANTED FOR  
MODELING AND CONTROL OF FLEXIBLE SPACE SYSTEMS

JIGUAN GENE LIN  
CONTROL RESEARCH CORP.  
LEXINGTON, MASSACHUSETTS

WORKSHOP ON COMPUTATIONAL ASPECTS IN THE CONTROL OF FLEXIBLE SYSTEMS  
JULY 12-14, 1988  
WILLIAMSBURG, VIRGINIA

ADDITIONAL SOFTWARE DEVELOPMENTS **URGENTLY** WANTED

● ACCURACY-PRESERVING COMPUTATIONALLY EFFICIENT  
COORDINATE REDUCTION OF FINITE-ELEMENT MODELS,

TO ENABLE

1. PRE-DESIGN OPEN-LOOP DYNAMIC ANALYSIS OF  
REALISTIC, LARGE, FLEXIBLE SPACE STRUCTURES  
AND
2. POST-DESIGN **FULL-ORDER CLOSED-LOOP** EVALUATION OF  
CONTROL SYSTEMS FOR SUCH STRUCTURES

● ANALYTICAL SELECTION OF CONTROL AND STATE WEIGHTS,

TO AID

DESIGN OF LINEAR-QUADRATIC REGULATORS DESIRED FOR  
VIBRATION CONTROL OF FLEXIBLE SPACE STRUCTURES

ACCURACY-PRESERVING COMPUTATIONALLY EFFICIENT  
COORDINATE REDUCTION OF FINITE-ELEMENT MODELS

DESIGN OF RELIABLE CONTROL SYSTEMS FOR FLEXIBLE SPACE SYSTEMS

NEEDS

1. CAREFUL PRE-DESIGN OPEN-LOOP DYNAMIC ANALYSIS OF  
THE SPACE STRUCTURE, AND
2. CAREFUL POST-DESIGN FULL-ORDER CLOSED-LOOP EVALUATION OF  
CONTROL SYSTEMS FOR THE STRUCTURE

NEEDS PRE-DESIGN OPEN-LOOP DYNAMIC ANALYSIS

- TO ASSESS EFFECTS OF DISTURBANCES ON SYSTEM PERFORMANCE,  
E.G., POINTING STABILITY, LINE-OF-SIGHT ERRORS, ...
- TO IDENTIFY STRUCTURAL MODES NEEDING ACTIVE CONTROL
- TO FORM A COMPUTATIONALLY FEASIBLE  
REDUCED-ORDER CONTROL-DESIGN MODEL
- TO ASSESS EFFECTIVENESS OF CONTROL ACTUATORS AND SENSORS

NEEDS POST-DESIGN FULL-ORDER CLOSED-LOOP EVALUATION

- TO DETECT POSSIBLE INSTABILITY INTRODUCED BY  
REDUCED-ORDER CONTROL DESIGN
- TO VERIFY ACTUAL TIME-DOMAIN PERFORMANCE
- TO TEST ROBUSTNESS TO MODELING ERRORS, PARAMETER VARIATIONS, ...

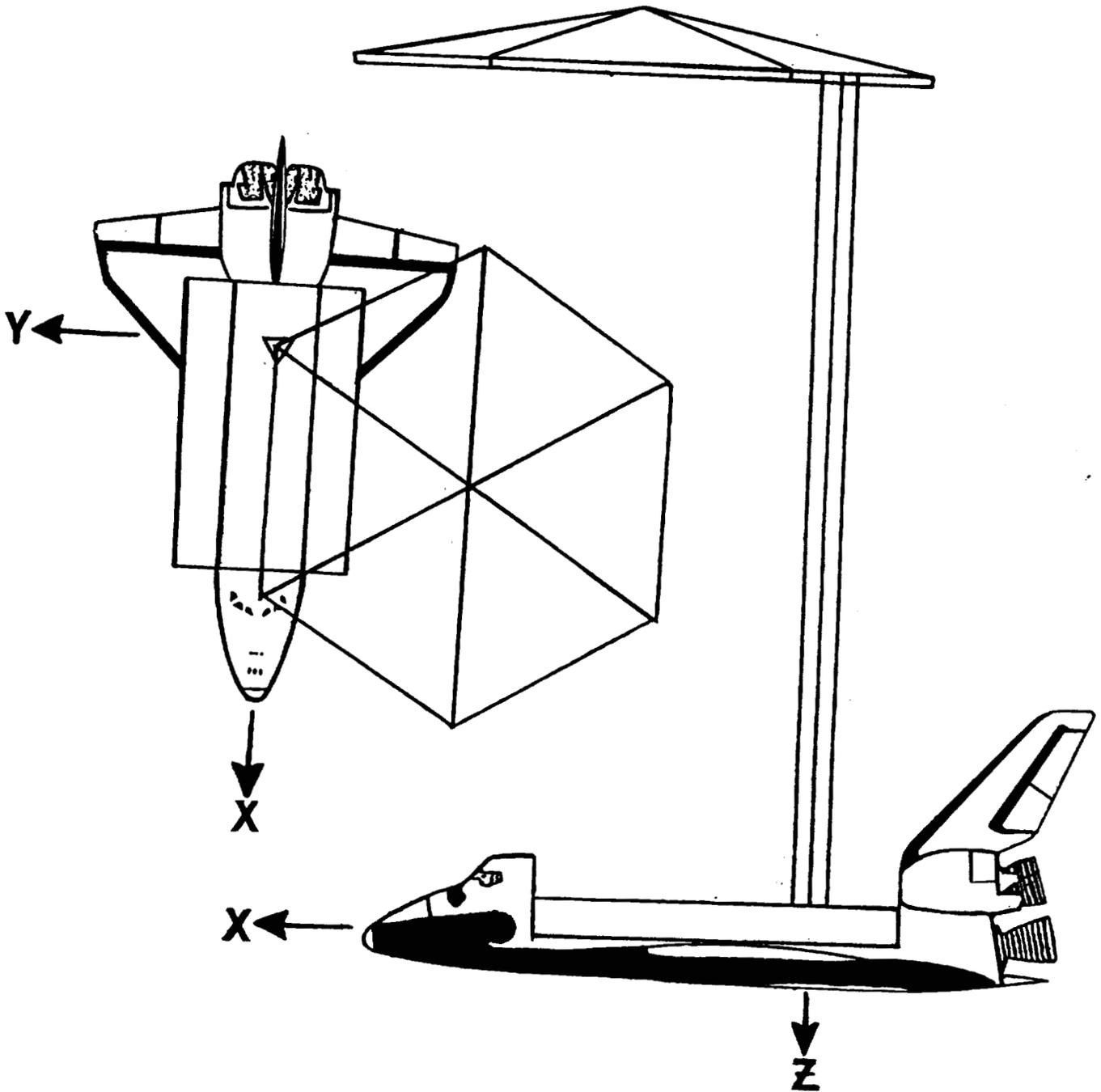


Fig. 1-1 Spacecraft Control Laboratory Experiment (SCOLE)--  
the orbital Shuttle-Mast-Antenna configuration.

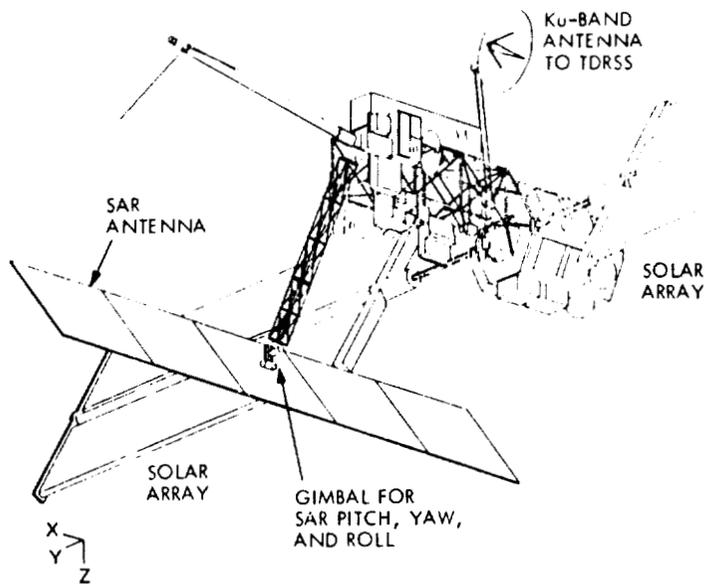


Figure 1. A Possible Eos Polar Platform Configuration

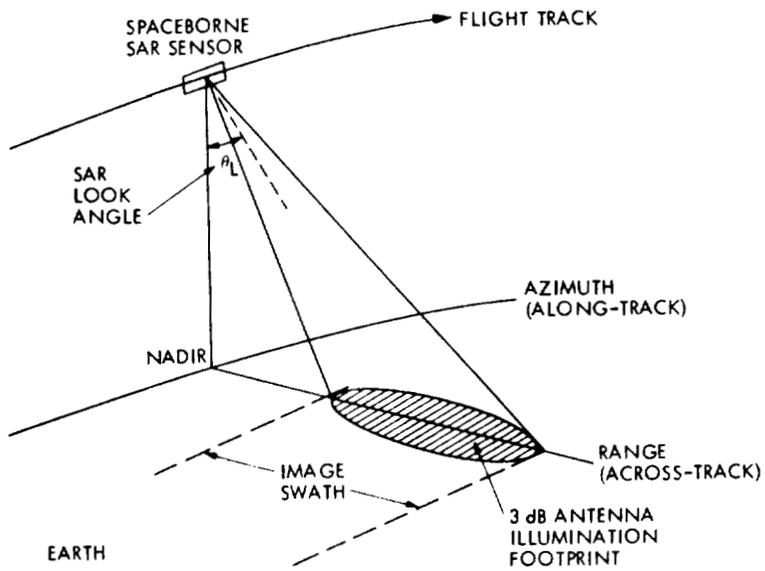


Figure 2. SAR Imaging Geometry

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AIAA-87-0022 PAPER:

"EARTH OBSERVING SYSTEM INSTRUMENT POINTING CONTROL  
MODELING FOR POLAR ORBITING PLATFORM,"

BY H.C. BRIGGS, T.KIA, S.A. McCABE, AND C.E. BELL

- NASTRAN STRUCTURAL DYNAMICS MODEL OF PLATFORM: 200 NODES  
-- 600 TO 1,200 DOFS  
-- 40 MODES BELOW 10 Hz

- SLEW OF SAR ANTENNA ABOUT ITS BOOM AXIS WAS SIMULATED  
-- TO ASSESS THE POINTING CONTROL AND STABILITY  
OF INSTRUMENTS MOUNTED ON THE CARRIER STRUCTURE  
DURING SLEWING OF ADJACENT INSTRUMENTS.

- "LARGE ANGLE TIME-DOMAIN SIMULATIONS CAN PRESENTLY BE CONDUCTED USING DISCOS, BUT DUE TO EXECUTION EXPENSE AND THE DIFFICULTY OF USER INTERFACE THIS APPROACH IS IMPRACTICAL FOR EOS STUDIES.

NEXT GENERATION SIMULATION TOOLS WHICH REDUCE THE NUMBER OF NUMERICAL OPERATIONS FROM ORDER  $N^4$  (DISCOS) TO  $N^3$  (TREETOPS) AND BEYOND TO ORDER  $N$  ARE URGENTLY NEEDED TO EFFICIENTLY AND COST EFFECTIVELY VERIFY THE PERFORMANCE OF LARGE SYSTEMS OF MULTIPLE ARTICULATED AND ROTATING ELEMENTS SUCH AS EOS PLATFORMS."

## SOME SERIOUS TECHNICAL PROBLEMS

CURRENT REDUCED-ORDER MODELS ARE GENERALLY NOT APPROPRIATE FOR REALISTIC, LARGE, FLEXIBLE SPACE STRUCTURES:

### ● MODEL ACCURACY VS COMPUTATIONAL CAPABILITY

IF COMPUTATIONALLY FEASIBLE TO SIMULATE ON THE COMPUTER, THEN LIKELY NOT ACCURATE ENOUGH TO REPRESENT THE FLEXIBLE STRUCTURE;

IF SATISFACTORILY ACCURATE, THEN MOSTLY TOO LARGE FOR EFFECTIVE DYNAMIC SIMULATION ON COMPUTER

### ● COMPUTATIONAL EXPENSE AND ACCUMULATED ERRORS

COMPUTING A VERY LARGE NUMBER OF NORMAL MODES IS VERY EXPENSIVE;

ACCUMULATED COMPUTATIONAL ERRORS IN THE NATURAL FREQUENCIES AND MODE SHAPES GROW VERY RAPIDLY,

### ● WASTED EXPENSIVE MODAL COMPUTATIONS

MANY USELESS MODES COMPUTED,  
THEN IGNORED IN CONTROL DESIGN OR EVALUATION

-- UN-RELATED TO DISTURBANCES CONCERNED,  
OR CONTROL ACTUATIONS CONSIDERED

### ● ACCURACY-SACRIFICING COORDINATE REDUCTION

POPULAR GUYAN REDUCTION TECHNIQUE IS OFTEN USED FIRST  
TO REDUCE THE LARGE FINITE-ELEMENT STIFFNESS AND MASS MATRICES

-- LARGE ERRORS INTRODUCED THEREBY;  
INCREASE AS THE **SQUARE** OF FREQUENCIES OR HIGHER

## INNOVATIVE RAYLEIGH-RITZ METHOD

- A TREND TOWARDS SOME INNOVATIVE USE OF NON-NORMAL MODES (SUCH AS RITZ OR LANCZOS VECTORS) FOR REPRESENTING THE STRUCTURES BY A MUCH SMALLER NUMBER OF GENERALIZED COORDINATES
- AVAILABLE RESULTS INTERESTING AND PROMISING,
- ADDITIONAL DEVELOPMENT AND EXTENTION EFFORTS NEEDED.

### ● RAYLEIGH-RITZ METHOD

- ASSUMED SHAPES:  $q_1, q_2, \dots, q_N$  (SMALL N)
- APPROXIMATE THE STRUCTURAL DISPLACEMENT VECTOR X:

$$x = z_1 q_1 + z_2 q_2 \dots + z_N q_N = Q z$$

$$Q = [q_1, q_2, \dots, q_N], \quad z = (z_1, z_2, \dots, z_N)$$

- REDUCE ORIGINAL FINITE-ELEMENT MODEL:  $M \frac{D^2 x}{DT^2} + K x = F(T)$

TO

$$\boxed{(Q^T M Q) \frac{D^2 z}{DT^2} + (Q^T K Q) z = Q^T F(T)}$$

- ORIGINAL LARGE MATRICES M AND K NOW REDUCED TO SMALLER ONES:

$$M_C = Q^T M Q, \quad K_C = Q^T K Q$$

### ● WILSON-YUAN-DICKENS ALGORITHM

- ASSUME  $F(T) = B U(T)$ ,  $U(T) = A$  SCALAR FUNCTION
- GENERATE AND ORTHOGONALIZE THE ASSUMED SHAPES SEQUENTIALLY:

$$\begin{aligned} K q_1^* &= B & \text{=====}& \rightarrow q_1 \\ K q_2^* &= M q_1 & \text{=====}& \rightarrow q_2 \\ \dots & & & \\ K q_N^* &= M q_{N-1} & \text{=====}& \rightarrow q_N \end{aligned}$$

## ADDITIONAL DEVELOPMENT AND EXTENTION EFFORTS WANTED

### ● COMPUTATIONAL PROBLEMS WITH ORTHOGONALIZATION

1. ACCUMULATED ROUND OFF ERRORS CAN DESTROY THE ORTHOGONALITY OF THE RITZ VECTORS THUS GENERATED
  - NEED TO RE-ORTHOGONALIZE WHENEVER ORTHOGONALITY IS LOST
  
2. COMPUTATIONAL INTENSIVE: PERFORM GRAM-SCHMIDT ORTHOGONALIZATION EVERY TIME A VECTOR  $Q_I^*$  IS GEANERATED
  - NOUR-OMID AND CLOUGH'S SOLUTION WAS TO ORTHOGONALIZE ONLY WITH RESPECT TO TWO PREVIOUS VECTORS.
  - THE MOST TROUBLESOME DRAWBACK OF THE LANCZOS ALGORITHM REAPPEAR:
    - EASY LOSS OF ORTHOGONALITY OF THE LANCZOS VECTORS;
    - RE-ORTHOGONALIZATION REQUIRED WHEN ORTHOGONALITY IS LOST

### ● EXTENSION BEYOND THE SPECIAL CASE OF SCALAR FORCES

- THE WILSON-YUAN-DICKENS ALGORITHM WAS FORMULATED FOR SCALAR FORCES;
  - NOT DIRECTLY APPLICABLE TO THE GENERAL CASE OF MULTIPLE SIMULTANEOUS DISTURBANCE (OR CONTROL) FORCES
- SO WAS NOUR-OMID AND CLOUGH'S VERSION USING LANCZOS VECTORS
- BUT, SPACE SYSTEMS LIKELY BE SUBJECT TO MULTIPLE DISTURBANCES NOT ONE AT A TIME, BUT SIMULTANEOUSLY
- ALSO MOST CONTROL SYSTEMS USE MULTIPLE INDEPENDENT ACTUATORS TO APPLY FORCES/TORQUES TO THE STRUCTURES SIMULTANEOUSLY,

## LINEAR-QUADRATIC REGULATORS (LQR) FOR FLEXIBLE SPACE STRUCTURES

- TRUNCATED MODAL MODEL OF THE FLEXIBLE STRUCTURE

$$\ddot{\eta}_I + 2 \zeta_I \omega_I \dot{\eta}_I + \omega_I^2 \eta_I = \phi_I^T B_F U \quad I = 1, \dots, N$$

- PUTTING INTO STATE-SPACE FORM

$$\dot{x} = A x + B u$$

WITH  $x = \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix}$   $n = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix}$

- LQR DESIGN:

FIND A FEEDBACK GAIN MATRIX  $K$  SUCH THAT

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

IS MINIMIZED WITH  $u = K x$

- GIVEN THE CONTROL AND STATE WEIGHTING MATRICES  $R$  AND  $Q$ , ANY "MODERN CONTROL" DESIGN PROGRAM, SUCH ORACLS, CTRL-C, CAN PRODUCE AN OPTIMAL SOLUTION  $K$  VIRTUALLY AUTOMATICALLY

DESIGN OF LINEAR-QUADRATIC REGULATORS FOR  
ACTIVE AUGMENTATION OF SPECIFIED DAMPING TO SPECIFIC MODES

APPROACH 1. CONSTRAINED OPTIMIZATION

OPTIMIZE THE PERFORMANCE INDEX  $J$  WITH THE SPECIFIED DAMPING RATIOS  
AS CONSTRAINTS.

- CONSTRAINED OPTIMIZATION IS PARTICULARLY COMPLICATED  
WHEN DYNAMIC EQUATIONS ARE INVOLVED

APPROACH 2. ALPHA-SHIFT

SHIFT ALL POLES TO THE LEFT OF THE IMAGINARY AXIS BY A CONSTANT  $\alpha$ .

- SOME MODES MAY NOT GET ENOUGH DAMPING TO BE CLOSE TO THE SPECIFIED,  
WHILE SOME OTHERS MAY GET TOO MUCH MORE THAN THE SPECIFIED.

APPROACH 3. TRIAL AND ERROR ON THE CONTROL AND STATE WEIGHTS

START WITH DIAGONAL  $R$  AND  $Q$  WITH SOME ARBITRARY NUMBERS, E.G., 1;  
CARRY OUT THE DESIGN OF THE CORRESPONDING LQR;  
EVALUATE THE CLOSED-LOOP POLES, AND HENCE THE DAMPING RATIOS.

TRY OTHER CONTROL AND STATE WEIGHTS,  
REPEAT THE DESIGN-EVALUATION CYCLE,  
UNTILL THE RESULTS ARE SATISFACTORY.

- THE CONTROL AND STATE WEIGHTS USED MOSTLY ARE AD HOC;  
THE TRIAL-AND-ERROR PROCESS IS MOSTLY ENDLESS,  
VERY TIME CONSUMING

## ADDITIONAL SOFTWARE DEVELOPMENT WANTED

SOFTWARE MODULES FOR AIDING DESIGNERS IN MAKING GOOD INITIAL CHOICES,  
AND INTERMEDIATE ADJUSTMENTS, OF THE CONTROL AND STATE WEIGHTS  
SO THAT,

THE RESULTING DESIGN OF LINEAR-QUADRATIC REGULATORS  
CAN, WITHIN ONLY A FEW ITERATIONS, SATISFY CLOSELY  
THE DESIGN SPECIFICATIONS,  
E.G., ON DAMPING AUGMENTATION, STIFFNESS AUGMENTATION,  
LINE-OF-SIGHT POINTING ACCURACY, ETC.

ADDITIONAL SOFTWARE DEVELOPMENTS **URGENTLY** WANTED

● ACCURACY-PRESERVING COMPUTATIONALLY EFFICIENT  
COORDINATE REDUCTION OF FINITE-ELEMENT MODELS,

TO ENABLE

1. PRE-DESIGN OPEN-LOOP DYNAMIC ANALYSIS OF  
**REALISTIC, LARGE, FLEXIBLE** SPACE STRUCTURES  
AND
2. POST-DESIGN **FULL-ORDER CLOSED-LOOP** EVALUATION OF  
CONTROL SYSTEMS FOR SUCH STRUCTURES

● ANALYTICAL SELECTION OF CONTROL AND STATE WEIGHTS,

TO AID

DESIGN OF LINEAR-QUADRATIC REGULATORS DESIRED FOR  
VIBRATION CONTROL OF FLEXIBLE SPACE STRUCTURES

**SESSION II - SURVEY OF AVAILABLE SOFTWARE**

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N90-10084

**FLEXIBLE STRUCTURE CONTROL EXPERIMENTS USING A REAL-TIME  
WORKSTATION FOR COMPUTER-AIDED CONTROL ENGINEERING**

By

Michael E. Stieber  
Communications Research Centre  
Ottawa, Ontario, CANADA

**ABSTRACT**

A Real-Time Workstation for Computer-Aided Control Engineering has been developed jointly by the Communications Research Centre (CRC) and Ruhr-Universitaet Bochum (RUB), West Germany. The system is presently used for the development and experimental verification of control techniques for large space systems with significant structural flexibility.

The Real-Time Workstation (cf. Attachment 1) essentially is an implementation of RUB's extensive Computer-Aided Control Engineering package "KEDDC" on an INTEL micro-computer running under the RMS real-time operating system. The portable system supports system identification, analysis, control design and simulation, as well as the immediate implementation and test of control systems. A wealth of classical and modern control analysis and design methods are available to the user who interacts with KEDDC through a friendly dialog. The workstation can be configured both with analog and digital interfaces to the "real world" for data acquisition and control.

The Real-Time Workstation is currently being used by CRC to study control/structure interaction on a ground-based structure called "DAISY" (cf. Attachment 2), whose design was inspired by a reflector antenna. DAISY emulates the dynamics of a large flexible spacecraft with the following characteristics: rigid body modes, many clustered vibration modes with low frequencies and extremely low damping. DAISY presently has seven control actuators and eight sensors which are all "spacecraft-like."

The class of control algorithms currently investigated by experiments is "robust LQG" control. The Real-Time Workstation was found to be a very powerful tool for experimental studies, supporting control design and simulation, and conducting and evaluating tests within one integrated environment. It has dramatically increased the flexibility and turnaround of the experiments. As the Workstation all but eliminates the barriers between ideas on control systems and their experimental evaluation, analytical and experimental development can take place essentially simultaneously.

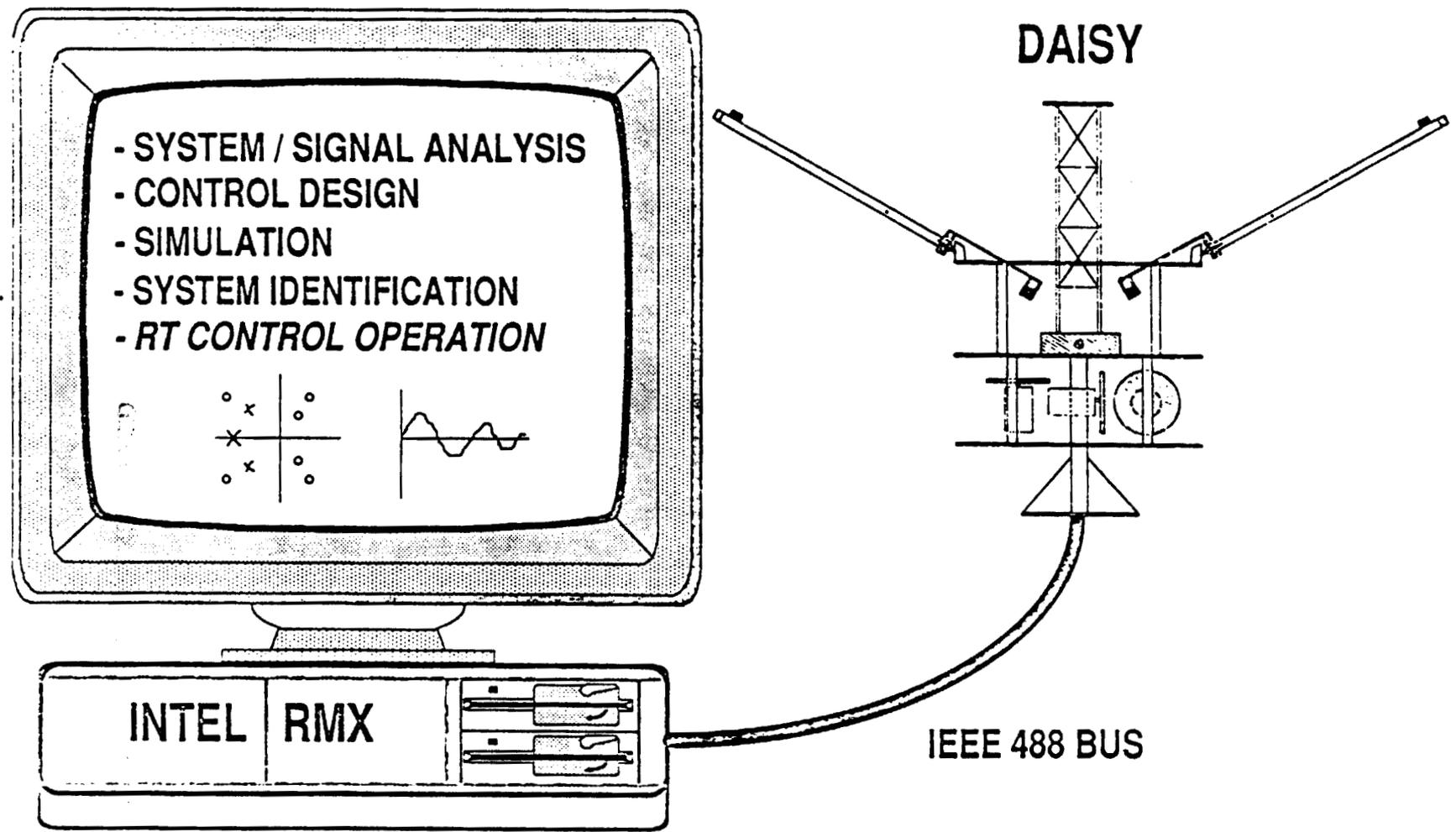
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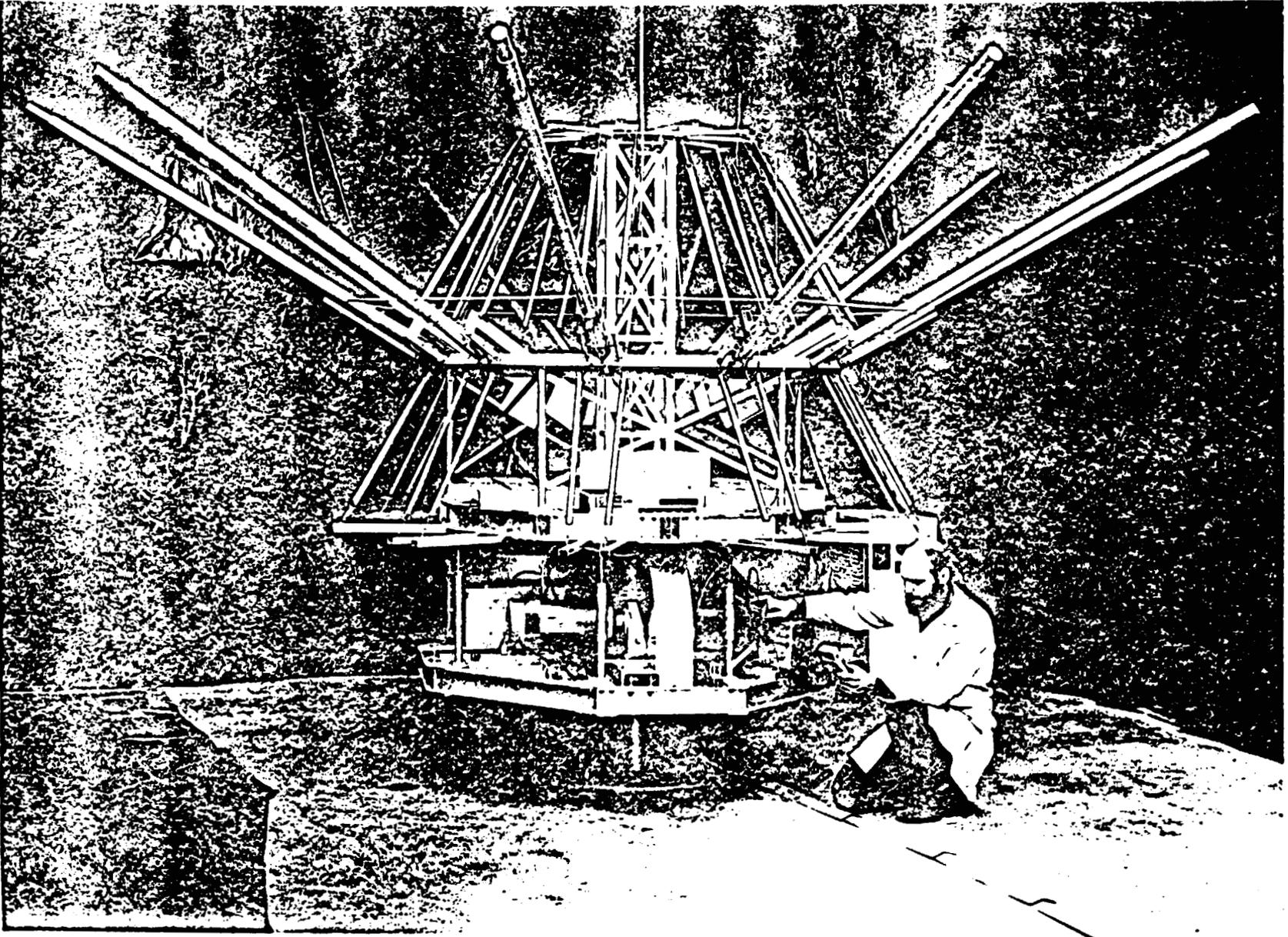
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# REAL-TIME WORKSTATION FOR COMPUTER-AIDED CONTROL ENGINEERING

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**FLEXIBLE STRUCTURE CONTROL EXPERIMENTS  
USING A REAL-TIME WORKSTATION FOR  
COMPUTER-AIDED CONTROL ENGINEERING**

**MICHAEL E. STIEBER**

**SPACE MECHANICS DIRECTORATE**

**COMMUNICATIONS RESEARCH CENTRE, OTTAWA, CANADA**

**SPONSORED BY: SPACE-BASED RADAR PROGRAM  
DEPARTMENT OF NATIONAL DEFENCE, CANADA**

**NASA WORKSHOP ON COMPUTATIONAL ASPECTS IN THE CONTROL OF FLEXIBLE STRUCTURES, JULY 12-14, 1988**

# OUTLINE

## 1. INTRODUCTION

## 2. REAL-TIME WORKSTATION

- CAPABILITIES
- HOST ENVIRONMENT

## 3. FLEXIBLE STRUCTURE CONTROL EXPERIMENT

- CHARACTERISTICS
- APPLICATION OF REAL-TIME WORKSTATION

## 4. SUMMARY & CONCLUSIONS



# TECHNOLOGY DEVELOPMENT FOR CONTROL OF FLEXIBLE SPACE STRUCTURES

- **ANALYTICAL STUDIES**

  - DEVELOPMENT OF NEW TECHNIQUES

  - APPLICATION TO STRAWMAN PROBLEMS (SIMULATIONS)

- **GROUND-BASED EXPERIMENTS**

  - VALIDATION AND DEMONSTRATION OF ANALYTICAL RESULTS

- **FLIGHT TEST**

***SUPPORT BY CAD SYSTEMS ?***

# **HOW DO CAD PACKAGES SUPPORT CONTROL SYSTEM TECHNOLOGY DEVELOPMENT ?**

## **MANY SUPPORT ANALYTICAL STUDIES**

- NUMERICAL ANALYSIS
- GRAPHICS

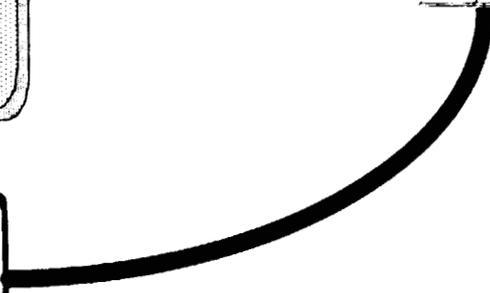
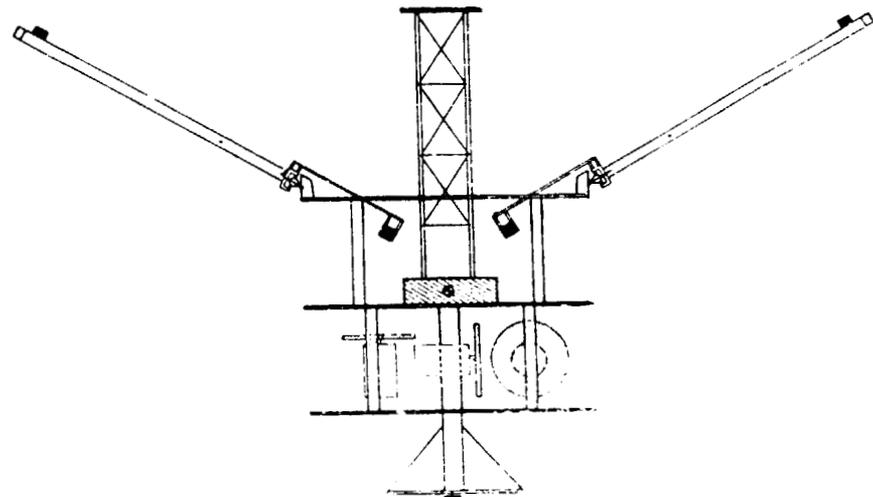
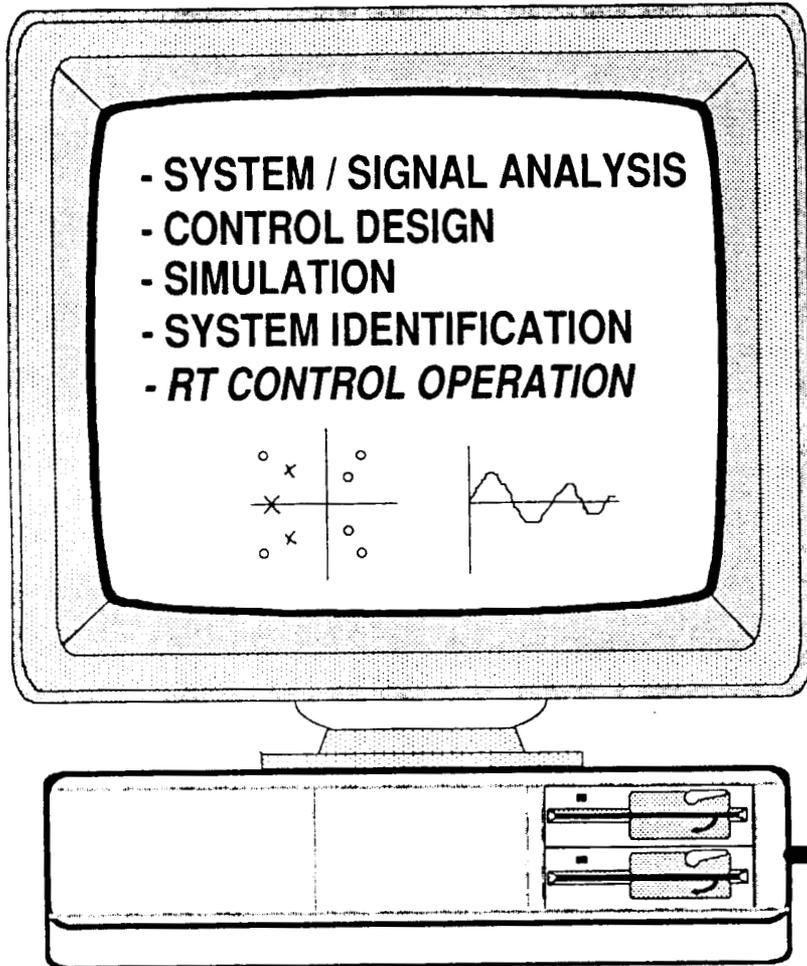
## **FEW DIRECTLY SUPPORT EXPERIMENTAL STUDIES, WHICH REQUIRES:**

- INTERFACE TO THE REAL WORLD
- DATA ACQUISITION
- IMPLEMENTATION & TEST OF REAL-TIME CONTROL SYSTEMS

# REAL-TIME WORKSTATION

# FLEXIBLE STRUCTURE CONTROL EXPERIMENT

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# REAL-TIME WORKSTATION SOFTWARE

## UNDERLYING CAD PACKAGE: *KEDDC*

- DEVELOPED BY DR. CHRISTIAN SCHMID
- AT RUHR-UNIVERSITY, BOCHUM, WEST GERMANY
- RT WORKSTATION A JOINT PROJECT OF RUHR-U. AND CRC

## FEATURES

- MATURE
- COMPREHENSIVE
- PORTABLE (RUNNING UNDER 12 OPERATING SYSTEMS)
- MODULAR, OPEN SYSTEM

# KEDDC

## CORE MODULES

- MATRIX MANAGER
- SYSTEM MANAGER
- FREQUENCY MANAGER
- SIGNAL MANAGER
- POLYNOMIAL MATRIX MANAGER
- GRAPHICS MANAGER

## CAPABILITY OF CORE PACKAGE

- INTERACTIVE 'CALCULATOR' -TYPE ENVIRONMENT
- 250 COMMANDS
- EXTENDED BY APPLICATIONS MODULES

# HOST ENVIRONMENT

## REQUIREMENTS FOR SELECTION

- REAL-TIME MULTI-TASKING OPERATING SYSTEM
- PORTABLE COMPUTER
- COMPATIBLE WITH FUTURE MICRO-PROCESSORS

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## SYSTEM CHOSEN (IN 1985): INTEL 286/310

- OPEN SYSTEM (MULTIBUS 1)
- CPU: INTEL 80286/80287
- OPERATING SYSTEM: INTEL RMX86
- UPGRADE TO 386-BASED RMX286 SYSTEM PLANNED

# HOST ENVIRONMENT (CONT'D)

## PERIPHERALS

- GRAPHICS TERMINAL (780 X 1024 RESOLUTION)
- DOT MATRIX PRINTER

## REAL-TIME SIGNAL INTERFACE FOR DATA ACQ. AND CONTROL

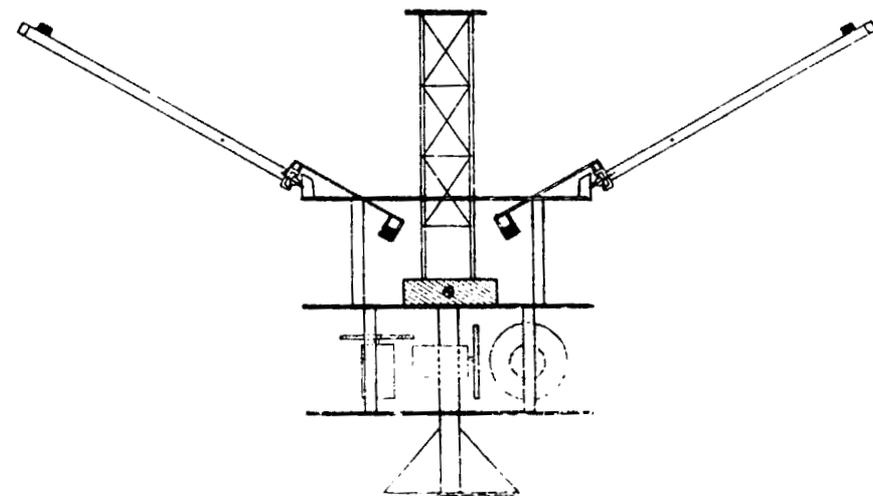
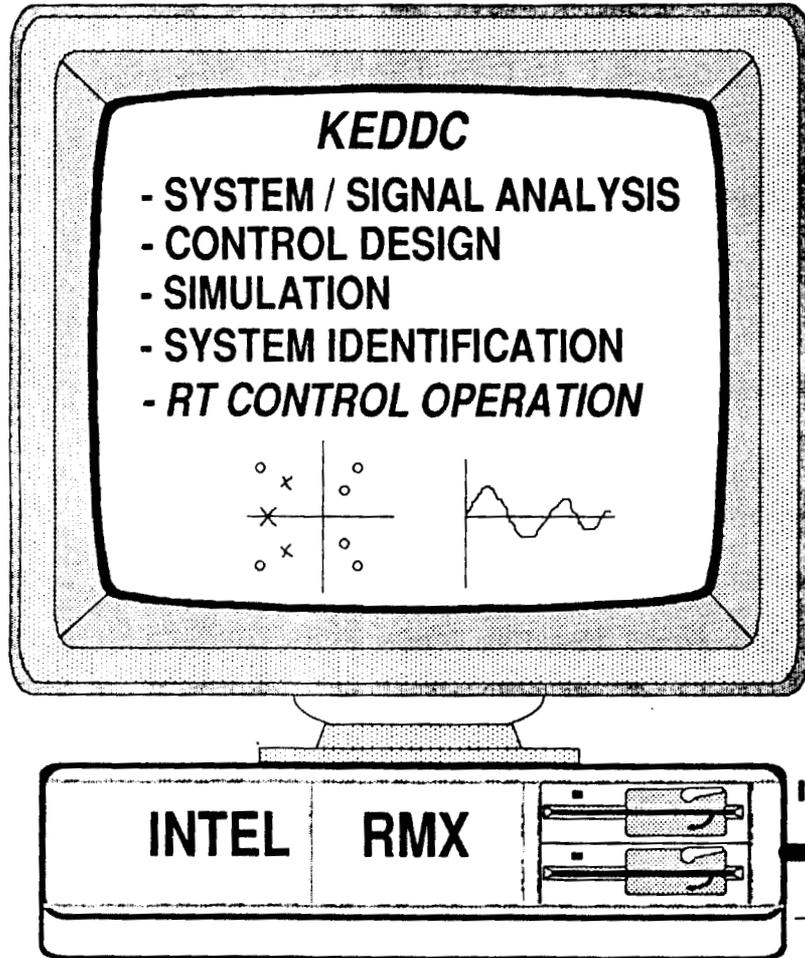
- IEEE 488 GPIB (USED IN FLEXIBLE STRUCTURE CONTROL EXPERIMENT)
- ANALOG SIGNALS

## DATA LINK TO REMOTE MAINFRAME

# REAL-TIME WORKSTATION

# FLEXIBLE STRUCTURE CONTROL EXPERIMENT "DAISY"

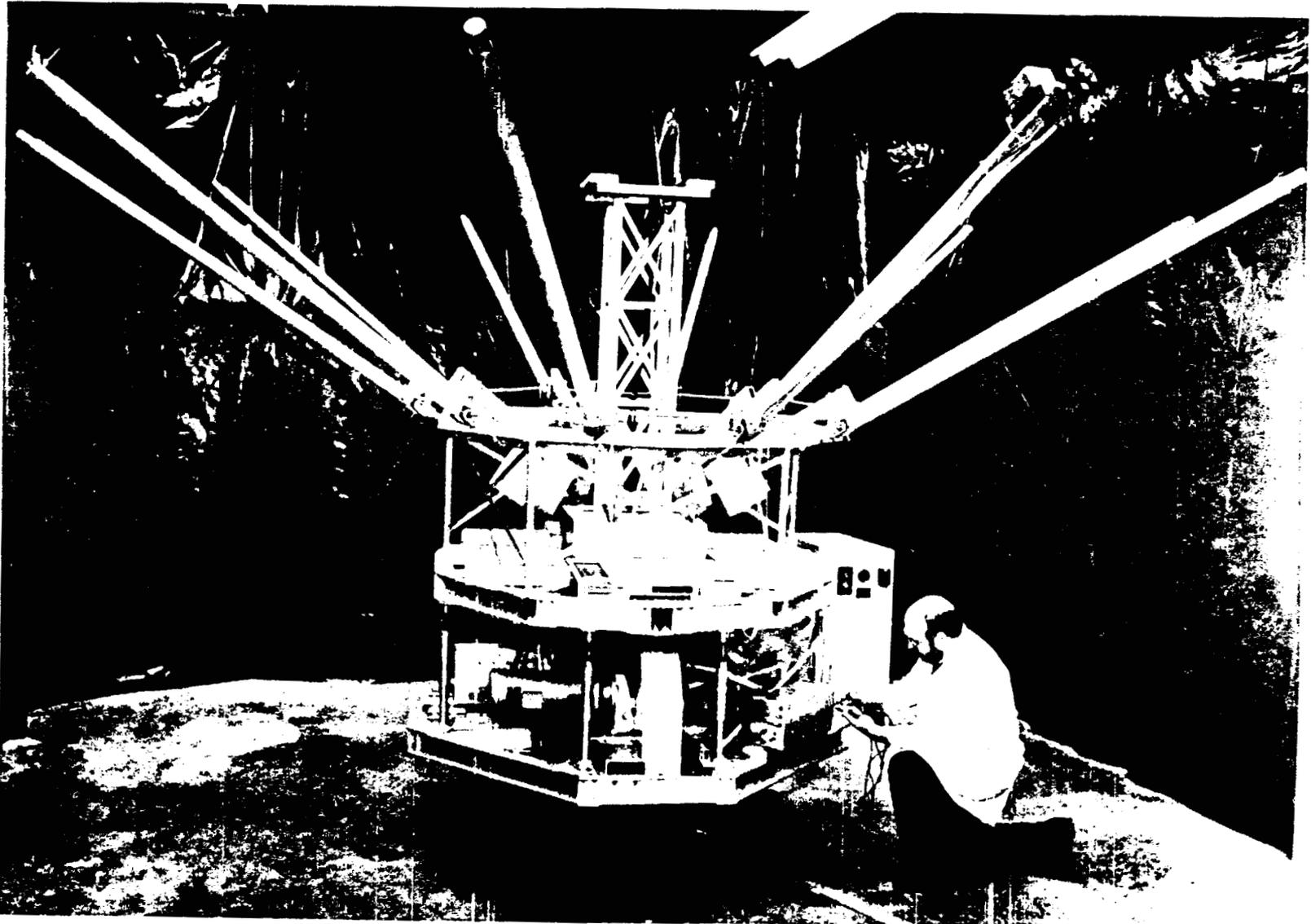
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IEEE 488 BUS

PHONE LINE

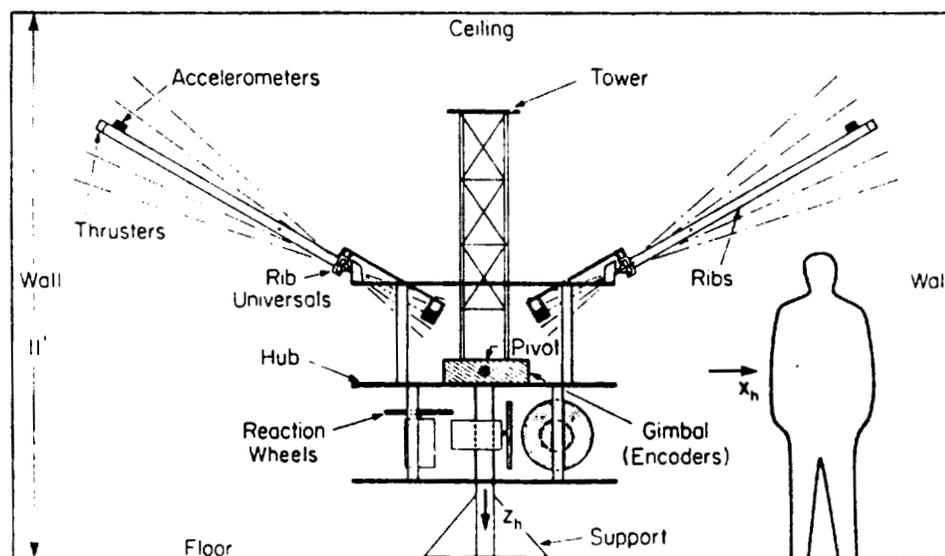
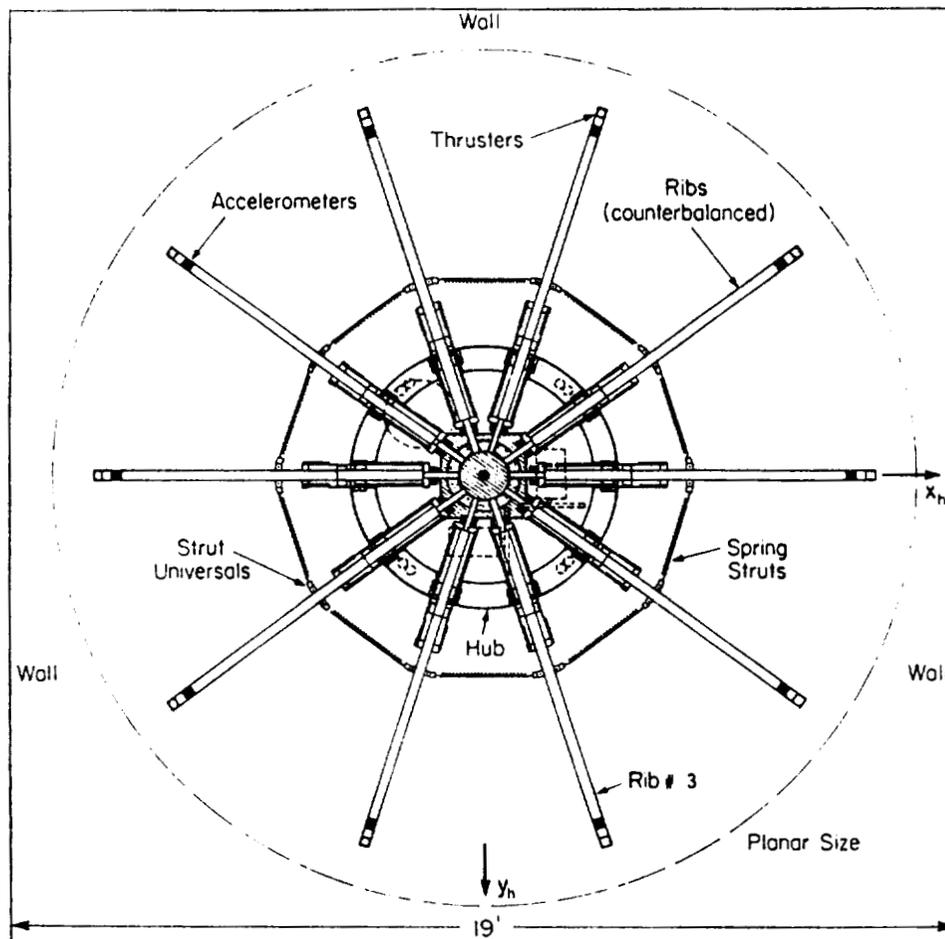




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# DAISY: A FLEXIBLE SPACECRAFT EMULATOR



# DAISY

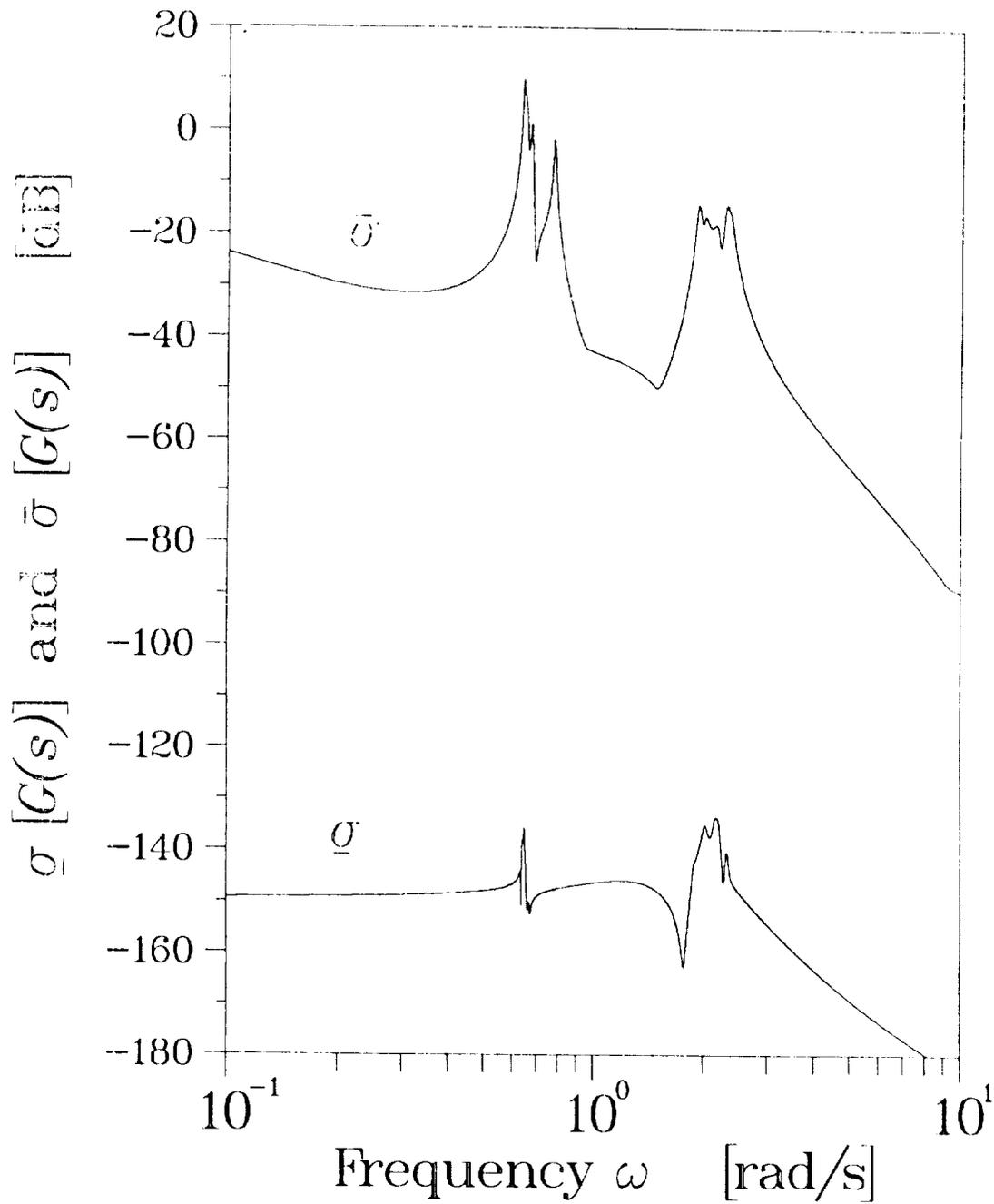
## EMULATES DYNAMICS OF A LARGE FLEXIBLE SPACE STRUCTURE

- 3 RIGID-BODY MODES  
(SLIGHT PENDULOSITY IN 2 RIGID-BODY MODES)
- 20 FLEXIBLE BODY MODES,  
LOW FREQUENCIES: 0.07 ... 0.11 Hz, IN CLUSTERS
- LOW DAMPING RATIO ACHIEVED  
RIBS: 0.008, HUB: 0.01 ... 0.05

## SPACECRAFT - LIKE SENSORS AND ACTUATORS

- 3 REACTION WHEELS ON HUB
- THRUSTERS ON RIB(S)
- ENCODERS ON HUB GIMBAL
- ACCELEROMETERS ON RIB(S)

# *Principal Gains of DAISY*



# EXPERIMENTAL RESEARCH USING DAISY

## PRESENT OBJECTIVE

DEVELOPMENT AND DEMONSTRATION OF  
ROBUST CONTROL ALGORITHMS FOR FLEXIBLE STRUCTURES

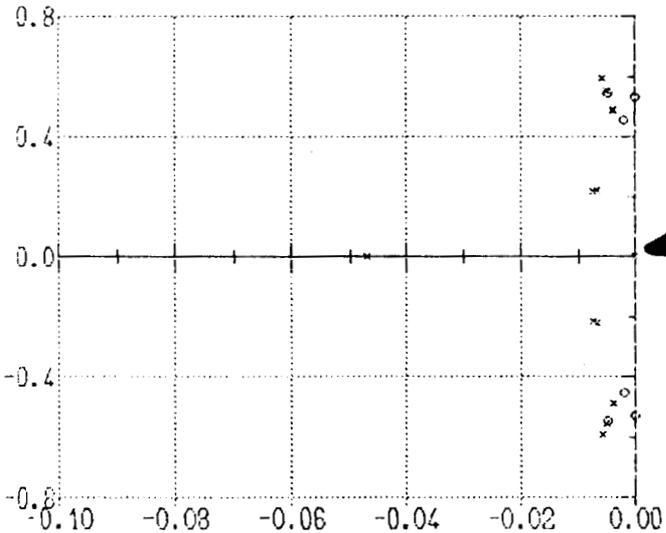
## STEPS (NOT NECESSARILY IN THIS ORDER)

- GIVEN: ANALYTICAL DYNAMICS MODEL
- SYSTEM-ORDER REDUCTION
- MODEL DISCRETIZATION
- SYNTHESIS OF CONTROL ALGORITHM
- SIMULATION
- EXPERIMENT
- EVALUATION OF ALGORITHM

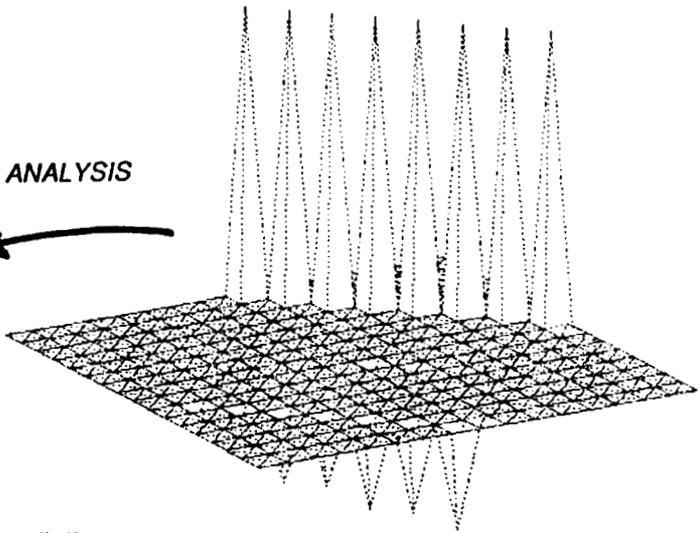
TURNAROUND: 40 MIN

# DESIGN EXAMPLE

SYSTEM EIGENVALUES AND TRANSMISSION ZEROS



MODEL (SYSTEM MATRIX)



ANALYSIS

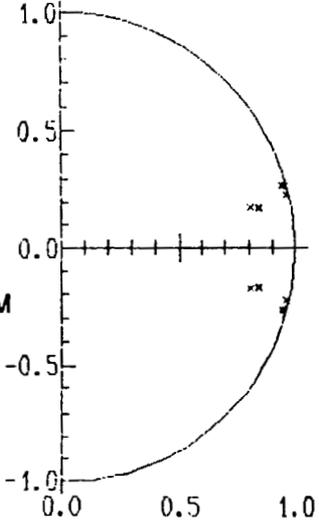


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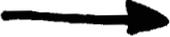
DESIGN



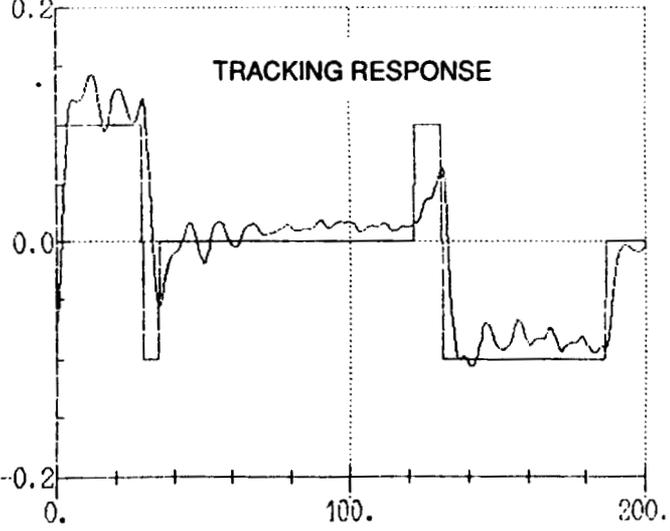
EIGENVALUES OF CLOSED-LOOP SYSTEM



EXPERIMENT



TRACKING RESPONSE



# REAL-TIME CONTROL OPERATION

## INTERACTIVE MONITOR

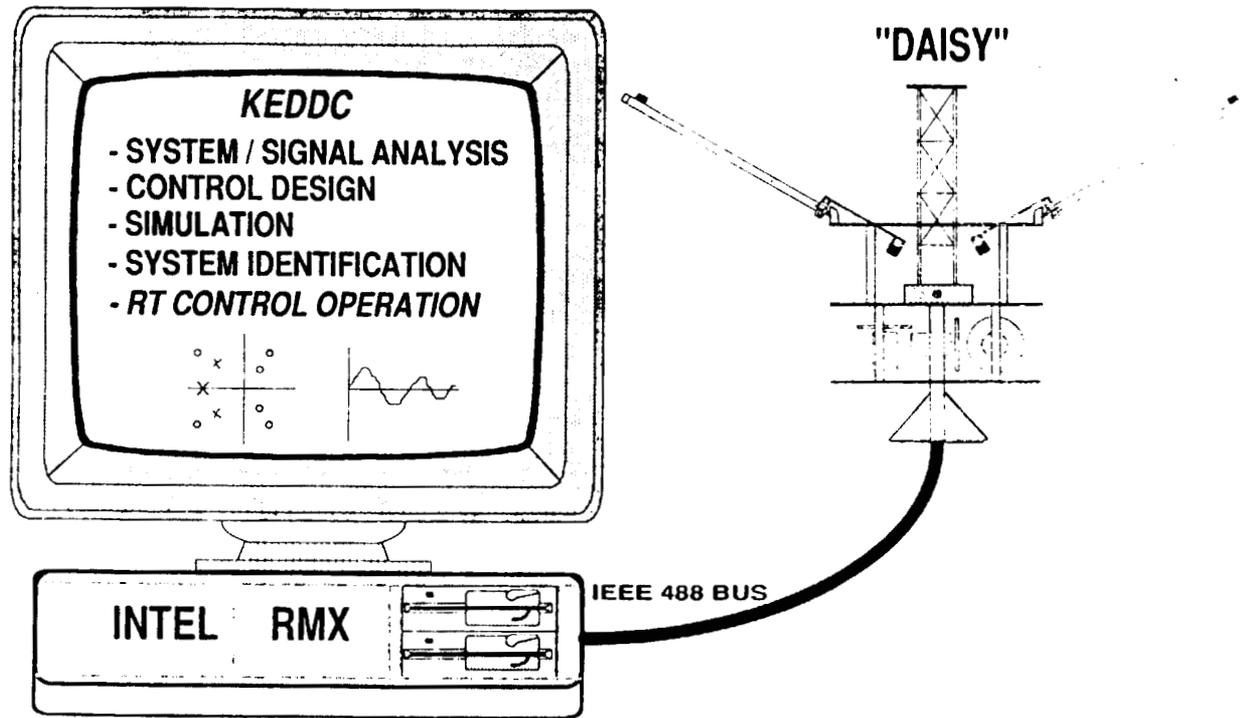
- INTERFACE BETWEEN USER AND REAL-TIME CONTROL ALGORITHM
- CONFIGURATION AND CONTROL OF REAL-TIME ALGORITHM
- DISPLAY AND RECORDING OF EXPERIMENTAL RESULTS  
SIGNALS: PLANT INPUT/OUTPUT, SETPOINTS, OBSERVER STATES, ...
- COMPLETE ENVIRONMENT FOR EFFICIENT EXPERIMENTATION

## REAL-TIME CONTROL ALGORITHM

- EXECUTION TIME  
EXTREMES: 5 MILLISEC WITH 5<sup>TH</sup>-ORDER OBSERVER  
1.2 SEC WITH 50<sup>TH</sup>-ORDER OBSERVER, 10 INPUTS, 10 OUTPUTS  
TYPICAL FOR DAISY APPLICATION (20<sup>TH</sup>-ORDER, 5 INP, 5 OUTP): 20 MILLISEC
- HOST FAST ENOUGH FOR REAL-TIME CONTROL OF DAISY  
SAMPLING INTERVAL: 0.2 SEC ... 1 SEC

# SUMMARY

## EXPERIMENTAL RESEARCH ON CONTROL OF FLEXIBLE STRUCTURES



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# CONCLUSION

REAL-TIME WORKSTATION BRIDGES GAP BETWEEN THEORY AND EXPERIMENT!

N90-10085

**CONSOLE: A CAD TANDEM FOR OPTIMIZATION-BASED DESIGN INTERACTING  
WITH USER-SUPPLIED SIMULATORS**

By

Michael K. H. Fan, Li-Sheng Wang, Jan Koninckx and Andre L. Tits  
University of Maryland  
College Park, Maryland

**ABSTRACT**

The most challenging task when designing a complex engineering system is that of coming up with an appropriate system "structure." This task calls extensively upon the engineer's ingenuity, creativity, intuition and experience. After a structure has been (maybe temporarily) selected, it remains to determine the "best" value of a number of "design parameters." The engineer's input is still essential here, as multiple tradeoffs are bound to appear. However, except in the simplest cases, achieving anything close to optimal would be impossible without the support of numerical optimization. Providing such support while emphasizing tradeoff exploration through man-machine interaction is the purpose of interactive optimization-based design packages such as CONSOLE (Proceedings of American Control Conference 1988). A requirement for CONSOLE is that the parameters to be optimally adjusted vary over a continuous (as opposed to discrete) set of values.

CONSOLE employs a recently developed design methodology (International Journal of Control 43:1693-1721) which provides the designer with a congenial environment to express his problem as a multiple objective constrained optimization problem and allows him to refine his characterization of optimality when a suboptimal design is approached. To this end, in CONSOLE, the designer formulates the design problem using a high-level language and performs design task and explores tradeoff through a few short and clearly defined commands.

The range of problems that can be solved efficiently using a CAD tools depends very much on the ability of this tool to be interfaced with user-supplied simulators. For instance, when designing a control system one makes use of the characteristics of the plant, and therefore, a model of the plant under study has to be made available to the CAD tool. CONSOLE allows for an easy interfacing of almost any simulator the user has available.

To date CONSOLE has already been used successfully in many applications, including the design of controllers for a flexible arm and for a robotic manipulator and the solution of a parameter selection problem for a neural network (all under P. S. Krishnaprasad at the University of Maryland at College Park), the design of an RC controller for a radar antenna (under F. Emad at the University of Maryland at College Park), and the design of power filters (at the Westinghouse Defense and Electronics Center). In the case of the neural network application, CONSOLE was coupled to the nonlinear system simulator SIMNON.

CONSOLE



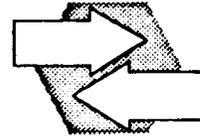
**CONSOLE :**

A CAD Tandem for Optimization-Based  
Design Interacting with User-Supplied  
Simulators

Michael K.H. Fan  
Li-Sheng Wang  
Jan Koninckx  
André L. Tits

Systems Research Center  
University of Maryland, College Park

CONSOLE



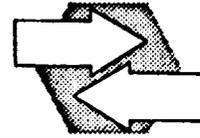
### HISTORY

DELIGHT (Nye, Polak, Sangiovanni-Vincentelli, Tits) 1980 -  
general purpose interactive package  
+ optimization algorithms

DELIGHT.MaryLin (Fan, Nye, Tits) 1985 -  
interactive optimization-based design package  
for linear time-invariant systems

CONSOLE (Fan, Wang, Koninckx, Tits) 1987 -  
interactive optimization-based design package  
for engineering systems (with user-supplied  
simulators)

CONSOLE



## PARAMETRIC OPTIMIZATION IN DESIGN

Assume *structure* already chosen

Examples :

Circuit → Topology

Control System → Controller Structure

Earthquake Proof Building → Number and Position  
of Beams

Remain to choose *best* value of finitely many parameters

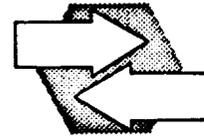
Examples :

Circuit → R, C, W, A, ...

Control System → Controller Gains,  
LQR/LQG Weighting Matrices,  
Q-parameterization, ...

Earthquake Proof Building → Beam Thickness,  
Amount of Steel, ...

CONSOLE

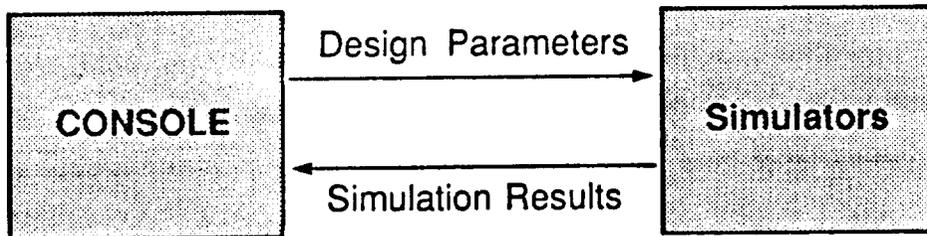


COMPONENTS FOR PARAMETERIC OPTIMIZATION

1. Design Methodology (Nye, Tits)

Problem Formulation  
Optimal in what Sense ?  
Optimization Algorithm  
User-Machine Interaction } → CONSOLE

2. Model and Simulation Tool → Simulators



CONSOLE



## PROBLEM FORMULATION

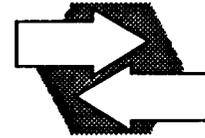
### Types of Specifications

Objectives - The smaller (larger) the better.

Soft Constraints - Aim for a *target* value. If unachievable,  
the smaller (larger) the better.

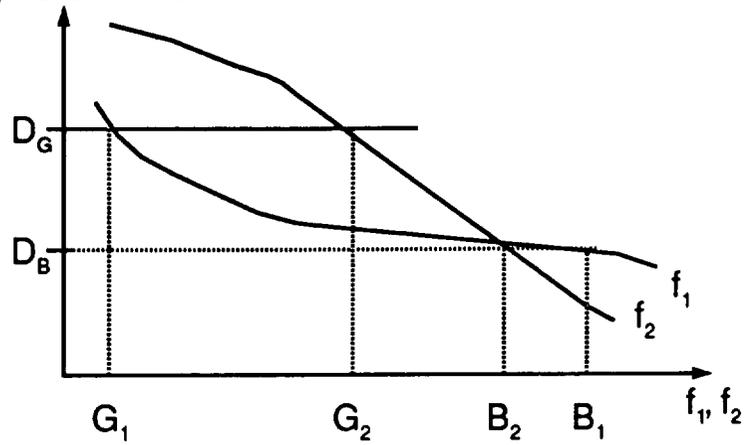
Hard Constraints - Specified value *must* be achieved.

CONSOLE

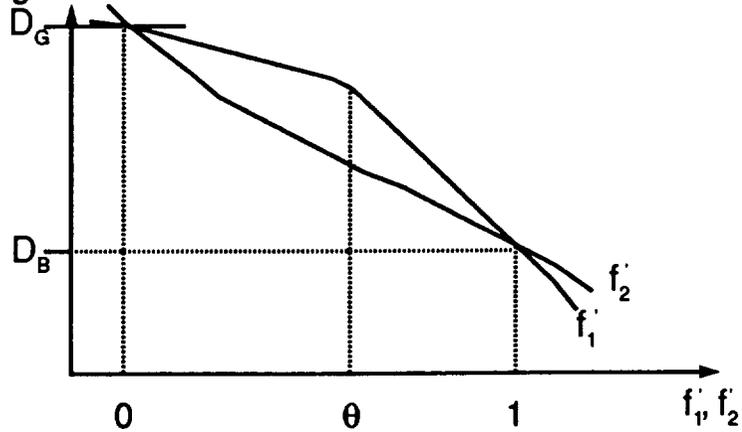


OPTIMAL IN WHAT SENSE ?

Degree of Satisfaction



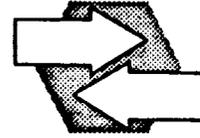
Degree of Satisfaction



$$f'_i = \frac{f_i - G_i}{B_i - G_i}$$

$$\min_x \max_i f'_i(x)$$

CONSOLE



OPTIMIZATION ALGORITHM

**Three Phase Feasible Direction Algorithm**

Phase 1 (until all hard constraints are satisfied)

attempt to satisfy hard constraints (HC)

*minimax* on HC

Phase 2 (until all good values are achieved)

improve objectives (O) and soft constraints (SC)

*minimax* on O and SC

subject to satisfying HC

Phase 3

improve objectives

*minimax* on O

subject to satisfying HC and SC

CONSOLE



$$\min_x \max_i f_i(x)$$

subject to

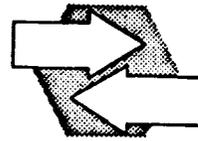
$$g_k(x) \leq 0, \quad \forall k$$

where

$$f_i(x) = \max_{\omega} \varphi_i(x, \omega)$$

$$g_k(x) = \max_{\omega} \psi_k(x, \omega)$$

CONSOLE



## USER-MACHINE INTERACTION

### **Purpose**

Progressively refine problem definition

### **Means**

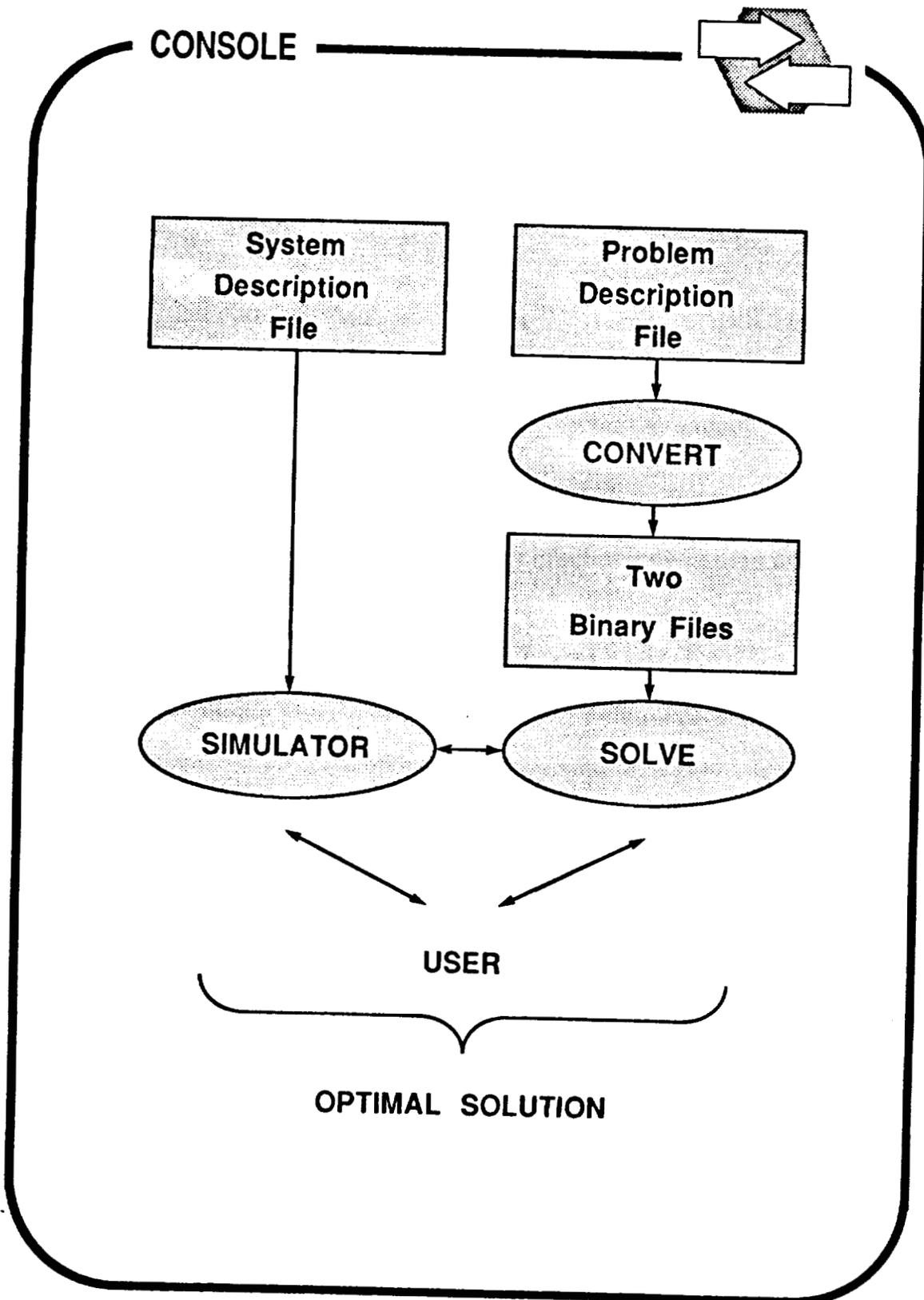
- Information on status of design conveyed graphically to user (Pcomb, Ecomb).
- User steers design to *his* optimal solution by adjusting good/bad values/curves.

CONSOLE



CONSOLE =

CONvert + SOLvE

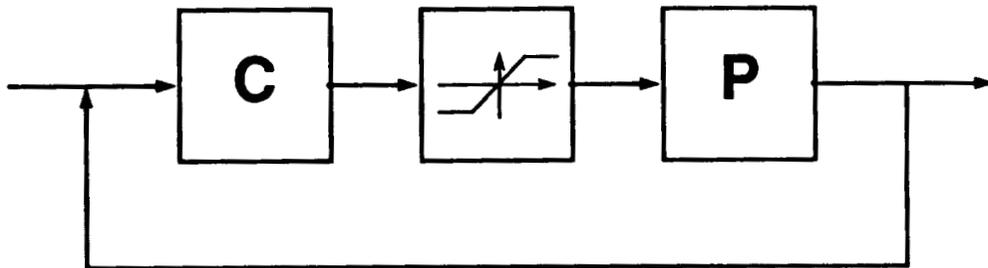


CONSOLE



A SIMPLE DESIGN EXAMPLE

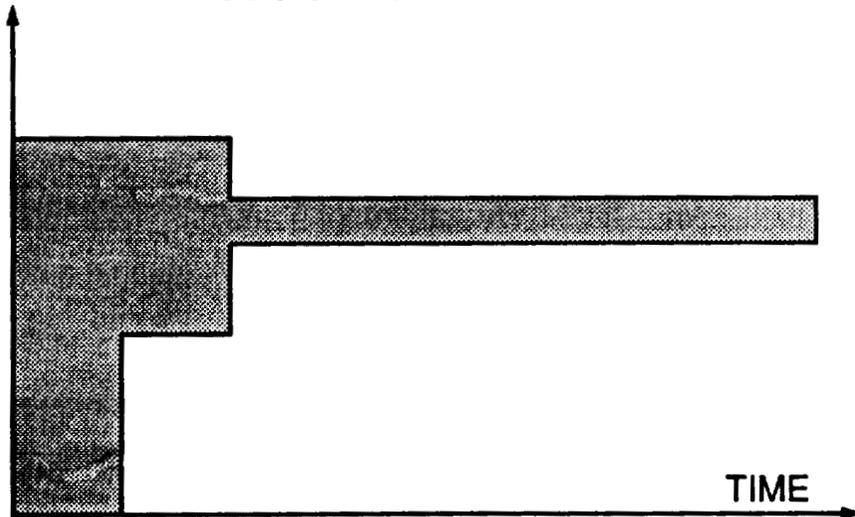
CONTROL SYSTEM



$$C(s) = K_p + \frac{K_I}{s}$$

$$P(s) = \frac{s+1}{s^2}$$

DESIGN SPECIFICATION



CONSOLE

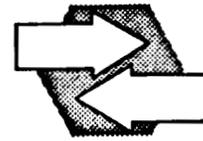


SYSTEM DESCRIPTION FILE FOR THE EXAMPLE  
(SIMNON\*)

```
CONTINUOUS SYSTEM servo
STATE x1 x2 x3
DER dx1 dx2 dx3
x1:0
x2:0
x3:0
dx1 = x2
dx2 = if (e > 0.4) then 0.4
      else if (e < -0.4) then -0.4
      else e
dx3 = r - y
e = (r - y)*Kp + x3*Ki
y = x1+x2
r:1
Kp:0
Ki:0
END
```

\*SIMNON was developed at the Lund Institute of Technology, Lund, Sweden

CONSOLE

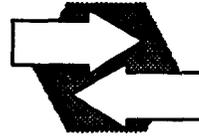


PROBLEM DESCRIPTION FILE FOR THE EXAMPLE

```
design_parameter Kp init=1 variation=5
design_parameter Ki
```

```
functional_objective "overshoot"
for t from 0 to 20 by 0.1
minimize {
    double simnon_time_response();
    return simnon_time_response(Kp,Ki,"y",t);
}
good_curve={
    if (t <= 4) return 1.05;
    else      return 1.01;
}
bad_curve ={
    if (t <= 4) return 1.1;
    else      return 1.02;
}
```

```
functional_objective "settling time"
for t from 2 to 20 by .1
maximize {
    ...
```



### MAIN FEATURES OF CONSOLE

Problem formulation is closely related to the character of a design problem.

Problem formulation syntax is strict, but easy to use.

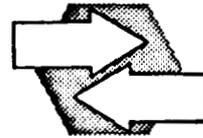
Efficient iteration between **CONVERT** and user for debugging the PDF.

**SOLVE** is interactive, with short and clearly defined commands providing efficient communication between the program and the user.

Interactive graphics provide the user with easy-to-interpret information on the current design (Pcomb, Ecomb).

User-supplied simulators can easily be linked with **SOLVE** .

CONSOLE



GLANCE AT APPLICATIONS

Design of a copolymerization reactor controller  
(Butala, Choi, Fan)

Design of controllers for a flexible arm  
(Wang, Krishnaprasad)

Design of a controller for a robotic manipulator  
(Chen, Krishnaprasad)

H-infinity Design of a Sampled-Data Control Systems  
(Yang, Levine)

Solution of a parameter selection problem  
for a neural network  
(Pati, Krishnaprasad *et al.*)

Design of an RC controller for a radar antenna  
(Emad)

Design of power filters  
(Glover, Walrath at Westinghouse Defense  
and Electronics Center)

... and soon

Design of earthquake proof buildings  
(Austin)

Design of controllers for X29 aircraft  
(Reilly, Levine)

Design of circuits  
(Westinghouse)

CONSOLE



DESIGN OF A COPOLYMERIZATION  
REACTOR CONTROLLER

(CONSOLE + Copoly) (Butala, Choi, Fan)

Objectives and Constraints

Molecular Weight  
Composition  
Final Volume  
Temperature  
Feed Flowrate

Manipulated Variables

Temperature =  $a_1 + a_2t + a_3t^2 + a_4t^3$   
Feed Flowrate =  $b_1 + b_2t + b_3t^2 + b_4t^3$

Design Parameters =  $a_i$ 's and  $b_i$ 's

Results

Pcomb (Iter= 22) (Phase 2) (MAX\_COST\_SOFT= 0.0766327)

SPECIFICATION	PRESENT	GOOD		G	B	BAD
FD1 (MN-MNs)^2	1.92e+06	0.00e+00	*****			2.50e+07
FD2 (CC-CCs)^2	3.88e-03	0.00e+00	*****			6.06e-02
C1 final vol	3.47e+00	4.00e+00	<--			4.10e+00
FC1 upper temp	3.53e+02	3.63e+02	<--			3.64e+02
FC2 lower temp	3.45e+02	3.28e+02	<-----			3.23e+02
FC3 upper flow	9.70e-03	7.00e-02	<--			7.50e-02
FC4 lower flow	6.00e-03	0.00e+00	<-----			-5.00e-03

CONSOLE



DESIGN OF A DC DIRECT DRIVE MOTOR

(CONSOLE + Simnon) (Wang, Krishnaprasad)

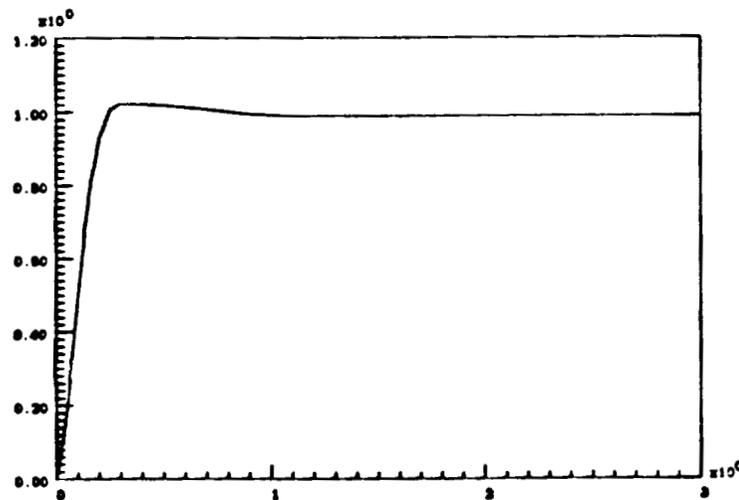
Objective

Position Profile

Design Parameters

Feedback Gains

Results



CONSOLE



FUTURE ENHANCEMENTS

User Interface

More Powerful Optimization Algorithms

Gradient Computation

**N90-10086**

**THE APPLICATION OF TSIM SOFTWARE TO ACT DESIGN AND ANALYSIS ON  
FLEXIBLE AIRCRAFT**

By

Ian W. Kaynes  
Royal Aerospace Establishment  
Farnborough, United Kingdom

**ABSTRACT**

The TSIM software is described. This is a package which uses an interactive FORTRAN-like simulation language for the simulation on nonlinear dynamic systems and offers facilities which include: mixed continuous and discrete time systems, time response calculations, numerical optimization, automatic trimming of nonlinear aircraft systems, and linearization of nonlinear equations for eigenvalues, frequency responses and power spectral response evaluation.

Details are given of the application of TSIM to the analysis of aeroelastic systems under the RAE Farnborough extension FLEX-SIM. The aerodynamic and structural data for the equations of motion of a flexible aircraft are prepared by a preprocessor program for incorporation in TSIM simulations. Within the simulation the flexible aircraft model may then be selected interactively for different flight conditions and modal reduction techniques applied. The use of FLEX-SIM is demonstrated by an example of the flutter prediction for a simple aeroelastic model.

By utilizing the numerical optimization facility of TSIM it is possible to undertake identification of required parameters in the TSIM model within the simulation. The optimizer is applied to the minimization of error between predicted and measured time responses of the system; while possibly not so efficient as dedicated identification software this has the great advantages that the identification is made directly involving the simulation model without further reprogramming or data transfer and it may be applied directly to nonlinear models. Examples are given of this analysis applied to aircraft measured responses and to simulated responses of a controlled aircraft with nonlinearities.

THE APPLICATION OF TSIM SOFTWARE TO ACT DESIGN  
AND ANALYSIS ON FLEXIBLE AIRCRAFT

by

IAN KAYNES

ROYAL AEROSPACE ESTABLISHMENT  
Farnborough, England

Head, Theoretical Dynamics Section,  
Structural Dynamics Division,  
Materials and Structures Department

PROGRAMME OBJECTIVES

1. Improvement of aeroelastic modelling techniques
2. ACT Design methods for structural applications
3. Assessment of structural impact of ACT

2.

RAE FLEX-SIM

RAE EXPERIMENTAL PROGRAMMES

1. Flight data from flexible aircraft  
(VC10, Tornado)
2. Wind tunnel experiments  
(GARTEUR, 'flying model', spoiler tests)

3. RAE FLEX-SIM

AEROELASTIC MODELLING INPUT

- a) STRUCTURAL MODAL DATA  
Calculated from mass and stiffness data by  
finite element or beam models AND/OR  
derived from ground resonance tests.  
Model reduction techniques used as appropriate.
- b) AERODYNAMIC LOADINGS  
Calculated from geometric data by vortex lattice  
or RAE methods for steady and unsteady flow.
- c) SENSOR and ACTUATOR DATA.  
Linearity assumed in these models.

4. RAE FLEX-SIM

AEROSERVOELASTIC MODEL

Combination of structural, aerodynamic, sensor and actuator data with the control system model.

Expressed in a first order form compatible with stability and control representations to allow integration between the aeroelastician and the S&C specialists.

Software required for response prediction and control design activities on these models.

5. RAE FLEX-SIM

TSIM

Time SIMulation

Non-linear dynamic simulation package

Originated and developed at RAE since late 1970s

Now documented, supported and developed as a commercial product by Cambridge Control

Used in RAE and in research organisations, aerospace industry and universities in Britain and overseas

6. RAE FLEX-SIM

### TSIM FACILITIES

Interactive program using FORTRAN-like simulation language and facilitating modification of model

Simulation of linear and non-linear equations

Mixed continuous and discrete time systems

Time response calculation

Linearisation of non-linear equations for:

Eigen values

Frequency responses

RMS response evaluation

Numerical optimisation

Automatic trimming of non-linear aircraft

Communication with other control design packages

7.

RAE FLEX-SIM

### SAMPLE OF TSIM SERIAL INTERACTION

SIM>

SIM>; Assign values to some TSIM variables:-

SIM> ZPOSA 0.9 DAMPA 0.7 RTB 15

SIM>

SIM>; Enter the time response set-up module and

SIM>; define the required parameters:-

SIM> SET TIME\_RESP

SIM>

SET TIME\_RESP: OUTPUT 1 NZB 2 BMR 3 TWG

SET TIME\_RESP: SCALE 2 -0.8 0.8

SET TIME\_RESP: RKUTTA 0.4, 0.002, 0.01

SET TIME\_RESP: STEP BGO 0.0, -0.1, -0.6

SET TIME\_RESP:

SIM>; Now run the time response module:-

SIM> RUN TIME\_RESP

8.

RAE FLEX-SIM

FLEX-SIM: APPLICATION OF TSIM TO FLEXIBLE AIRCRAFT

PRE-PROCESSING FUNCTIONS:

- a) structural data processing
- b) aerodynamics calculations and modification
- c) loads, actuator and sensor modelling
- d) model reduction and combination
- e) TSIM model generation

TSIM-CONCURRENT FUNCTIONS:

- f) generation of aeroelastic input functions
- g) order reduction and changes of flight conditions in the flexible aircraft model
- h) flight loads and sensor response calculation
- i) presentation of results

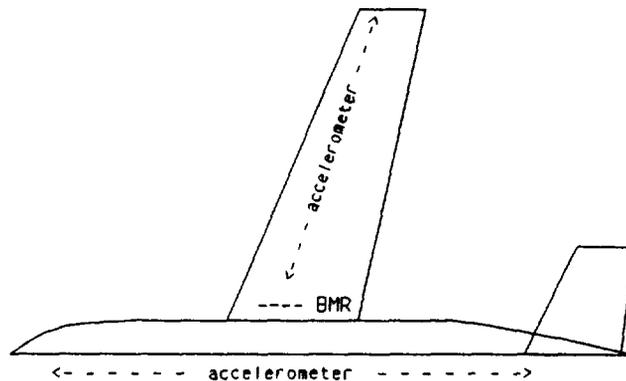
POST-PROCESSING FUNCTION:

- j) analysis of aeroservoelastic results

9.

RAE FLEX-SIM

DEMONSTRATION LOAD ALLEVIATION - AIRCRAFT

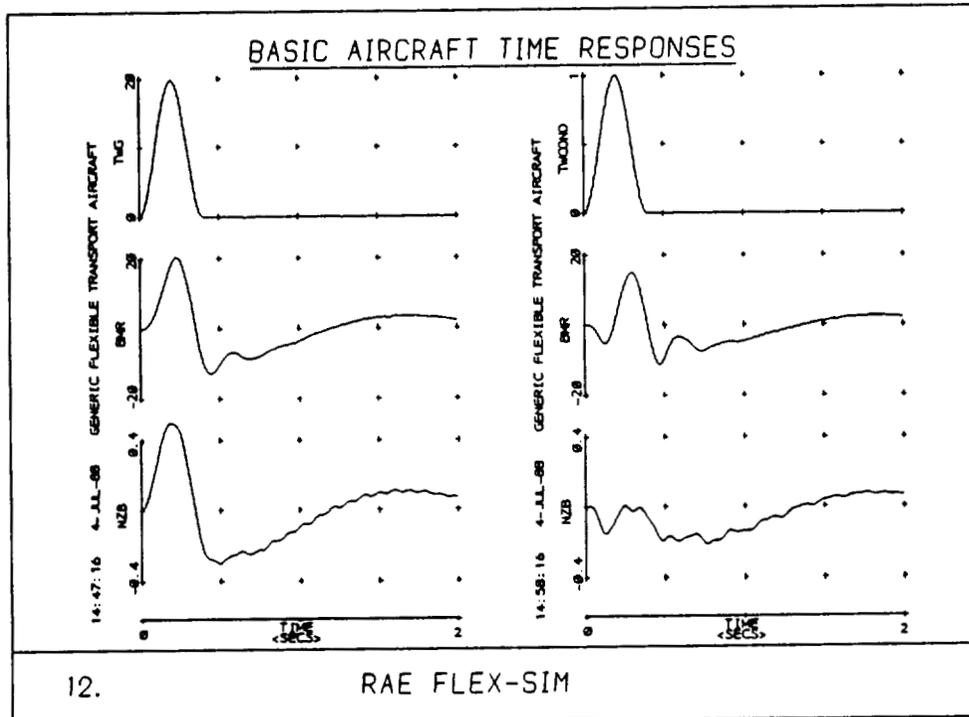
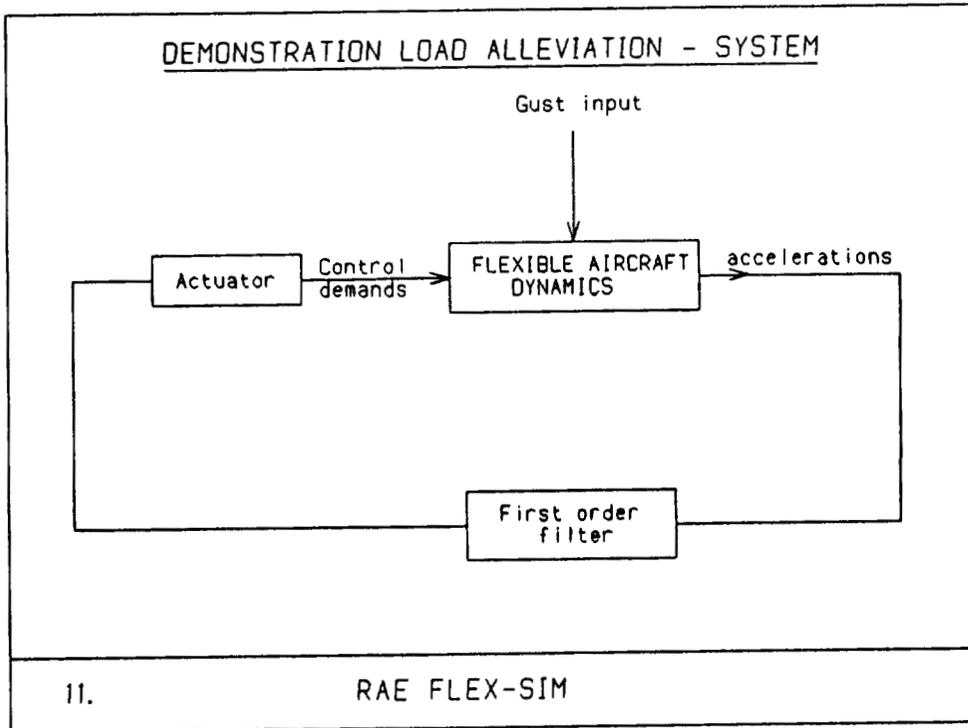


OBJECTIVE: reduction of wing loads in turbulence through outboard wing controls

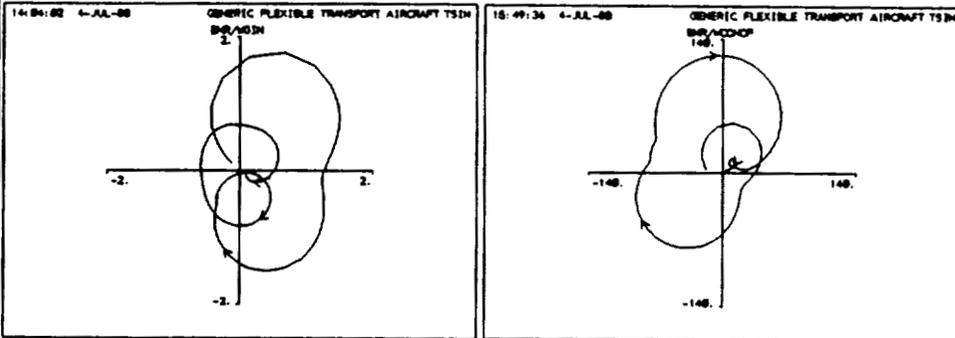
INVESTIGATION: sensor location and combination

10.

RAE FLEX-SIM



### BASIC AIRCRAFT FREQUENCY RESPONSES



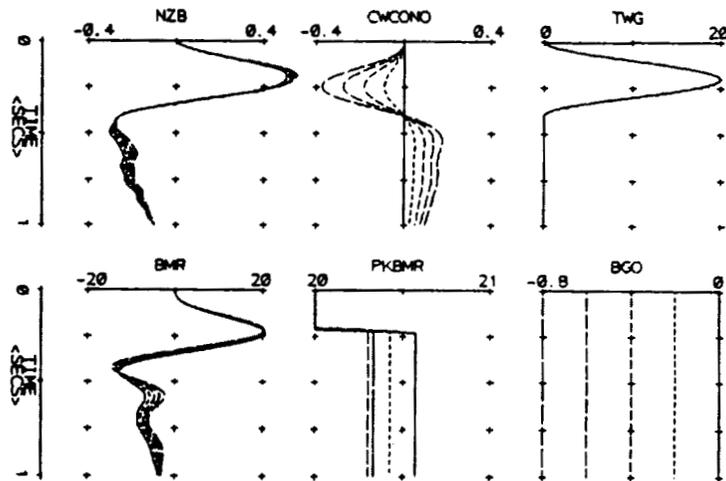
13.

RAE FLEX-SIM

### GLA WITH ACCELEROMETER AT CG

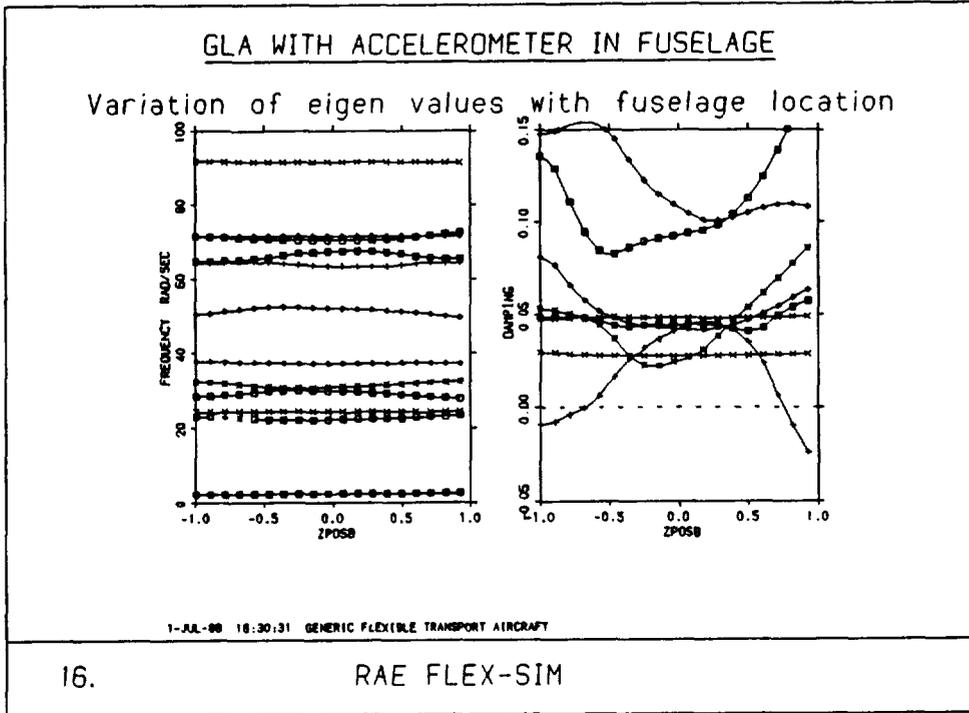
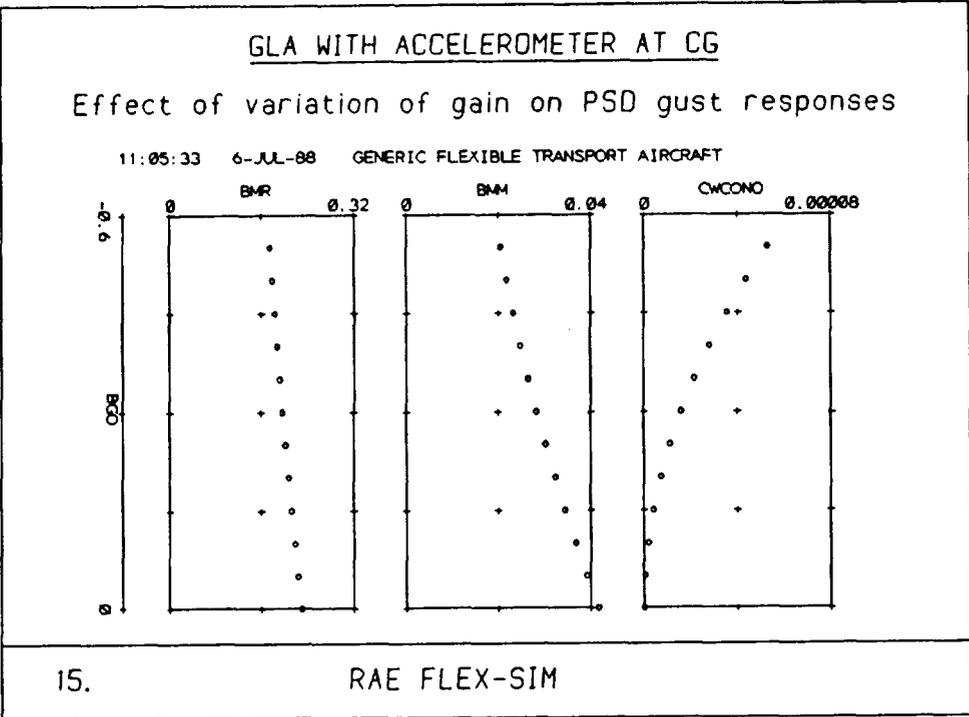
Effect of variation of gain on gust responses

16:24:37 4-JUL-88 GENERIC FLEXIBLE TRANSPORT AIRCRAFT



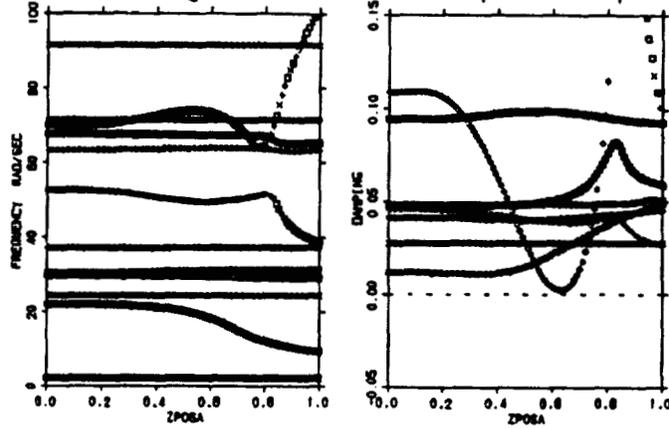
14.

RAE FLEX-SIM



GLA WITH ACCELEROMETERS ON WING AND AT CG

Variation of eigen values with spanwise position



5-JUL-88 15:58:51 GENERIC FLEXIBLE TRANSPORT AIRCRAFT

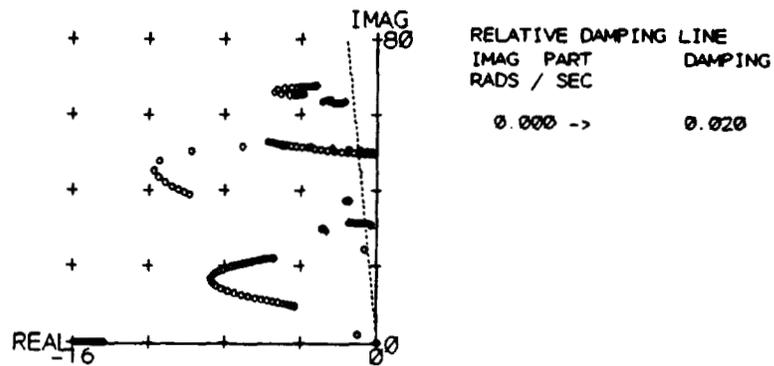
17.

RAE FLEX-SIM

GLA WITH ACCELEROMETERS ON WING AND AT CG

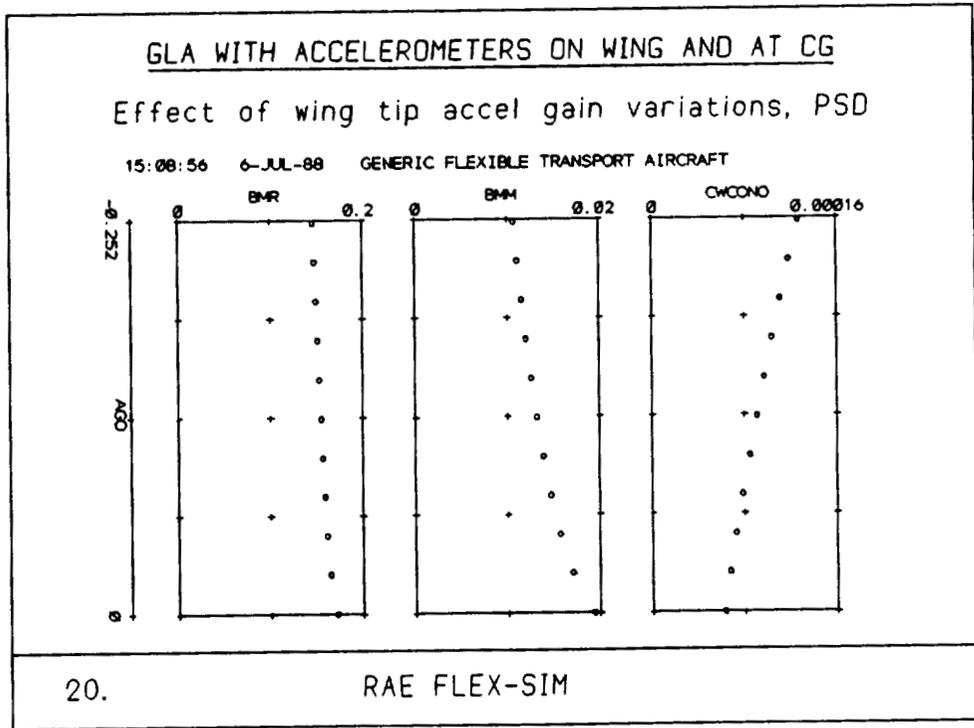
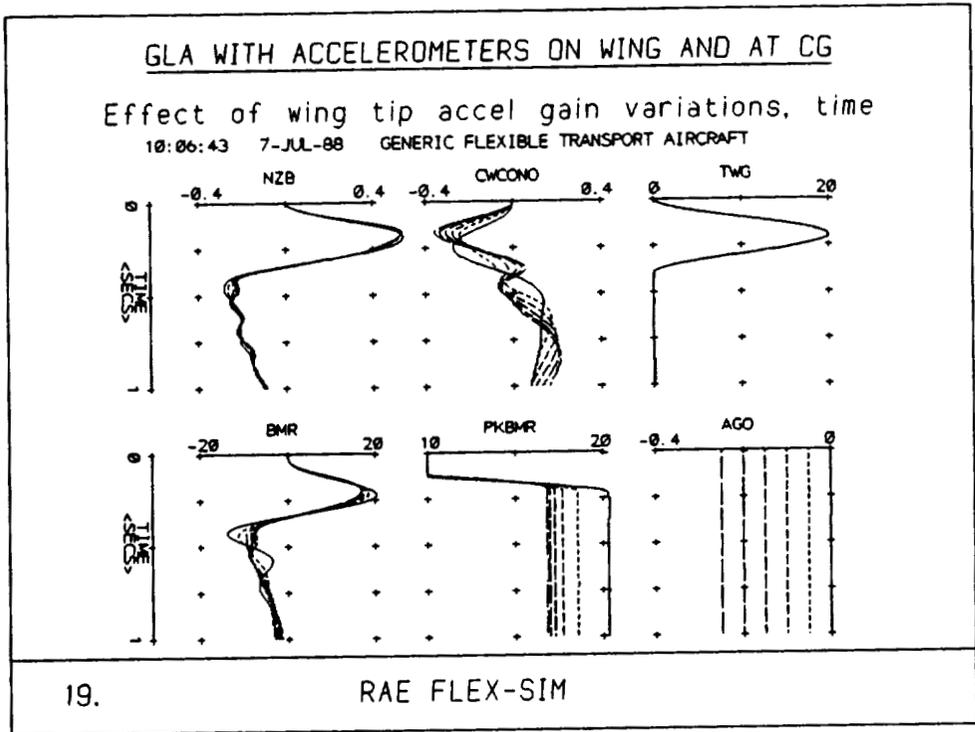
Root locus with spanwise position

11:34:34 5-JUL-88 GENERIC FLEXIBLE TRANSPORT AIRCRAFT



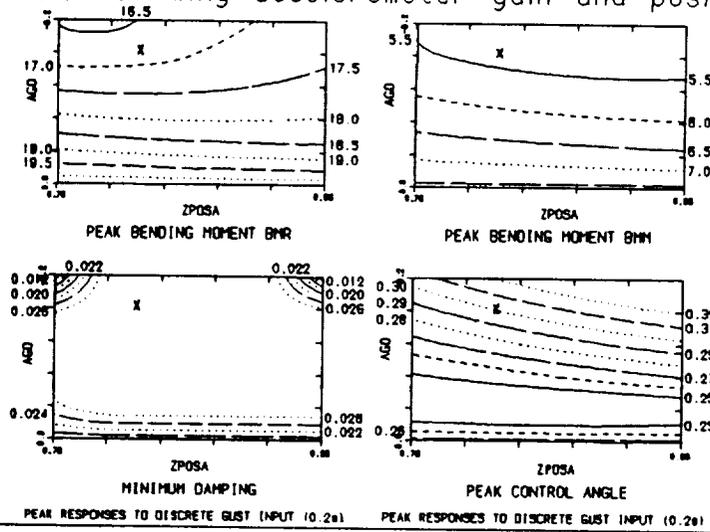
18.

RAE FLEX-SIM



GLA WITH ACCELEROMETERS ON WING AND AT CG

Variation of wing accelerometer gain and position

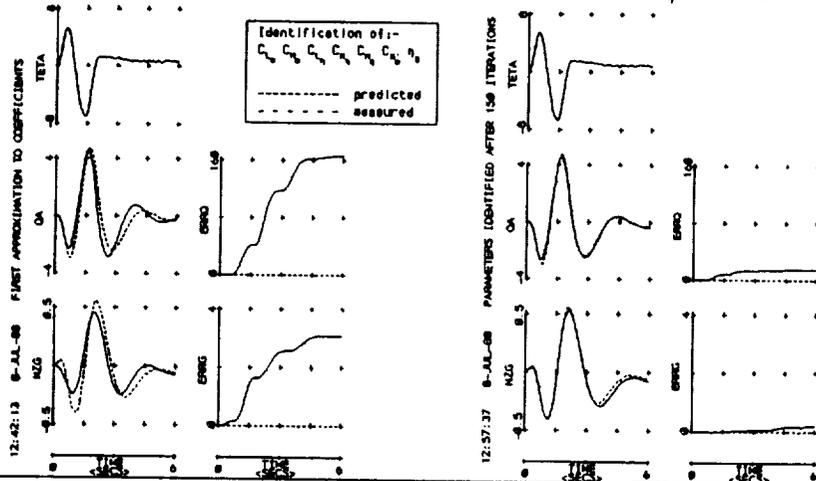


21.

RAE FLEX-SIM

PARAMETER IDENTIFICATION VIA TSIM

Numerical optimisation to minimise G and D errors between predicted and measured VC10 responses



22.

RAE FLEX-SIM

N90-10087

**CONTROL/STRUCTURE INTERACTION METHODS FOR SPACE STATION POWER SYSTEMS**

By

Paul Blelloch  
Structural Dynamics Research Corporation  
San Diego, California

**ABSTRACT**

The Structural Dynamics Research Corporation and the NASA Lewis Research Center have been working together to develop tools and methods for the analysis of control/structure interaction problems related to the space station power systems. Flexible modes of the solar arrays below 0.1 Hz, suggest that even for relatively slow control systems, the potential for control/structure interaction exists. The emphasis of the effort has been to develop tools which couple NASTRAN's powerful capabilities in structural dynamics with EASY5's powerful capabilities in control systems analysis. One product is an interface software package called CO-ST-IN for Control-Structure-Interaction. CO-ST-IN acts to translate data between NASTRAN and EASY5, facilitating the analysis of complex coupled problems. Interfaces to SDRC I-DEAS and MATRIXx are also offered. Beside transferring standard modal information, CO-ST-IN implements a number of advanced methods. These include a modal ordering algorithm that helps eliminate uncontrollable or unobservable modes from the analysis, an implementation of the more accurate mode acceleration algorithm for recovery of element forces and stresses directly in EASY5 and an implementation of fixed interface modes in NASTRAN, which reduces the error in the closed-loop model due to the use of truncated mode sets. A brief overview of the program will be presented, along with description of some of the methods used to facilitate rapid and accurate analyses.

# **CONTROL/STRUCTURE INTERACTION METHODS FOR SPACE STATION POWER SYSTEMS**

**presented by**

**Paul Belloch, Ph.D.  
SDRC WRO  
San Diego, CA**

**supported by**

**NASA Lewis Research Center  
Cleveland, OH**

**July 11, 1988**

## AGENDA

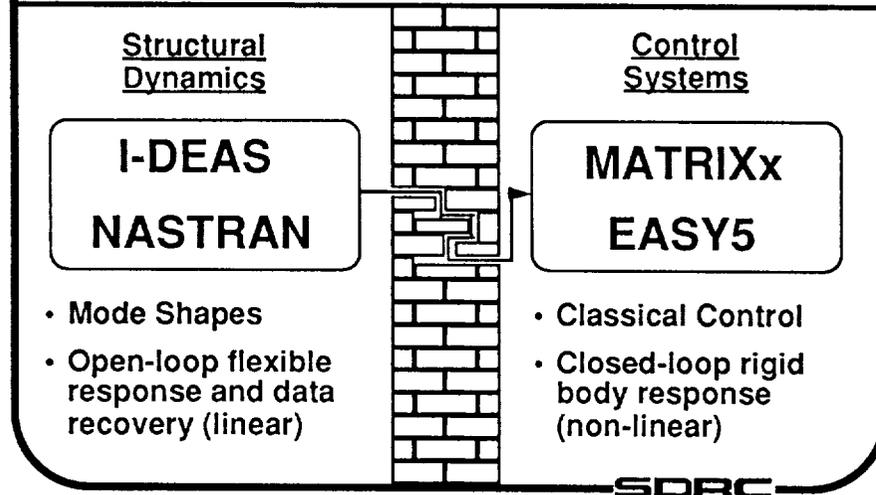
- Quick Overview of CO-ST-IN Program
- Alternate Modal Representations
- Discussion

SDRC

## NOTES

SDRC has been working with the NASA Lewis Research Center to develop methods for the study of control/structure interaction problems related to space station power systems. We will discuss the software developed for this project, (CO-ST-IN) and if we have time we will briefly mention the important area of alternate modal representations to improve the accuracy of closed-loop models.

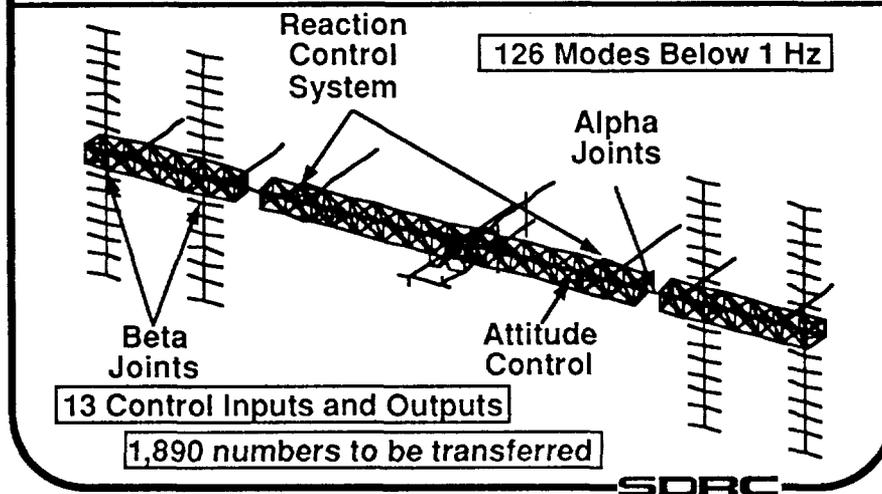
## DATA NEEDS TO BE TRANSFERRED FROM STRUCTURES TO CONTROLS



## NOTES

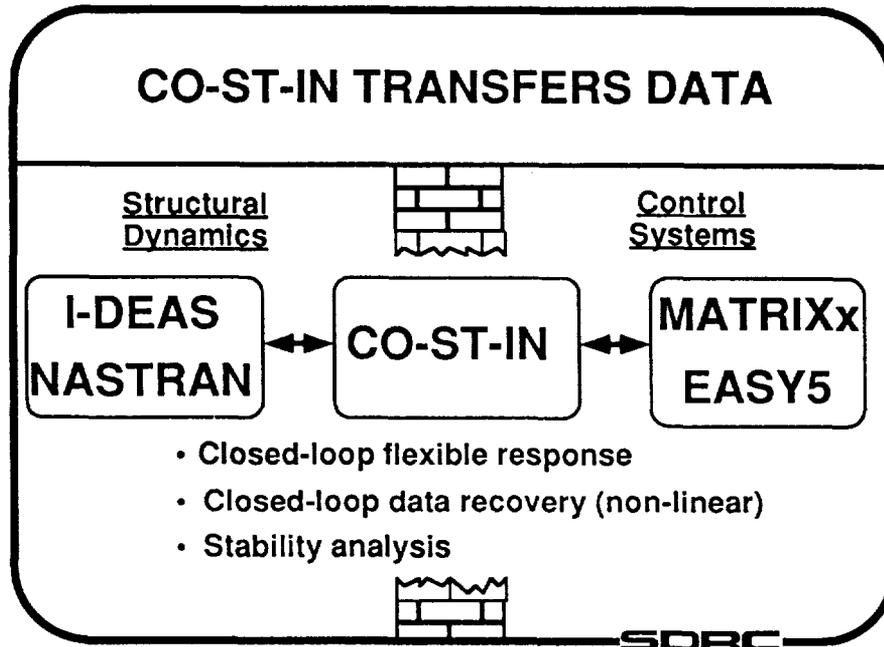
Standard approaches to Control/Structure Interaction problems combine two separate disciplines, structural dynamics and control systems. Data is often passed manually from engineers in one group to engineers in the other. Furthermore, each group uses its own analysis tools. We use I-DEAS and NASTRAN for structural dynamics and MATRIXx and EASY5 for control systems.

## SPACE STATION MODEL TOO LARGE FOR MANUAL TRANSFER OF DATA



## NOTES

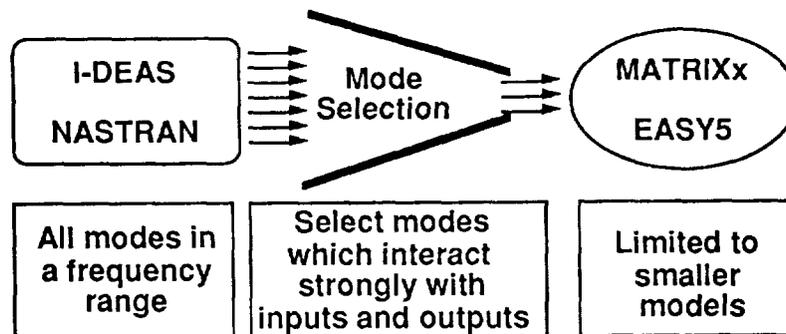
The space station is a large complex structural system with a large number of closely spaced, low frequency modes and a large number of structural inputs and outputs. The size of the model makes manual transfer of data impractical. Model size also puts a large emphasis on practical model reduction algorithms.



## NOTES

**CO-ST-IN** stands for **Control-Structure-Interaction**. It automates the transfer of data back and forth among **I-DEAS**, **NASTRAN**, **MATRIXx** and **EASY5**. **CO-ST-IN** implements a number of special (non-standard) capabilities as well as the automated transfer of modal data.

## MODAL ORDERING REDUCES MODEL SIZE

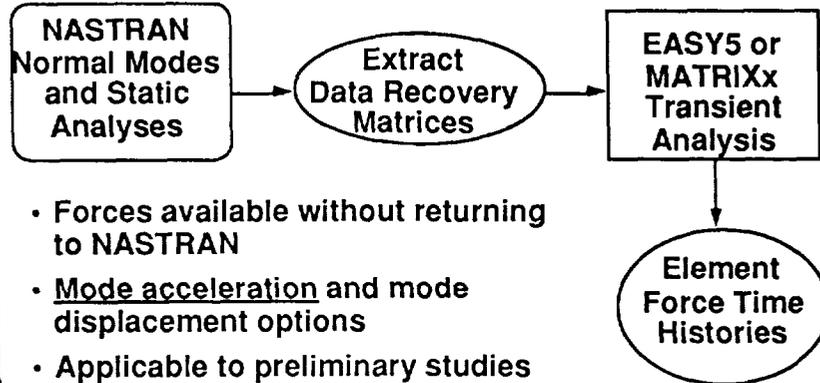


SDRC

## NOTES

Since many control system algorithms fail for large models it is essential to select as small a model as possible. Large structural models usually contain a number of modes which do not interact significantly with inputs and outputs. Modal ordering can help eliminate these modes, resulting in an accurate reduced order model. We implement both Skelton's modal cost and the approximate balanced singular value as measures of modal influence. Inputs and outputs can be scaled to reflect their relative importance, and modes can be grouped when modal frequencies are close.

## ELEMENT FORCES CAN BE CALCULATED IN EASY5 OR MATRIXx

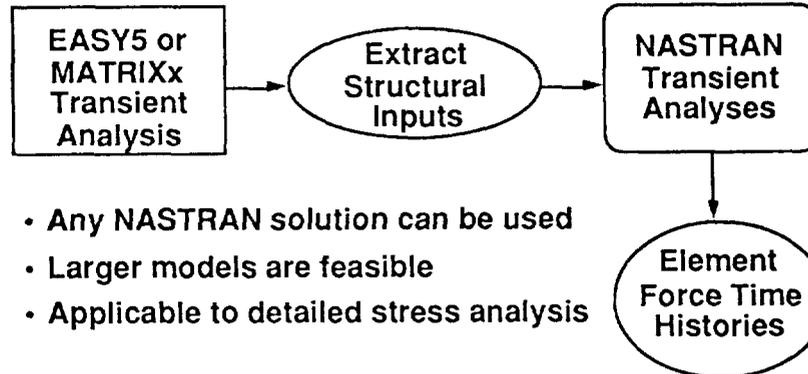


SDRC

## NOTES

Calculating element forces and stresses directly in the control system routine can greatly accelerate turn around time. We transfer the appropriate matrices from NASTRAN to let us implement a mode acceleration technique. The mode acceleration formulation adds a static correction term to the standard mode displacement formulation which improves accuracy when using truncated mode sets. This approach is applicable to parameter studies, where quick turn around time is paramount.

## ELEMENT FORCES CAN BE CALCULATED IN NASTRAN

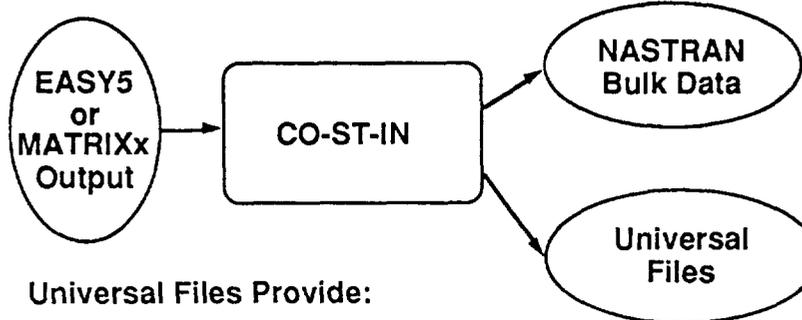


SDRC

## NOTES

Detailed stress analyses fall into the realm of structural dynamicists. In order to facilitate the direct application of NASTRAN to this problem we extract the structural input forces from the controls routine and write these as NASTRAN bulk data. This allows the structural dynamicist the flexibility to choose any NASTRAN transient technique (including a direct transient) to recover element forces and stresses. This method increases turn around time, but is applicable to a detailed stress analysis after control system parameters have been fixed.

## CO-ST-IN TRANSLATES EASY5 AND MATRIXx OUTPUT



Universal Files Provide:

- Flexible plotting options
- Data management (SYSTAN)

SDRC

## NOTES

As well as provide input to NASTRAN, other reasons for translating control system output include the requirement for more flexible plotting and data management. By translating time simulation output to I-DEAS Universal file format, we can store functions in a database, facilitating the application of powerful data management and plotting capabilities.

## **CO-ST-IN IS A TOOL FOR CONTROL/STRUCTURE INTERACTION**

- Transfers data between structural dynamic (NASTRAN and I-DEAS) and control system software (EASY5 and MATRIXx).
- Uses modal ordering to reduce model size.
- Data recovery performed directly in controls routine using the more accurate mode acceleration method.
- EASY5 and MATRIXx output translated for plotting, data management and NASTRAN data recovery.

**SDRC**

## **NOTES**

CO-ST-IN is simply a tool for transferring data among otherwise incompatible analysis programs. CO-ST-IN tries to be smart in what it transfers by using modal ordering to reduce model size and a mode acceleration technique to recover element forces and stresses directly in the controls routine. Control routine output is translated to I-DEAS Universal file format in order to facilitate data management and plotting.

## AGENDA

- Quick Overview of CO-ST-IN Program
- Alternate Modal Representations
- Discussion

SDRC

## NOTES

The main focus of our investigation has been to determine the best possible structural models to use in control/structure interaction study. Here we define best to mean those which result in the most accurate closed-loop models while using a minimum number of dynamic states. This investigation has led us to examine the use of alternate (other than normal) modal representations.

## **FIXED INTERFACE MODES REPRESENT EFFECT OF INPUT FORCES**

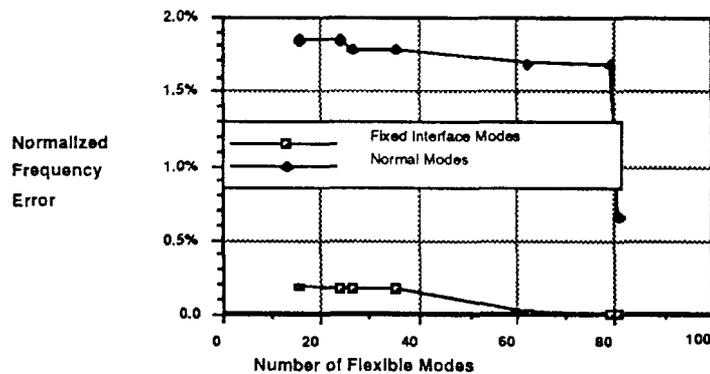
- Normal modes are calculated on basis of free-free boundary conditions
- Normal modes do not represent effect of input forces
- Fixed interface (cantilever, Craig-Bampton) modes result in more accurate closed-loop models
- Improvement even for "soft" controllers
- More pronounced for "stiffer" controllers

**SDRC**

## **NOTES**

The basic problem with normal modes is that they are calculated on the basis of free-free boundary conditions. The result is that these modes poorly represent the local effects of forces and moments applied by control actuators at these boundaries. The use of fixed interface modes (sometimes called cantilever or Craig-Bampton modes) can help alleviate this problem by providing an accurate static representation at the location of control inputs. We have found that the use of fixed interface modes does result in more accurate closed-loop models, even for control frequencies which lie well below flexible frequencies. For stiffer controllers the differences are even more pronounced.

## CLOSED-LOOP FREQUENCIES MORE ACCURATE

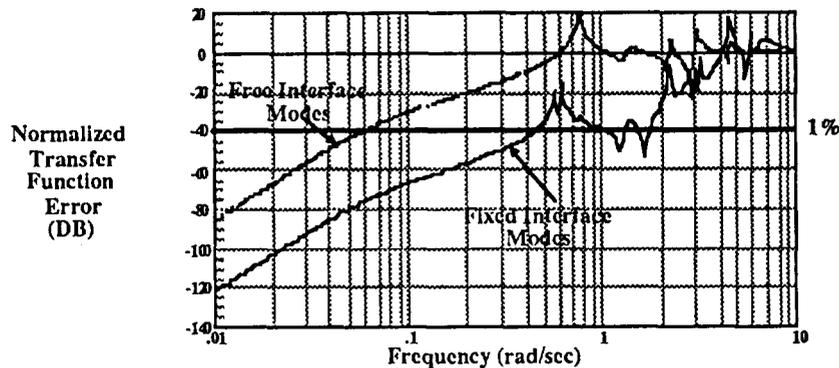


SDRC

## NOTES

One measure of accuracy for the closed-loop model is the accuracy of closed-loop frequencies. Normalized error is defined as the distance of the approximate frequency from the exact frequency, divided by the magnitude of the exact frequency. In this case we are examining the accuracy of an alpha joint control frequency as we increase the number of open-loop modes, using either a fixed interface or a normal modes representation. In this case control frequencies are close to an order of magnitude below flexible frequencies and the results are consistent with other models that we've looked at.

## CLOSED-LOOP FREQUENCY RESPONSE MORE ACCURATE



SDRC

## NOTES

Another measure of accuracy for the closed-loop system is accuracy of the closed-loop frequency response. Here we examine the frequency response from an attitude command about the y-axis (roll) to response about the same axis. Normalized error at each frequency is defined as the distance of the approximate to the exact frequency response, divided by the magnitude of the exact response. Note that both representations are inaccurate at high frequencies (where modes are neglected), but that the fixed interface representation is more accurate at lower frequencies. The control frequency in this case is more than an order of magnitude lower than the flexible frequencies and again the results are consistent with other models we've examined.

## SUMMARY

- CO-ST-IN transfers data between NASTRAN, IDEAS, EASY5 and MATRIXx
- Modal ordering reduces model size
- Mode acceleration data recovery performed in control simulation
- Fixed interface modal representations result in more accurate closed-loop models

SDRC

## NOTES

Our work with NASA Lewis is on-going, and we will be continuing to develop methods which facilitate fast and accurate closed-loop structural analyses. We will also continue to place emphasis on the selection of improved structural representations for control/structure interaction studies.

## A Brief List of CO-ST-IN Commands -

DAMP	- Define modal damping ratios.
DMPDAT	- Write all data to an unformatted file.
DRSC	- Scale data recovery outputs for modal ordering.
GYRO	- Define gyroscopic forces due to a spinning body.
INPT	- Define an absolute or relative structural input.
INPT1	- Define a generalized structural input.
INSC	- Scale inputs for modal ordering.
MACC	- Define an absolute or relative acceleration measurement.
MASC	- Scale acceleration measurements fro modal ordering.
MDSP	- Define an absolute or relative displacement measurement.
MDSC	- Scale displacement measurements for modal ordering.
MVEL	- Define an absolute or relative velocity measurement.
MVSC	- Scale velocity measurements for modal ordering.
OACC	- Define an absolute or relative acceleration output.
OASC	- Scale acceleration outputs for modal ordering.
ODSP	- Define an absolute or relative displacement output.
ODSC	- Scale displacement outputs for modal ordering.
OGRP	- Group modes for ordering.
ORDER	- Order modes on the basis of approximate balanced singular values.
ORDUSR	- User-defined modal ordering.
OVEL	- Define an absolute or relative velocity output.
OVSC	- Scale velocity measurements for modal ordering.
PARAM	- Define various problem parameters.
PID	- Define a PID controller.
PULSE	- Define a pulse train input.
REDDAT	- Read unformatted data file written by DMPDAT.
RCS	- Define a simple reaction control system for space station reboost.
RDRM2	- Read data recovery matrices from a NASTRAN Output2 file.
RDRM4	- Read data recovery matrices from a NASTRAN Output4 file.
RMN2	- Read modal data from a NASTRAN Output2 file.
RMU	- Read modal data from an I-DEAS Universal file.
RRESP	- Read response time histories from an EASY5 Plots file.
STITLE	- Define a problem subtitle.
TITLE	- Define a problem title.
WDRM4	- Write data recovery matrices in NASTRAN Output4 format.
WEAD	- Write an EASY5 Analysis Definition File.
WEMG	- Write an EASY5 Model Generation File.
WLODN	- Write NASTRAN FORCE and MOMENT cards for static solution.
WMATX	- Write matrices in MATRIXx format.
WRSPN	- Write structural input force response as NASTRAN Bulk Data.
WRSPU	- Write EASY5 responses in I-DEAS Universal File Format.

N90-10088

**FLEXIBLE MISSILE AUTOPILOT DESIGN STUDIES WITH PC-MATLAB/386**

By

Michael J. Ruth  
Johns Hopkins University/Applied Physics Laboratory  
Laurel, Maryland

**ABSTRACT**

Development of a responsive, high-bandwidth missile autopilot for airframes which have structural modes of unusually low frequency presents a challenging design task. Such systems are viable candidates for modern, state-space control design methods. The PC-MATLAB interactive software package provides an environment well-suited to the development of candidate linear control laws for flexible missile autopilots. The strengths of MATLAB include: (1) Exceptionally high speed -- MATLAB's version for 80386-based PC's offers benchmarks approaching minicomputer and mainframe performance; (2) Ability to handle large design models of several hundred degrees of freedom, if necessary; and (3) Broad extensibility through user-defined functions. To characterize MATLAB capabilities, a simplified design example is presented. This involves interactive definition of an observer-based state-space compensator for a flexible missile autopilot design task. MATLAB capabilities and limitations, in the context of this design task, are then summarized.

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**FLEXIBLE MISSILE AUTOPILOT DESIGN STUDIES  
WITH PC-MATLAB/386**

**Michael J. Ruth  
Johns Hopkins University / Applied Physics Laboratory  
Laurel, Maryland**

**Workshop on Computational Aspects in the Control  
of Flexible Systems**

**12-14 July 1988  
Williamsburg, Virginia**

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## **PRESENTATION OVERVIEW**

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- 1. Introduction**
- 2. MATLAB Background**
- 3. Characteristics of MATLAB Environment**
- 4. Classical Control Capabilities**
- 5. Modern Control Design Example**
- 6. Summary**

## INTRODUCTION

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- **JHU/APL acts as technical direction agent for US Navy weapon system programs**
- **A key task of APL's Guidance, Control, and Navigation Systems Group is the evaluation or conceptual design of missile guidance and control systems**
- **Analysis and design work requires a flexible, interactive linear modeling tool**
- **PC-MATLAB resident on 80386 engineering workstations provides such a tool**
- **Work presented here shows general attributes of MATLAB, demonstrating use of PC-MATLAB/386 for linear design of a flexible missile autopilot**

## MATLAB BACKGROUND

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- **MATLAB (MATrix LABoratory) provides an interactive, matrix-oriented environment**
- **MATLAB is based on the EISPACK and LINPACK routines for matrix computations**
- **PC-MATLAB/386 is a high-performance MATLAB implementation for 80386-based workstations**
- **MATLAB built-in functions, plus higher-level functions developed for control system calculations, allow for effective controls design studies**

## **HARDWARE AND SOFTWARE CONFIGURATION**

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- **COMPAQ 386/20 computer**
- **Weitek 1167 numeric coprocessor**
- **PC-MATLAB/386 with Control Systems Toolbox**

## **PC-MATLAB/386 ATTRIBUTES**

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- **Interactive, high-level command environment**
- **Very high processing speed**
- **Easy extensibility via user-defined functions**

## A MATLAB INTERACTIVE COMMAND LINE EXAMPLE

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```
>> k = lqr(a,b,q,rho*r); eig(a-b*k), y = step(a-b*k,b,c,d,1,t); plot(t,y);
```

- The single line above, typed at the MATLAB command line prompt, does several things:
  - Computes a quadratic regulator gain vector
  - Displays the closed-loop eigenvalues -- often useful for confirming that actuator bandwidth requirements are not excessive
  - Computes and plots a unit step response
  
- By varying the control cost ( $\rho$ ) above, a very large family of compensators may quickly be considered
  
- The above command line suggests the power and utility available from a high-level, interactive matrix language

## **PC-MATLAB/386 PROCESSING SPEED**

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- **MATLAB's LINPACK Benchmark: 460 double precision KFLOPS**
  
- **This processing speed is:**
  - **25 x faster than standard PC/AT**
  - **6 x faster than Mac II**
  - **3 x faster than MicroVax II**
  
- **Implication: the fast response time resulting from such performance allows for truly interactive design iterations on complex control laws**

## MATLAB EXTENSIBILITY

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- **User-defined functions may be developed through creation of simple text files**
  
- **Some typical user-defined functions:**
  - **Frequency-response plotting routines**
  - **Application-specific linear transformations**
  - **Multivariable Nyquist criterion**
  
- **Complex state-space or transfer-function models also defined through user text files**

## AN EXAMPLE OF A USER-DEFINED COMMAND FILE

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- Below command set calculates and plots the maximum and minimum singular values of a plant and observer-based compensator, for a loop broken at plant input

```
function [smin,smax] = svdinput(a,b,c,kcon,kobs,w);
%
jay = sqrt(-1);
[nn,xx]=size(a); i2=eye(nn); [ng,xx]=size(c*a*b); phi = '(s*i2-a)';
for i = 1:nc;
    s = w(i)*jay; phieval = eval(phi);
    gs = c/phieval*b; ks = kcon / (phieval+b*kcon+kobs*c) * kobs;
    xx=svd(ks*gs); smin(i)=xx(ng); smax(i)=xx(1);
end;
%
%
% convert to decibels and plot output
%
smin=20*log10(smin); smax=20*log10(smax);
semilogx(w,smin,w,smax,'r--'); grid;
title('Max and Min Singular Values; Loop Broken at Plant Input ');
xlabel('Frequency (rad/sec)'); ylabel('Magnitude (db)');
```

- Procedure requires only eleven lines of executable MATLAB code

## CLASSICAL CONTROL CAPABILITIES

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- **Frequency response**
- **Root locus**
- **Nyquist plots**
- **Development of dynamic compensators  
(lead-lag, notch filters, etc)**

## MODERN CONTROL DESIGN EXAMPLE

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- **Design plant describes tactical missile at a high-altitude flight condition**
- **Design plant includes single-plane rigid-body dynamics and effect of first flexible mode on sensed pitch rate**
- **Objective is to develop an autopilot to track commanded accelerations**
- **Design challenge is to achieve high closed-loop bandwidth in presence of low-frequency bending modes**

## DESIGN APPROACH

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- **Establish design goals for closed-loop responsiveness and stability**
- **Develop full-state feedback (LQR) gains for design plant**
- **Define linear observer to reconstruct full state vector**
  - **Use "robust observer" design (Doyle and Stein, 1979 IEEE Transactions on Automatic Control)**
  - **Adjust observer gains to recover original LQR loop transfer in desired frequency range**

## DESIGN PLANT MODEL

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- Fifth-order state vector  $\underline{x}$ ;  $\dot{\underline{x}} = \underline{A}\underline{x} + \underline{b}u$
- $\underline{x} = [q_r \quad q_r/s \quad a/s \quad q_f/s \quad q_f]$
- First three state variables are associated with rigid-body airframe; the last two describe flexible mode dynamics
- Rate gyro measurement:  $[1 \ 0 \ 0 \ 0 \ 1] * \underline{x}$
- (Integrated) accelerometer measurement:  $[0 \ 0 \ 1 \ 0 \ 0] * \underline{x}$

$$\underline{A} = \begin{bmatrix} 0 & -2.3557\text{e}+02 & 1.7967\text{e}+02 & 0 & 0 \\ 1.0000\text{e}+00 & 0 & 0 & 0 & 0 \\ 0 & 2.6158\text{e}+00 & -1.9951\text{e}+00 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000\text{e}+00 \\ 0 & 0 & 0 & -2.4649\text{e}+04 & -3.1400\text{e}+00 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} -2.8031\text{e}+02 \\ 0 \\ 9.2587\text{e}-02 \\ 0 \\ 3.0723\text{e}+02 \end{bmatrix}$$

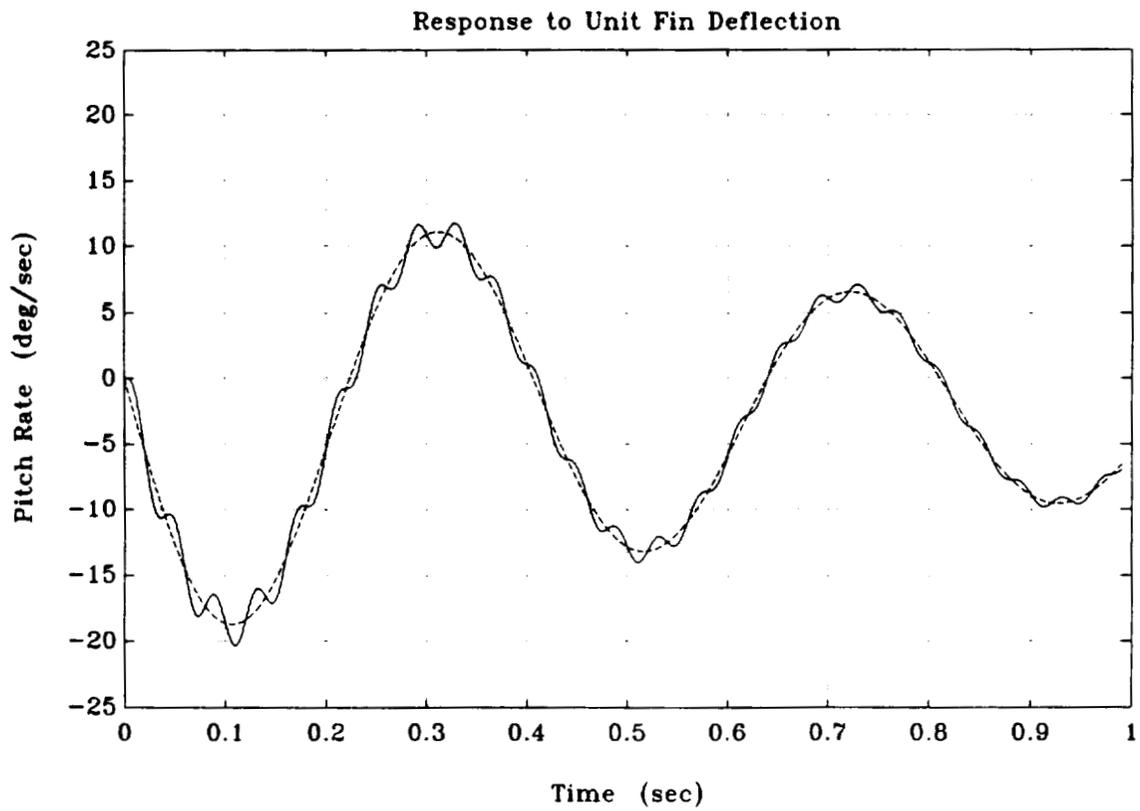
## **SOME OBSERVATIONS ON DESIGN PLANT MODEL**

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- **Feedback of the first three states describes a very standard (rigid-body) autopilot topology, used by tactical missiles since 1950's**
  
- **Open-loop plant is characterized by lightly damped airframe (weathercock) poles, and by bending mode poles**
  - **Airframe pole frequency lies at nominal 2.5 Hz**
  
  - **Bending mode has nominal 25 Hz natural frequency**
  
- **Desired autopilot crossover frequency here will lie near the bending mode frequency**

## EFFECT OF STRUCTURAL MODE ON SENSED PITCH RATE (RATE GYRO MEASUREMENT)

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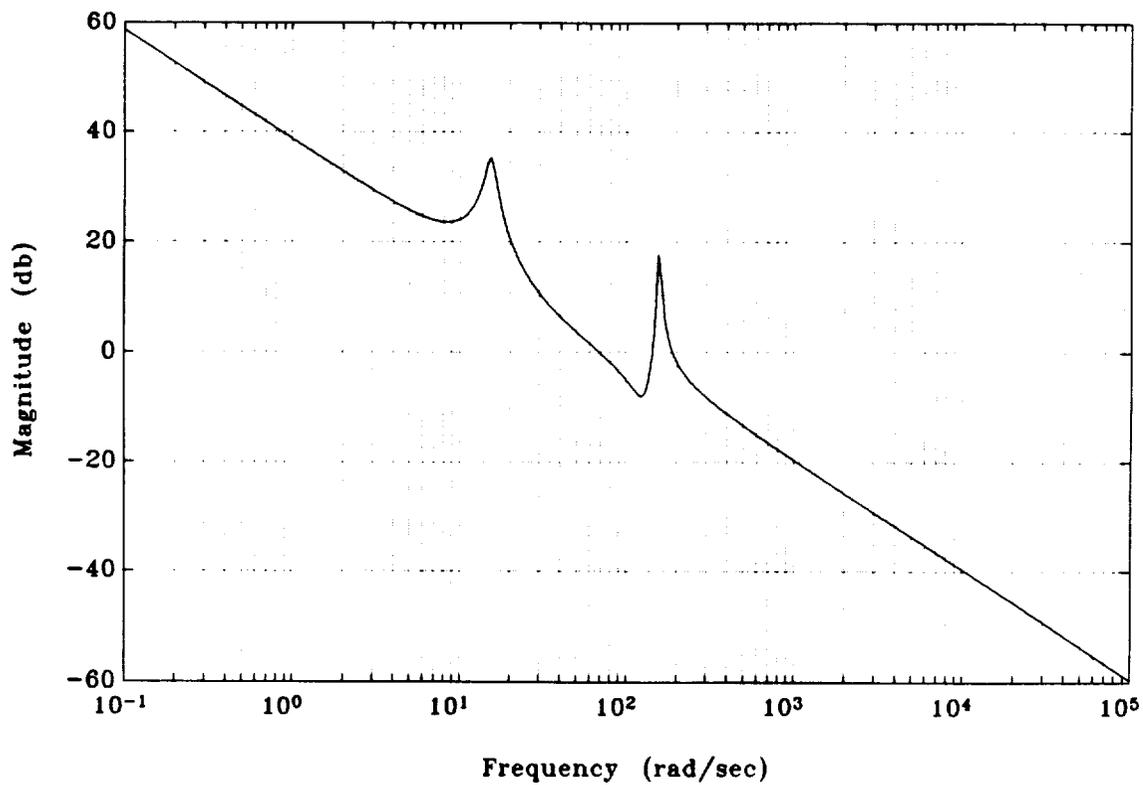
## CONTROLLABILITY AND OBSERVABILITY PROPERTIES OF PLANT

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- **System (A,b) is controllable**
- **System is unobservable if rate gyro alone, or accelerometer alone, is used as the measurement to reconstruct state vector**
- **Both sensor outputs thus should be used in the observer design**
- **Approach taken for this application:**
  - **Define a (non-square) design plant having one input (fin deflection) and two independent outputs (gyro and accelerometer)**
  - **Use extensions of loop transfer recovery (Williams and Madiwale, 1985 ACC) valid for non-square systems**

## FREQUENCY RESPONSE OF FULL-STATE FEEDBACK (LQR) SYSTEM (LOOP BROKEN AT PLANT INPUT)

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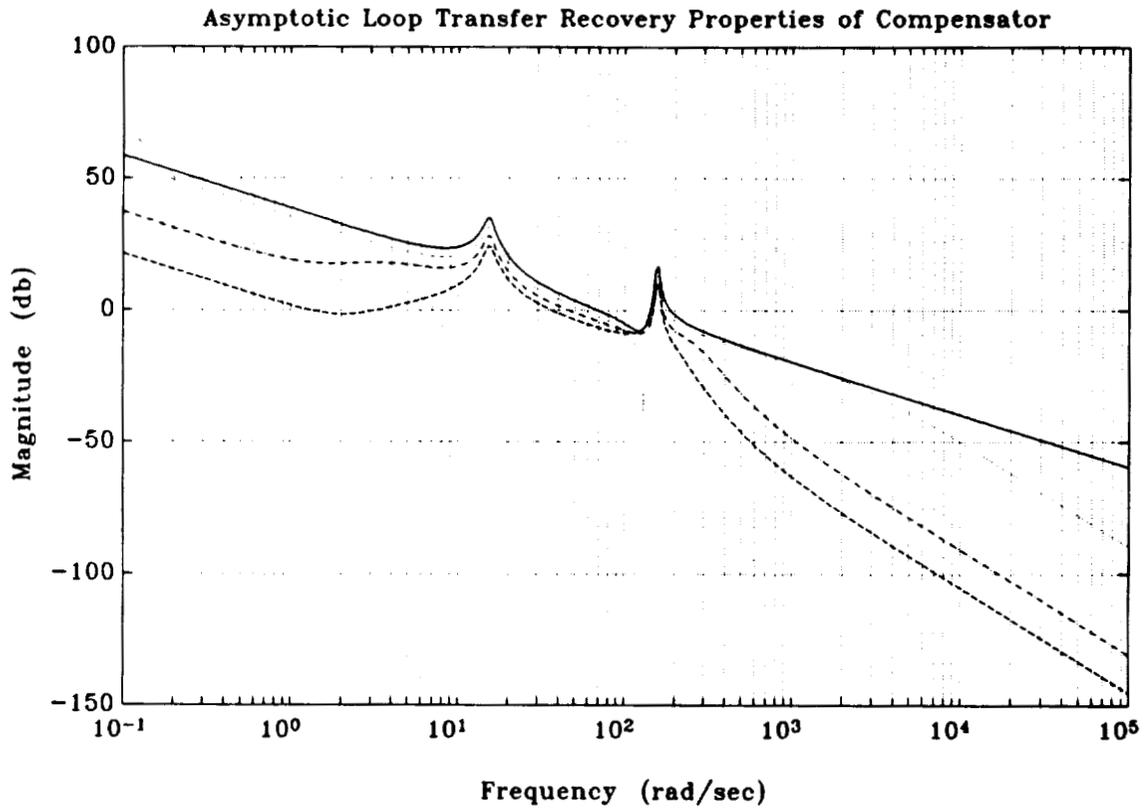
## OBSERVATIONS ON LOOP TRANSFER RECOVERY PROCEDURE

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- For this application, recovery at both the (rigid-body) airframe and bending mode frequencies may only be achieved with very high observer gains
- For practical ranges of observer gains, recovery at airframe frequencies is obtained at the cost of lessened robustness in the structural mode frequency range
- Use of a set of user-defined MATLAB files, to implement a range of observer gain calculations, makes evaluation of this robustness tradeoff straightforward

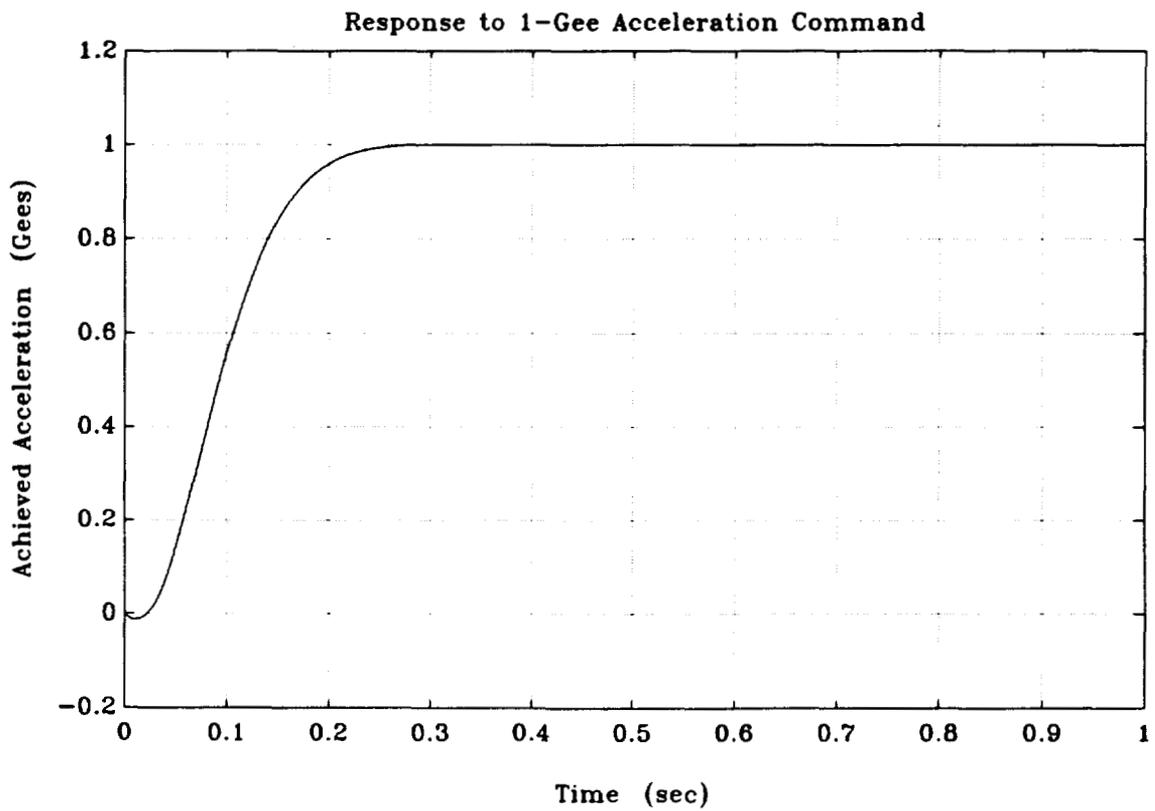
# RECOVERY OF DESIRED FULL-STATE FEEDBACK SYSTEM WITH MODEL-BASED COMPENSATOR

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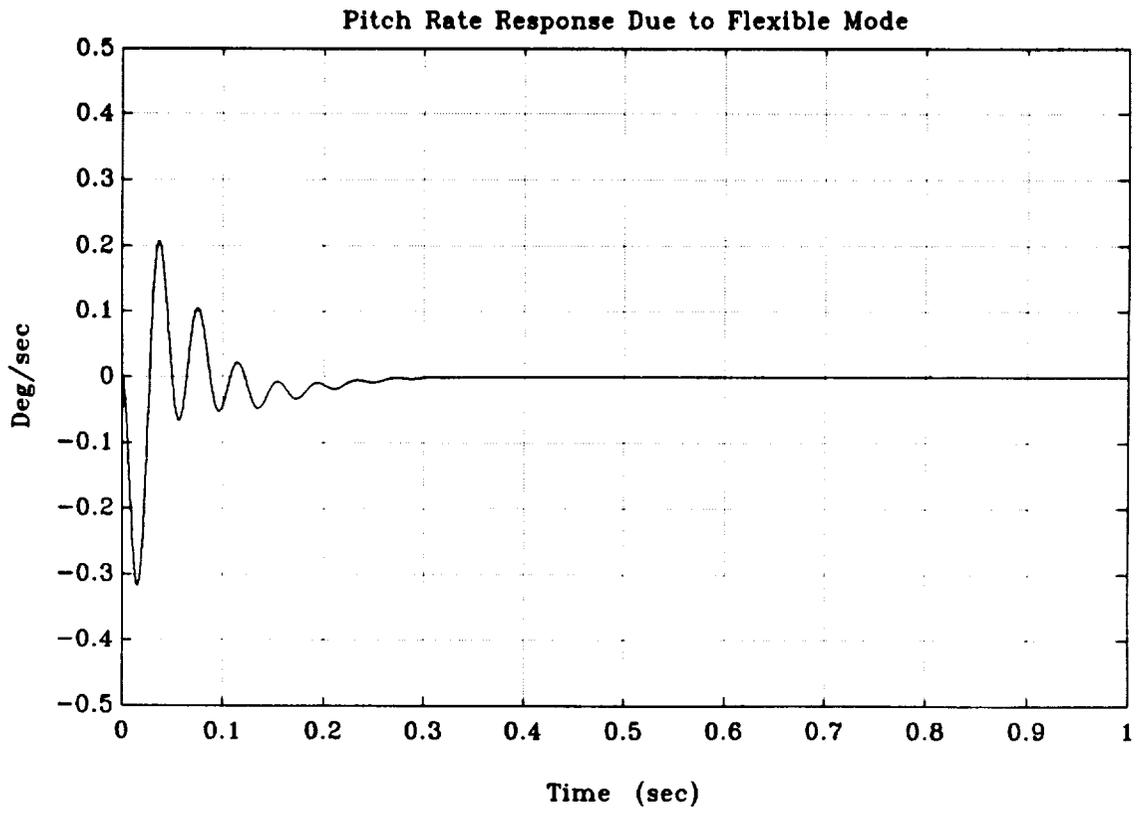
## ACCELERATION STEP RESPONSE OF FINAL COMPENSATOR DESIGN

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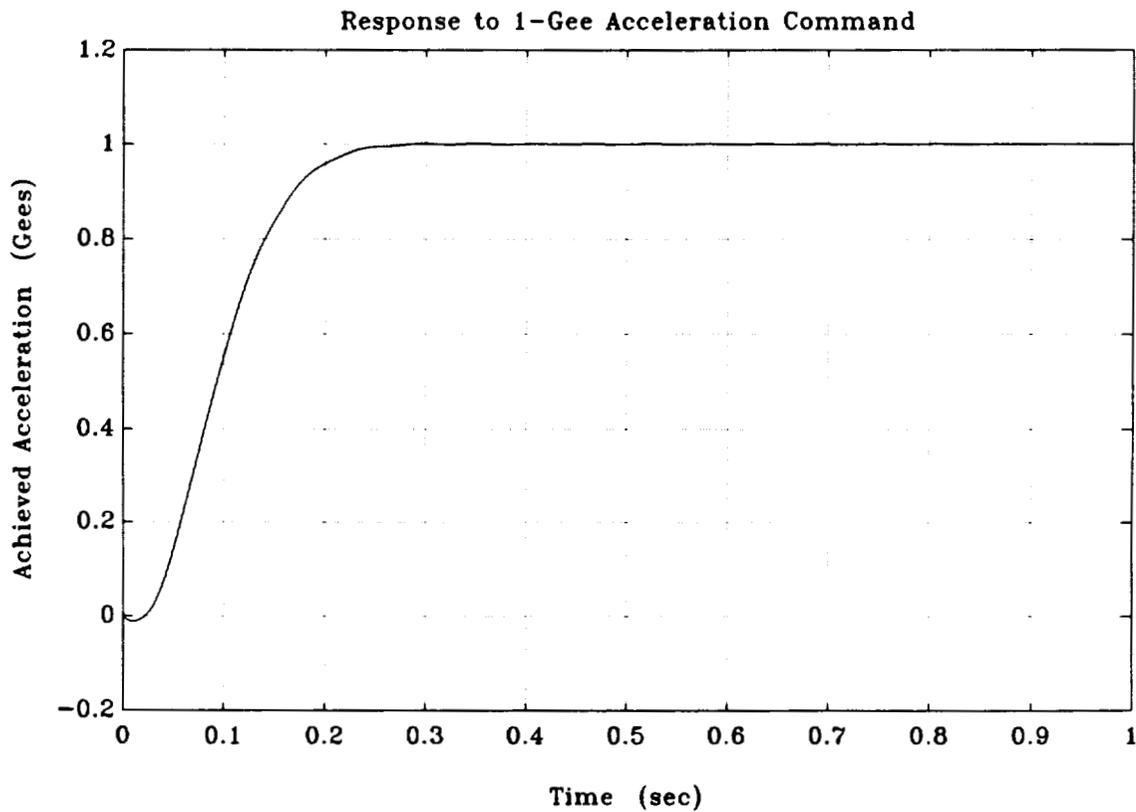
## RESPONSE OF FLEXIBLE MODE STATE DURING ACCELERATION STEP RESPONSE

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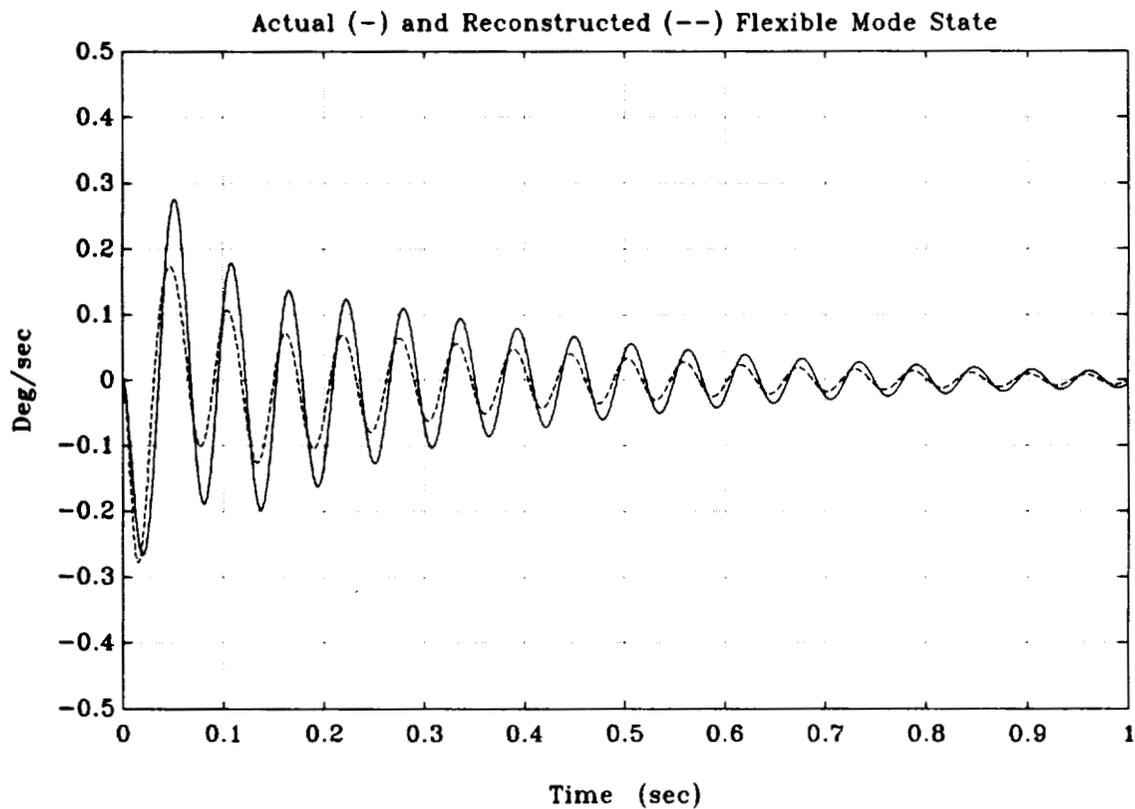


## ACCELERATION STEP RESPONSE FOR CASE WHEN BENDING MODE IS PERTURBED TO 25 % LOWER VALUE

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**COMPARISON OF ACTUAL AND RECONSTRUCTED FLEXIBLE  
MODE STATE DURING STEP RESPONSE -- BENDING MODE  
PERTURBED TO 25 % LOWER VALUE**



## SUMMARY OF DESIGN RESULTS

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- **Model-based compensator yields a high-bandwidth autopilot, which is robust to at least a 25% perturbation in bending mode frequency**
  
- **A number of issues still not addressed:**
  - **Detailed noise sensitivity assessment**
  - **Effect of higher-frequency structural modes**
  - **Phase lag from actuator dynamics**
  - **Effect of structural modes on accelerometer measurement**
  - **Tolerance to uncertainties in aerodynamics**
  
- **Above concerns could also be addressed using MATLAB**

## **SUMMARY: MATLAB APPLICABILITY FOR CONTROL DESIGN OF FLEXIBLE SYSTEMS**

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- **MATLAB provides the necessary tools for a variety of control system design techniques**
- **Extensibility of MATLAB allows development of tools to implement recent modern control design methods, including loop transfer recovery**
- **Implementation for 80386-based machines (PC-MATLAB/386) has very high performance, allowing for interactive control design of complex systems such as flexible structures**
- **Any flexible structures control problem which can be cast into a state-space framework may benefit from design work with MATLAB**

N90-10089

**DYSCO - A SOFTWARE SYSTEM FOR MODELING GENERAL DYNAMIC SYSTEMS**

By

Alex Berman  
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**ABSTRACT**

The DYSCO program has been under development since 1979. It has been funded by Army and Air Force laboratories and by the Kaman Aerospace Corporation. It is presently available at a number of government and nongovernment installations. It has been used to analyze a very broad range of dynamics problems.

A principle feature of the software design of DYSCO is the separation of the executive from the technology. The executive, which controls all the operations, is "intelligent" in the sense that it "knows" that its function is to assemble differential equations and to prepare them for solution. The "technology library" contains FORTRAN routines which perform standard functions, such as, computing the equation coefficients of an element (or "component") given the local state at any time. The technology library also contains algorithms and procedures for solving the coupled system equations.

The system was designed to allow easy additional of technology to the library. Any linear or nonlinear structural entity, control system, or set of ordinary differential equations may be simply coded and added to the library, as well as algorithms for time or frequency domain solution.

The program will be described with emphasis on its usefulness in easily modeling unusual concepts and configurations, performing analysis of damage, evaluating new algorithms, and simulating dynamic tests. Illustrations of several typical and illustrative applications will be presented. A summary of the technology presently residing in the technology libraries at the various sites will also be given.

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**KAMAN**  
AEROSPACE  
CORPORATION

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**DYSCO - A SOFTWARE SYSTEM  
FOR MODELING GENERAL DYNAMIC SYSTEMS**

**ALEX BERMAN  
KAMAN AEROSPACE CORPORATION**

**WORKSHOP ON COMPUTATIONAL ASPECTS IN THE CONTROL OF FLEXIBLE SYSTEMS  
WILLIAMSBURG, VIRGINIA, JULY 12-14, 1988**

- o DYNAMIC SYSTEM COUPLER (DYSCO)
- o INITIAL DEVELOPMENT - 1979
- o FUNDED BY ARMY, AIR FORCE, KAMAN
- o PRESENTLY OPERATIONAL ON IBM AND VAX
- o SIZE - 50000+ LINES OF CODE  
350+ SUBROUTINES  
4+ MEGABYTES OF STORAGE

DYSCO COUPLES AND SOLVES SECOND ORDER ODE

o  $M_I \ddot{X}_I + C_I \dot{X}_I + K_I X_I = F_I$  (COMPONENT I)

o  $X_I = T_I X_S$

o  $M_S \ddot{X}_S + C_S \dot{X}_S + K_S X_S = F_S$  (SYSTEM)

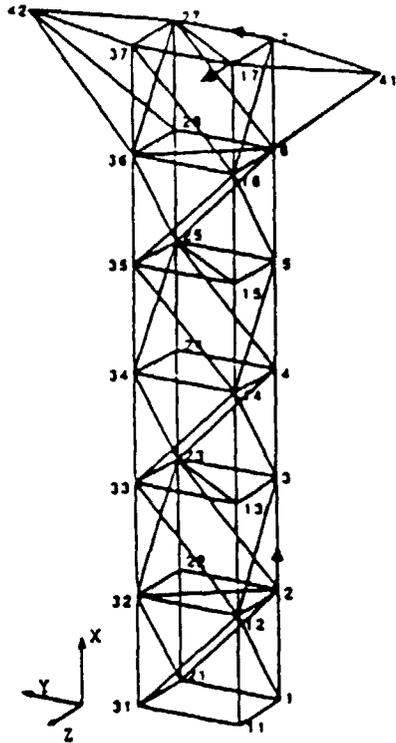
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DEFINITION OF COMPONENT

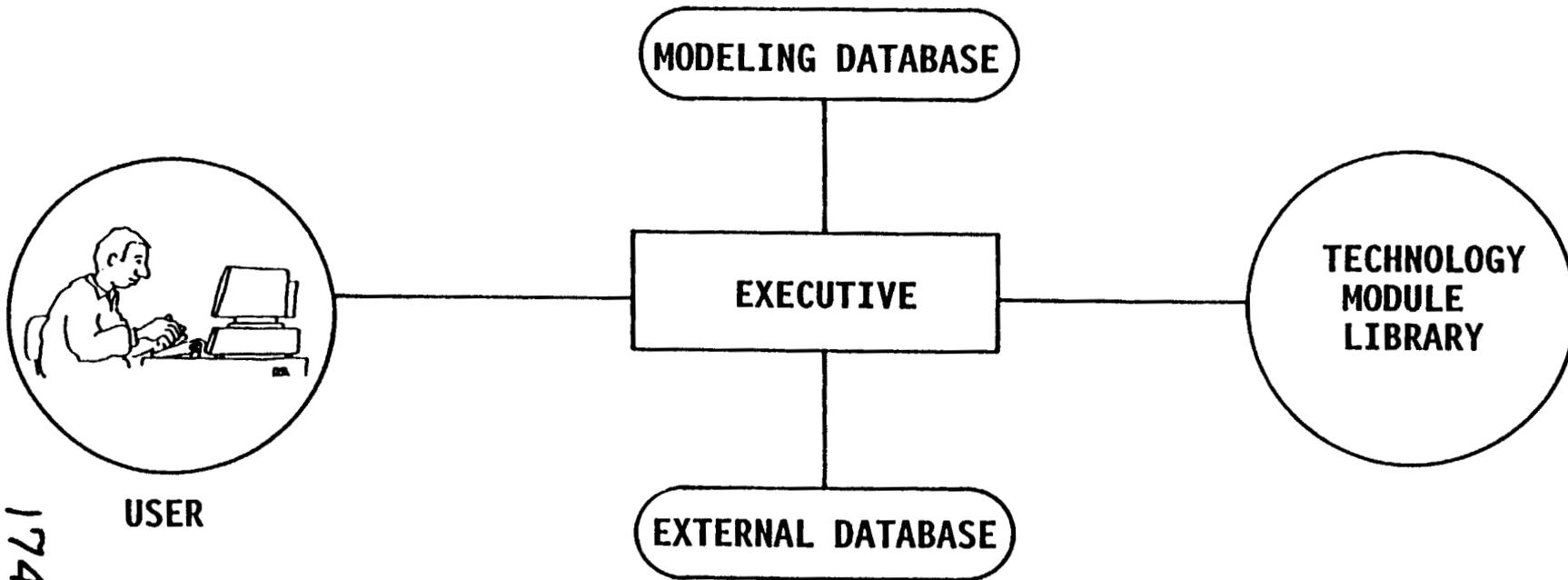
- o "COMPONENT" IS MORE GENERAL THAN "FINITE ELEMENT"
- o  $M_I, C_I, K_I, F_I =$  ARBITRARY FUNCTIONS OF STATE
- o  $X_I =$  ANY GENERALIZED DOF - PHYSICAL, MODAL, OTHER
- o COMPONENT MAY BE
  - FINITE ELEMENT
  - ASSEMBLY OF FINITE ELEMENTS (SUBSYSTEM, OUTPUT OF FE ANALYSIS)
  - SPECIAL SET OF EQUATIONS (E.G., HELICOPTER ROTOR, SPECIAL MECHANISM)
  - CONTROL ALGORITHM (MIMO, NON-SYMMETRICAL MATRICES, NONLINEAR)
  - FORCE ALGORITHM ( $M, C, K =$  NULL, AERO, ELECTROMAGNETIC)
  - ETC., ETC.

- o A MODEL IS A DESCRIPTION OF A COUPLED SET OF COMPONENT EQUATIONS
- o COMPONENT EQUATIONS ARE DEFINED BY
  - NAME OF THE ALGORITHM IN "TECHNOLOGY LIBRARY"
  - NAME OF DATA SET IN "MODELING DATABASE"
- o COMMAND "RUN" COUPLES EQUATIONS
- o NEXT STEP IS TO SPECIFY SOLUTION ALGORITHM

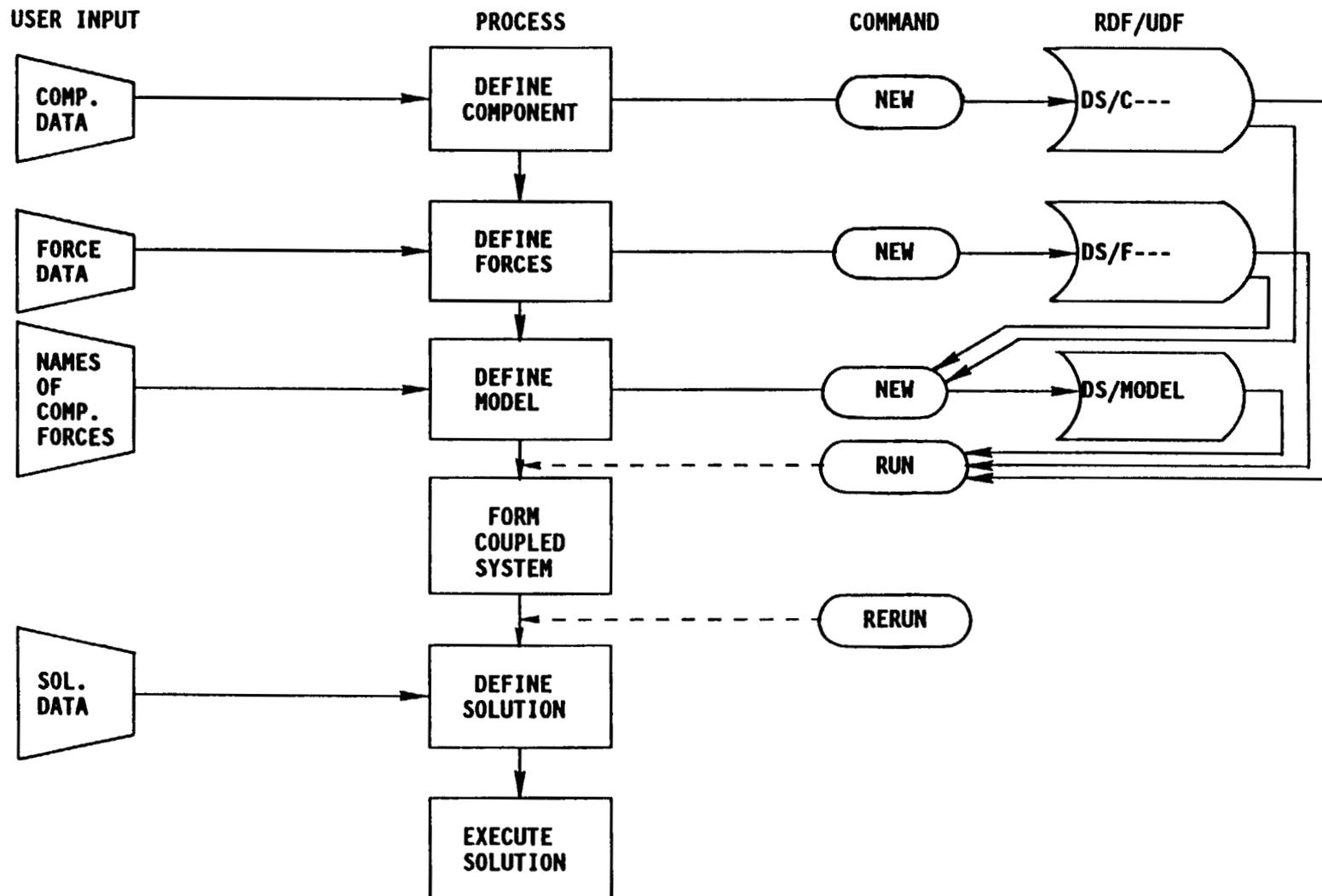
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<u>COMPONENT</u>	<u>NO.</u>	<u>DATA SET</u>	
1	CTR4	1	ABCD1
2	CTR4	3	ABCD1
3	CTR4	5	ABCD1
4	CTR4	2	ABCD2
5	CTR4	4	ABCD2
6	CTR4	6	ABCD2
7	CSF1		TOPR
8	CSF1		TOPL
9	CSF1		CONTR
10	CLC1		GROUND



MODELING SCENARIO AND COMMAND RELATIONSHIP



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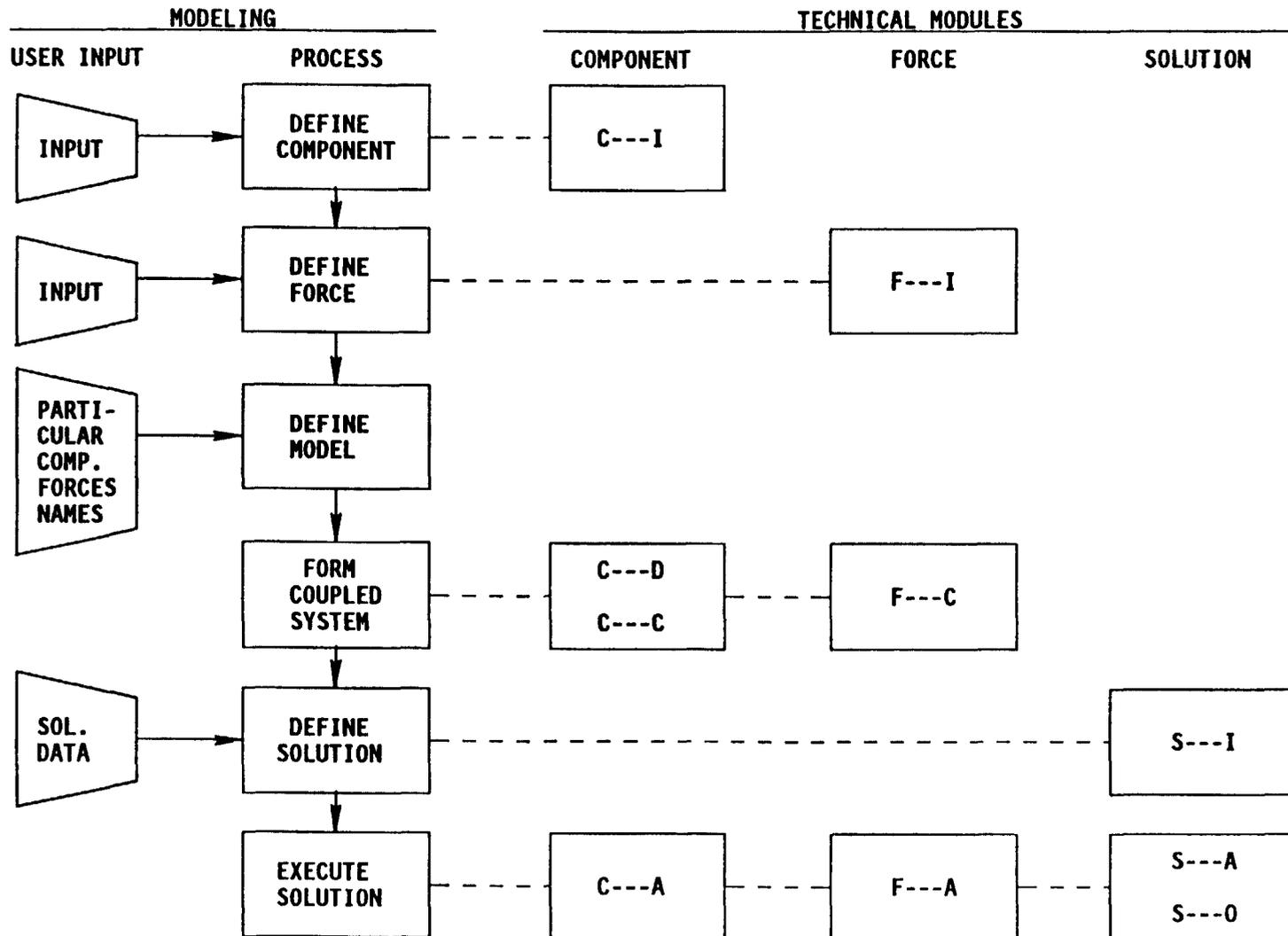
STANDARD TECHNICAL MODULES

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<u>FUNCTION</u>	<u>COMPONENT</u>	<u>FORCE</u>	<u>SOLUTION</u>
INPUT	C---I	F---I	S---I
DEFINITION	C---D	N/A	N/A
COEFFICIENT	C---C	F---C	N/A
ACTIVE	C---A	F---A	S---A
OUTPUT	N/A	N/A	S---0
LOADS	C---L	N/A	N/A

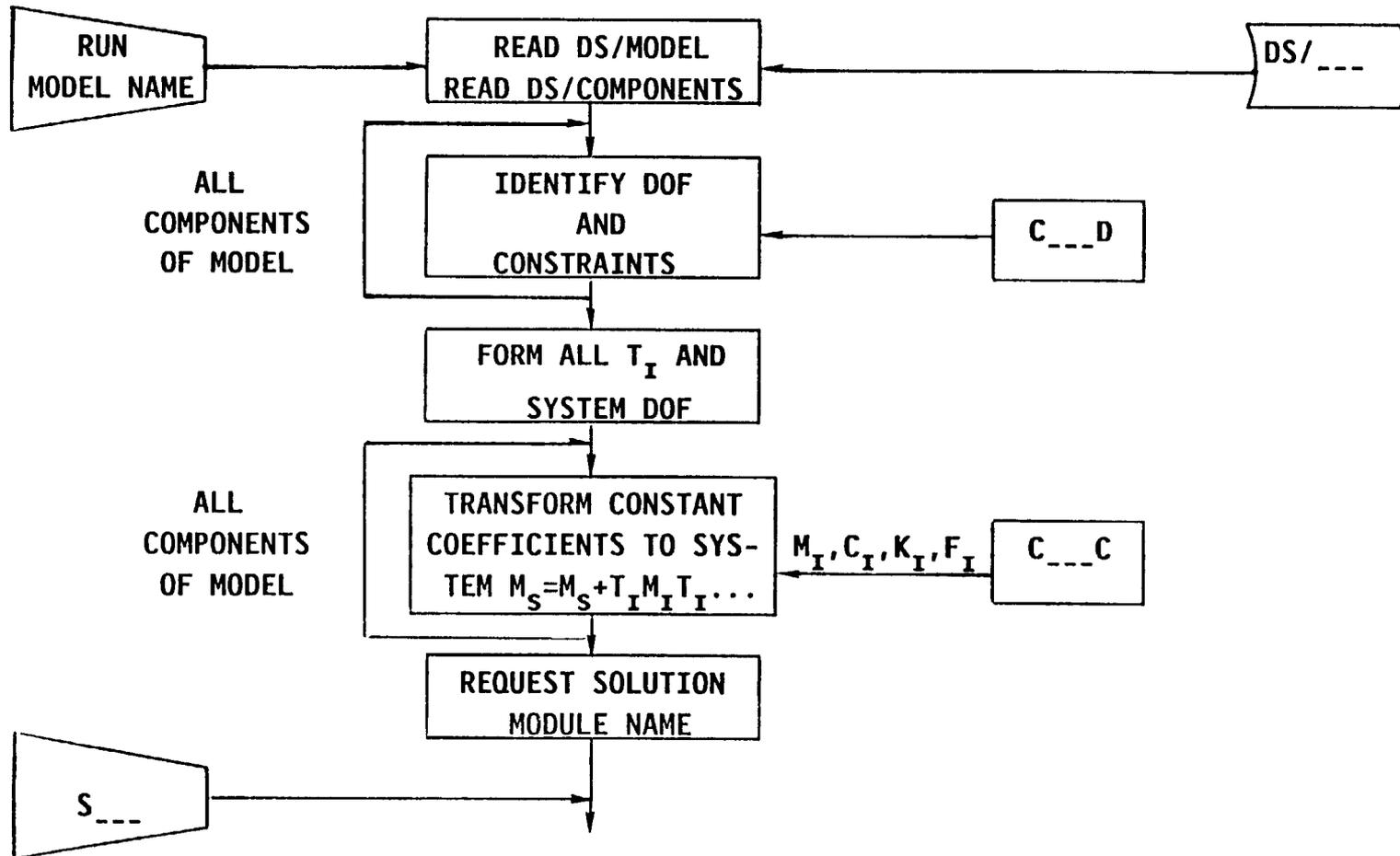
- I      INPUT DEFINITION
- D      DEFINE DEGREES OF FREEDOM
- C      COMPUTE CONSTANT COEFFICIENTS IN EQUATIONS
- A      COMPUTE NON-CONSTANT COEFFICIENTS, FUNCTION  
            OF TIME AND STATE
- O      OUTPUT
- L      INTERNAL LOADS, FUNCTION OF STATE

RELATIONSHIP BETWEEN MODELING  
SCENARIO AND TECHNICAL MODULES



**RUN COMMAND  
(ASSEMBLY OF MODEL)**

USER INPUT                      EXECUTIVE FUNCTION                      TECH. LIBRARY                      DATA LIBRARY

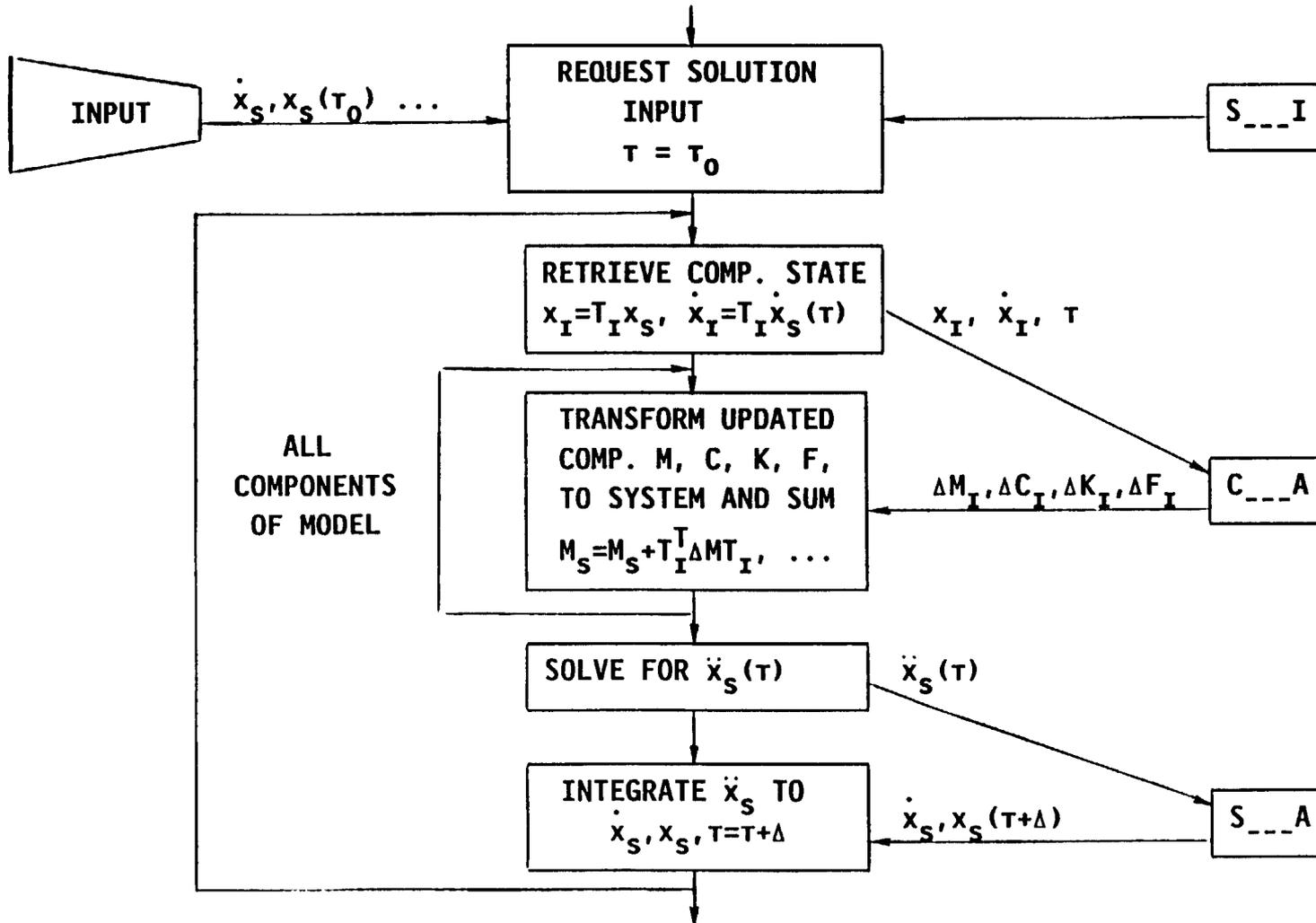


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USER INPUT

EXECUTIVE FUNCTION

TECH. LIBRARY



FEATURES OF EXECUTIVE

- o EXECUTIVE IS SPECIFICALLY BUILT TO MANAGE STRUCTURAL DYNAMIC ANALYSIS
- o IT UNDERSTANDS AND MANAGES
  - INPUT: IDENTIFICATION, STORAGE, EDITING
  - MODEL BUILDING: RETRIEVAL OF DATA, CALLS TO TECHNOLOGY LIBRARY
  - ASSEMBLY OF EQUATIONS: APPLIES MPC, SPC
  - SOLUTION OF EQUATIONS: CALLS TO TECHNOLOGY LIBRARY, RETRIEVAL OF LOCAL STATES, INTERFACE LOADS
- o EXECUTIVE INDEPENDENT OF ANY PARTICULAR AREA OF TECHNOLOGY
  - UNIFORM ABSTRACT INTERFACES TO TECHNOLOGY LIBRARY

- o NEW TECHNOLOGY EASILY ADDED
  - COMPONENT, FORCE, SOLUTION
  - UNIFORM INTERFACES TO EXECUTIVE
  - FORTRAN CODING
  
- o COMPONENTS ARE ANY SECOND ORDER ODE, SUCH AS,
  - SINGLE SPRING, DAMPER, OR MASS
  - ANY FINITE ELEMENT
  - COMPLETE NASTRAN MODEL
  - HELICOPTER ROTOR
  - MIMO CONTROL ALGORITHM
  
- o SOLUTIONS ACT ON MODEL EQUATIONS, E.G.
  - EIGENANALYSIS
  - FREQUENCY RESPONSE
  - TIME HISTORY
  - HELICOPTER TRIM (PERIODIC SHOOTING)
  - PERIODIC SYSTEM STABILITY
  - STATE FEEDBACK OPTIMIZATION

OTHER FEATURES

- o **VALIDATED INPUT AND EDITING**
  - **USES KNOWLEDGE TABLE: TYPE, CHARACTERISTICS, EXISTENCE, RANGE**
  - **PROMPTED INPUT**
  - **INSTANTANEOUS VALIDATION**
  - **ASSURED COMPLETE AND CONSISTENT DATA**
  
- o **SIMPLE EDITING OF MODEL**
  - **CONFIGURATION CHANGES**
  - **PARAMETER VARIATION**
  - **DAMAGE ANALYSIS**
  
- o **INTELLIGENT COUPLING PROCEDURES**
  - **RECOGNITION OF DOF NAMES**
  - **MPC OPTIONALLY AUTOMATICALLY FORMED**
  - **GENERAL MPC SOLVED FOR DOF EQUATIONS**

- o CSF1 - LINEAR FINITE ELEMENT  
USER SUPPLIES: NAMES OF DOF  
M, C, K, F
- o CFM3 - 3D MODAL STRUCTURE  
RIGID BODY, ELASTIC MODES (ALL OPTIONAL)  
DOF NAMES AUTOMATICALLY GENERATED  
AUTOMATIC COUPLING AT SPECIFIED NODES
- o CSB2 - GENERAL BAR ELEMENT\* (NOT AVAILABLE IN GOVT VERSION)  
MAY BE USED AS A BEAM OR ROD ELEMENT  
SHEAR FACTORS, CONSISTENT MASS, RAYLEIGH DAMPING  
UP TO 12 DOF

- o CES1 - ELASTIC STOP  
NONLINEAR SPRING, DAMPING, WITH GAP
- o CGF2 - GENERAL FORCE
  - POLYNOMIAL, FOURIER SERIES, OR TABULAR
  - PERIODIC
- o CLC0 - SINGLE POINT CONSTRAINTS
- o CLC1 - MULTIPOINT CONSTRAINTS
- o CLC2 - ADVANCED MULTIPOINT CONSTRAINT

- o SEA4 - EIGENANALYSIS, REAL
- o SEA5 - COMPLEX EIGENANALYSIS
- o STH4 - TIME HISTORY  
- CONDITION CODES
- o SFD1 - FREQUENCY DOMAIN MOBILITY  
- RESPONSE PER UNIT FORCE
- o STCO - OPTIMIZER FOR LINEAR STATE FEEDBACK\* (NOT  
AVAILABLE IN GOVT VERSION)  
- SOLVES MATRIX RICCATI EQUATION  
- INTEGRATES SYSTEM STATE EQUATIONS
- o SII3 - INTERFACE AND INTERNAL LOADS  
- RESIDUAL FORCES AT INTERFACES  
- FORCES, STRAIN ENERGY, BENDING MOMENTS

DYSCO - OTHER TECHNOLOGY MODULES

- o CRR2, CRR3 - HELICOPTER ROTOR
- o CCE0, CCE1 - ROTOR CONTROL SYSTEM
- o CRD3 - ROTOR DAMAGE
- o CFM2 - HELICOPTER FUSELAGE
- o CLG2 - NONLINEAR LANDING GEAR
- o CLS2 - LIFTING SURFACE
- o FRA0, FRA2, FRA3 - ROTOR AERODYNAMICS
- o FFA0, FFC2 - FUSELAGE AERODYNAMICS
- o STH3 - TIME HISTORY, HELICOPTER CONTROLS
- o STR3 - HELICOPTER TRIM
- o SSF3 - FLOQUET STABILITY

- o DYSCO DOES NOT COMPETE WITH FE CODES
- o DYSCO COMPLEMENTS FE CODES
- o FE ANALYSIS FOR DETAILED STRUCTURAL ANALYSIS
- o DYSCO CAN START WITH FE MODEL AND:
  - MODIFY CONFIGURATION
  - SIMULATE DAMAGE
  - ADD CONTROL ALGORITHMS
  - ADD SPECIAL COMPONENTS
  - PERFORM SOLUTIONS ON ALL MODIFICATIONS
  - STUDY EFFECTS OF CHANGE
  - ANALYZE CONFIGURATIONS NOT POSSIBLE (OR CONVENIENT) WITH FE CODES
- o DYSCO CAN ALSO MODEL STRUCTURES ON ITS OWN

- o SIMPLE PROBLEMS ARE EASY AND INEXPENSIVE TO SOLVE
- o PROBLEMS NOT CONVENIENTLY MODELED ELSEWHERE CAN BE SOLVED
- o PHENOMENA CAN BE BETTER UNDERSTOOD:
  - START WITH SIMPLE REPRESENTATION
  - GRADUALLY INCREASE COMPLEXITY
  - VARY PARAMETERS
  - VARY CONFIGURATION
- o NOVEL CONCEPTS CAN BE EASILY MODELED AND EVALUATED
- o NEW ALGORITHMS CAN BE TESTED AND EVALUATED

LIST OF ILLUSTRATIVE PROBLEMS

- o PACOSS TOWER DYNAMIC ANALYSIS
- o TRUSS STRUCTURE WITH ACTIVE ELEMENTS - VIBRATION CONTROL
- o PIEZOELECTRIC SENSORS/ACTUATORS ON BEAM - VARY CONTROL LAWS, ADD ELASTIC STOP, STABILITY, TIME, FREQUENCY DOMAIN
- o POINTING-TRACKING SYSTEM - MOTOR DRIVEN MIRRORS - MOVING, ACCELERATING TARGET, VARY CONTROL GAINS
- o ROTORCRAFT TRIM - DAMAGED BLADE - INTERNAL LOADS
- o RAIL GUN PNEUMATIC ACCELERATOR - GAS PRESSURE - BOLT MOTION
- o ALGORITHM EVALUATION - REDUCED MODELS, SYSTEM IDENTIFICATION, SIMULATE EFFECTS OF MEASUREMENT ERRORS

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- o DYSCO COUPLES THE EQUATIONS OF INDIVIDUAL COMPONENTS TO FORM THE EQUATIONS OF A MODEL
- o EACH COMPONENT AND MODEL ARE OF THE FORM
$$M\ddot{X} + C\dot{X} + KX = F$$
$$M, C, K, F = F(T, \dot{X}, X)$$
- o M, C, K, F MAY BE ARBITRARY FUNCTIONS OF TIME OR STATE
- o X MAY REPRESENT PHYSICAL, MODAL, OR ANY GENERALIZED DOF
- o EACH COMPONENT IS REPRESENTED BY FORTRAN SUBROUTINES IN THE TECHNOLOGY LIBRARY

- o DYSCO USES AN "INTELLIGENT" PROCEDURE FOR COUPLING DEGREES OF FREEDOM
- o DEGREES OF FREEDOM OF COMPONENTS MAY BE:
  - PHYSICAL COORDINATES
  - MODAL DISPLACEMENTS
  - ANY GENERALIZED COORDINATES
- o COUPLING INCLUDES
  - PHYSICAL TO PHYSICAL
  - PHYSICAL TO MODAL
  - MODAL TO MODAL
  - SINGLE POINT CONSTRAINTS
  - MULTIPLE POINT CONSTRAINTS
  - ANY LINEAR RELATIONSHIPS
- o EFFECTS SIMULATED
  - RIGID PHYSICAL LINKAGES
  - OPTICAL BEAM COORDINATES
  - CONTROL ALGORITHMS

$X, X_I$  ARE VECTORS OF THE DOFS OF THE SYSTEM (MODEL) AND  
THE COMPONENTS

$T_I$  IS A TRANSFORMATION MATRIX

$$X_I = T_I X$$

THE EQUATION OF THE MODEL IS

$$MX + CX + KX = F$$

WHERE

$$M = \sum T_I^T M_I T$$

$$C = \sum T_I^T C_I T$$

$$K = \sum T_I^T K_I T$$

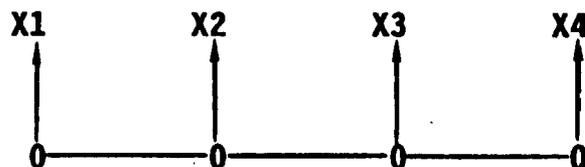
$$F = \sum T_I^T F_I$$

EACH  $T_I$  IS AUTOMATICALLY FORMED IN DYSCO

C-3

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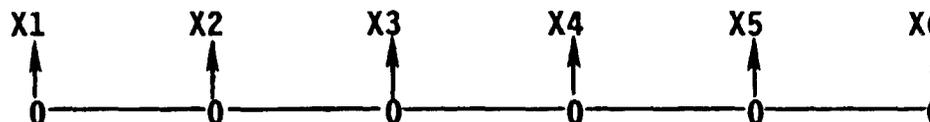
- o DYSCO USES A UNIQUE PROCEDURE WHERE VARIABLE NAMES (A4, I4) ARE RECOGNIZED AND PROCESSED
- o NAMES ARE AUTOMATICALLY FORMED OR USER SUPPLIED
- o LIKE NAMES IN COMPONENTS IMPLY CONNECTION
- o SIMPLE EXAMPLE:



TO CONSTRAIN X3 TO GROUND BY A SPRING,  $\kappa$ , USER SIMPLY ADDS COMPONENT TO MODEL WITH FOLLOWING INFORMATION

NO OF DOF = 1  
NAME = X3  
M = C = F = NULL  
K =  $\kappa$

SIMPLE CONTROL SYSTEM EXAMPLE



SENSOR LOCATIONS X1, X4, X6

ACTUATOR LOCATIONS X3, X4

$$F_{X3} = A \cdot X1 + B \cdot X4 + C \cdot X6 + D \cdot \dot{X4}$$

$$F_{X4} = E \cdot \dot{X1} + F \cdot X6 + G \cdot \dot{X6}$$

THIS MAY BE REPRESENTED BY COMPONENT WITH

$$\text{DOF} = [X1, X3, X4, X6]$$

$$M = 0$$

$$C = \begin{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \end{matrix} \\ - & \begin{matrix} 0 & 0 & D & 0 \\ E & 0 & 0 & G \\ 0 & 0 & 0 & 0 \end{matrix} \end{matrix}$$

$$K = \begin{matrix} & \begin{matrix} 0 & 0 & 0 & 0 \end{matrix} \\ - & \begin{matrix} A & 0 & B & C \\ 0 & 0 & 0 & F \\ 0 & 0 & 0 & 0 \end{matrix} \end{matrix}$$

SENSOR AND ACTUATOR LOCATIONS MAY BE CHANGED BY EDITING  
DOF NAMES. GAINS MAY BE CHANGED BY EDITING MATRICES.

- o COUPLING MAY ALSO USE OPTIONAL LINEAR RELATIONS
- o SINGLE POINT, MULTIPOINT, CONNECTIVITY CONSTRAINTS

$$\begin{aligned} X1 &= 1.0 * X2 \\ X10 &= 0 \end{aligned}$$

$$A * X10 + B * Y100 + C * Z20 = 0$$

- o REPRESENTATION OF MECHANICAL LINKAGES
- o CONVERSION TO CONVENIENT PARAMETERS
  - OPTICAL BEAM ANGLE AS FUNCTION OF MIRROR DOF
  - TIP DISPLACEMENT OF BEAM AS FUNCTION OF MODAL DOF

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DYSCO IS A DOMAIN EXECUTIVE CONTROL SYSTEM

THE DOMAIN IS "COUPLED DYNAMIC EQUATIONS"

IT EXECUTES TECHNICAL MODULES IN "PARALLEL"  
(RATHER THAN IN SEQUENCE)

SIMPLE COMMANDS PERFORM NUMEROUS MODULE  
EXECUTIONS (E.G., RUN)

INVALID COMMAND SEQUENCES ARE NOT ACCEPTED  
INVALID DATA USAGE IS NOT POSSIBLE

ALL DATA PLACED ON FILES OR EDITED IS ASSURED TO BE  
VALID (E.G., CONSISTENCY AND FORMAT)

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- o COMPONENT - ALGORITHM FOR COMPUTING M, C, K, F FOUND IN TECHNOLOGY LIBRARY

NAME: C---

- o DATA SET - SPECIFIC SET OF DATA TO BE USED WITH A COMPONENT FOUND IN DATA LIBRARY. INPUT BY USER. USER SUPPLIES DATA SET "NAME"
- o MODEL - COLLECTION OF COMPONENTS AND ASSOCIATED DATA SETS

SAMPLE MODEL

COMPONENT

CSF1  
CSF1  
CFM2  
CSF1  
CES1

DATA SET

STRUT1  
STRUT2  
FUSELAGE  
CONTROL 3  
K100

\*\*\*\*\* MODEL PATIE12 \*\*\*\*\*

CATSTAR: SAME AS PATIE11 BUT CLC1 REWRITTEN FOR PHIP1 IMPLICIT

INDEX	COMP	NO.	DATA SET	FORCE	DATA SET
1	CSF1		PRIMARY	NONE	
2	CSF1		BASE P-S	NONE	
3	CSF1		SECONDARY	NONE	
4	CSF1		BASE S-M	NONE	
5	CSF1		MOTOR	NONE	
6	CSF1		TRUSS-PR	NONE	
7	CSF1		TRUSS-DI	NONE	
8	CSF1		TRUSS-MO	NONE	
9	CFM2	.1	SHUTTLE	NONE	
10	CSF1		CONTROL	NONE	
11	CSF1		TARGET	NONE	
12	CSF1		NUMAL	NONE	
13	CLC1		OPTICS3	NONE	

\*\*\*\*\*

\*\*\*\*\*

GLOBAL VARIABLES

NO INPUT REQUIRED

\*\*\*\*\*

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```
*****          CONTROL1/CSF1          *****
CONTROL LAW 1 FOR GIMBALLED MIRROR

*****
INPUT FOR COMPONENT CSF1. FINITE ELEMENT

1 NCDF      - NUMBER OF DOF          =          3
2 CDFLI     - (DOF) DOF NAME
              PHIP1000  DSTR1000  MTH 1000
3 CM        - (REAL) MASS MATRIX VALUES
              NULL MATRIX

4 CC        - (REAL) DAMPING MATRX VALUES
              GENERAL MATRIX

ROW      1      NULL ROW
ROW      2
          3.70000E+02  0.00000E+00  0.00000E+00

ROW      3
          -3.70000E+02  0.00000E+00  0.00000E+00
5 CK       - (REAL) STIFFNESS MTRX VALUES
              GENERAL MATRIX

ROW      1      NULL ROW
ROW      2
          1.23300E+06  0.00000E+00  0.00000E+00
ROW      3
          -1.23300E+06  0.00000E+00  0.00000E+00
6 CF       - (REAL) FORCE VECTOR VALUES
          0.00000E+00  0.00000E+00  0.00000E+00
*****
```

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HELIUM1/CAG1  
HELIUM ACCUMULATOR FOR ET1 MODEL

\*\*\*\*\* HELIUM1 /CAG1 \*\*\*\*\*

HELIUM ACCUMULATOR FOR ET1 MODEL

\*\*\*\*\*  
INPUT FOR COMPONENT CAG1. ADIABATIC GAS

1 NCDF - NUMBER OF DOF = 3  
2 CDFLI - (DOF) DOF NAME  
PROJ1000 BOLT1000 MAGZ1000  
3 GVECT - (REAL) INITIAL GAS VECTOR  
4.55000E+03 4.00000E+00 1.66700E+00 2.07703E+00  
8.40000E+04  
4 AREA - (REAL) MATRIX FOR AREA CALC  
GENERAL MATRIX  
  
ROW 1  
2.00000E+00 2.00000E+00 3.00000E+00  
  
ROW 2  
1.00000E+00 1.00000E+00 6.66670E-01  
5 GOVDOF - (DOF) AREA EXIST CRITERIA  
PROJ1000 BOLT1000 BOLT1000  
6 AECV - (REAL) GOVDOF CRITICAL VALU  
-1.00000E+04 -1.00000E+04 1.00000E+00  
7 PECDOF - PRESSURE EX CRITERIA= PROJ1000  
8 PECVAL - CRITICAL VALUE = 7.75000E+00

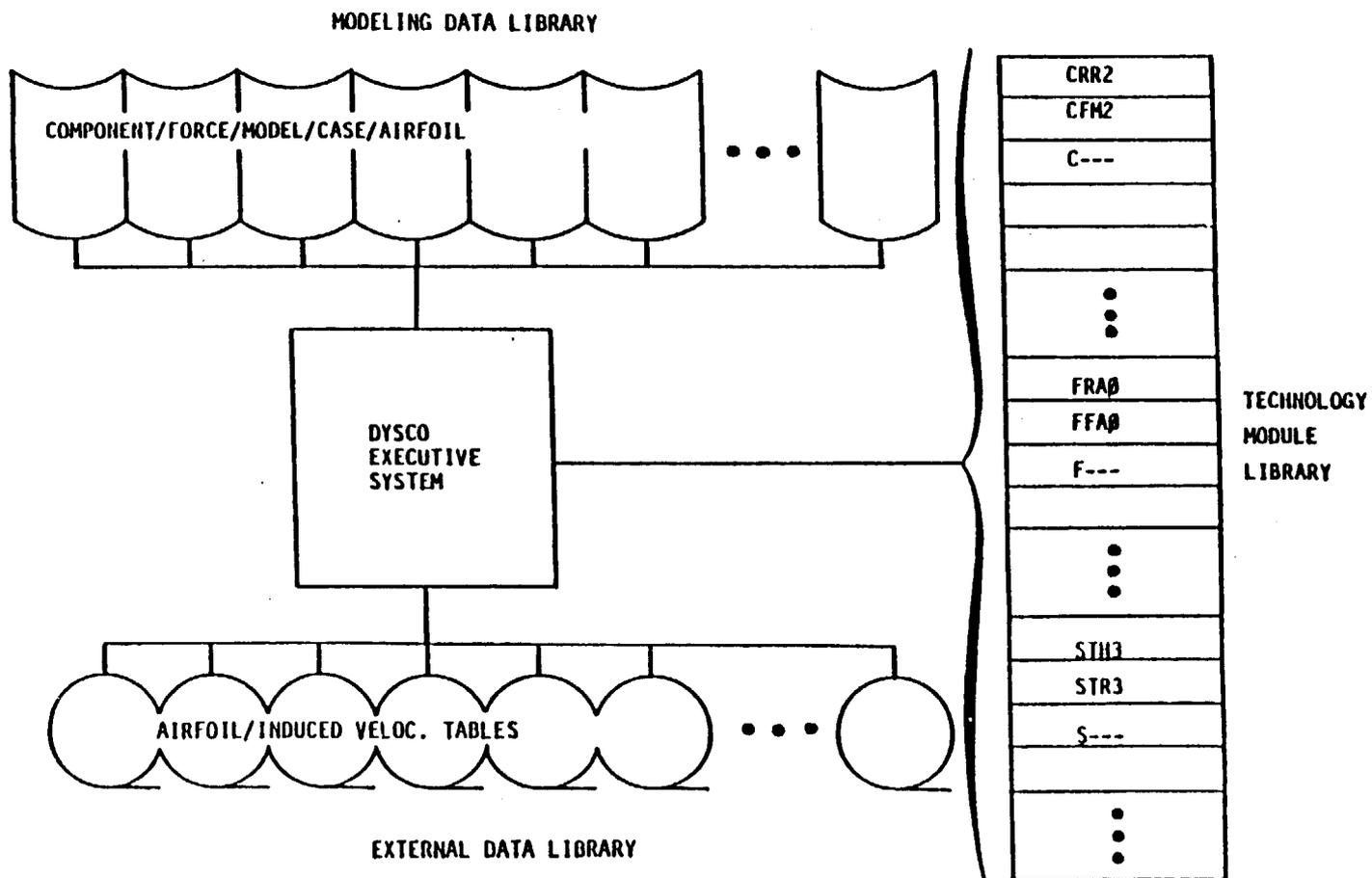
\*\*\*\*\*

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- o TECHNOLOGY LIBRARY CONTAINS
  - COMPONENT REPRESENTATIONS (C ...)
  - FORCE ALGORITHMS (F ...)
  - SOLUTION ALGORITHMS (S ...)
- o DATA LIBRARIES CONTAIN
  - DATA ASSOCIATED WITH PARTICULAR C ..., F ...
  - AND IDENTIFIED BY DATA SET NAME
  - SUPPLIED BY USER

DYSCO LIBRARIES

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ANY TECHNOLOGY MODULE MAY BE ADDED TO LIBRARY IF:

COMPONENT, FORCE

M, C, K, F MAY BE COMPUTED AS FUNCTIONS  
OF LOCAL STATE VECTOR AND TIME BY A  
FORTRAN PROGRAM

SOLUTION

ALGORITHM MAY BE WRITTEN IN FORTRAN, GIVEN  
SYSTEM M, C, K, F, AS ABOVE

COMMAND

NEW	ALLOWS USER TO MODIFY MODEL ALLOWS USER TO CREATE NEW COMPONENT/FORCE INPUT DATA
RERUN	NEW SOLUTION FOR MODEL JUST RUN
RUN	FORMS EQUATIONS OF MODEL AND EXECUTES A SOLUTION
EDIT	ALLOWS USER TO TO MODIFY MODEL AND PERFORM VALIDATED EDIT OF INPUT DATA

DYSCO ACHIEVES ITS MULTIPURPOSE CAPABILITY BY A COMPLETE SEPARATION OF ABSTRACT AND SPECIFIC DATA.

SINCE DYSCO DOES NOT TREAT SPECIFIC PHYSICAL COMPONENTS, FORCES, OR SOLUTIONS, IT CAN SOLVE PROBLEMS INVOLVING ANY COMPONENTS, FORCES, OR SOLUTIONS.

THE USER PERCEIVES A "MODEL" MADE UP OF COMPONENTS

HE THEN:

SELECTS APPROPRIATE REPRESENTATIONS FOR EACH  
COMPONENT FROM THE LIBRARY

SELECTS APPROPRIATE FORCE ALGORITHMS FOR EACH  
COMPONENT

SELECTS APPROPRIATE SOLUTION METHODS.

NOTE THAT THE ORIGINAL MODEL FORMULATION REQUIRES AN APPROPRIATE LEVEL  
OF ENGINEERING JUDGEMENT.

FOR EACH COMPONENT AND FORCE, THE DATA MUST BE IDENTIFIED AS RESIDING  
ON A USER FILE OR THE DATA MUST BE PLACED ON A FILE USING DYSCO.

THE MODEL AND VARIATIONS MAY THEN BE FORMED INTERACTIVELY AND SPECI-  
FIED SOLUTIONS MAY BE CARRIED OUT.

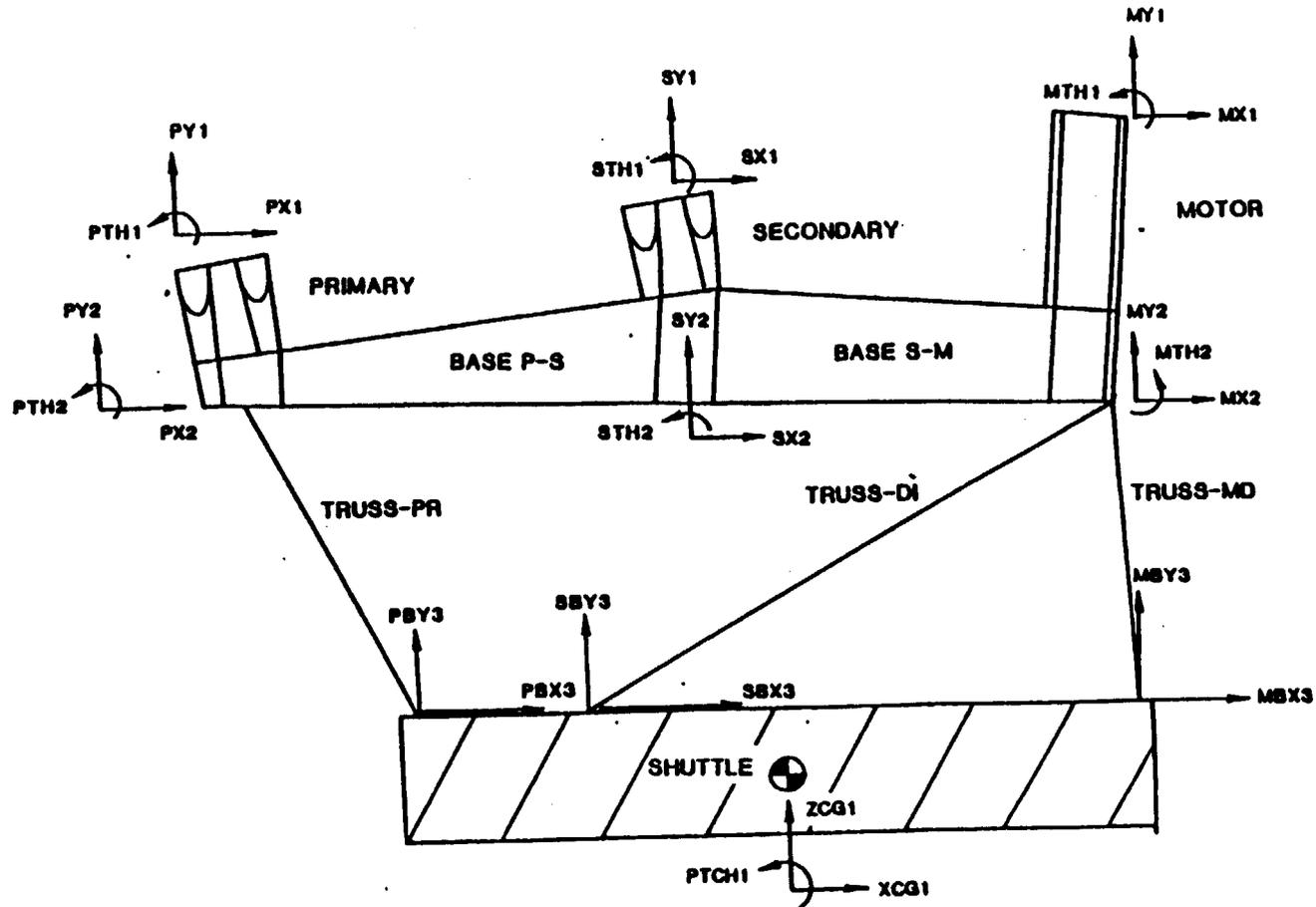
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DYSCO

ILLUSTRATIVE APPLICATIONS

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DYSCO 2D STRUCTURAL MODEL



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- MODEL 1 18 DOF (GROUNDED)
- 2-5 VARIOUS BASE SHAKES
- 6 21 DOF (3 BASE DOF)

- I. MAX TORQUE FOR MOTOR AT MAX RATE .05 R/S - 370 IN #
- II. DESIGN TORQUE  $T = 185$  IN #
- III. INITIAL TRIAL GAINS

1. DISPLACEMENT GAIN

IF  $\phi$  IS AT A MAX ALLOWABLE ERROR OF  $15 \mu R$ , LET MOTOR BE  
DRIVEN AT MAX SPEED

$$T = 185 \text{ IN \# AT } 15 \mu R$$

$$= 0 \text{ IN \# AT } 0$$

$$K_2 = 1.233 \times 10^7 \text{ IN \#/R OF } \phi$$

2. VELOCITY GAIN

IF  $\dot{\phi}$  IS AT MAX RECESSION OR APPROACH OF .05 R/S, LET  
MOTOR BE DRIVEN AT MAX SPEED

$$T = 185 \text{ IN \# AT } .05 \text{ R/S}$$

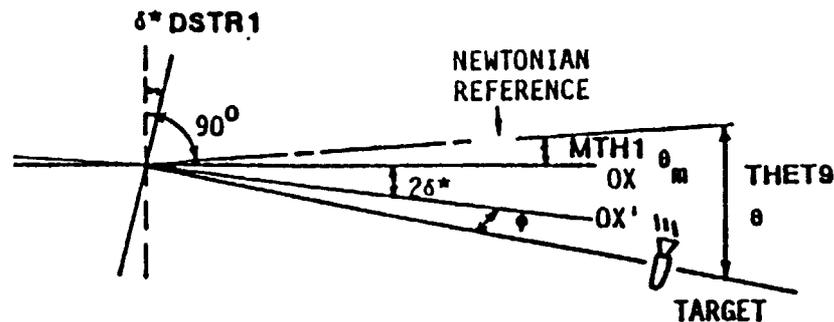
$$= 0 \text{ IN \# AT } 0$$

$$K_1 = 3.7 \times 10^3 \text{ IN \# SEC/R OF } \phi$$

TYPICAL STRUCTURAL VARIABLES MONITORED

I. MIRROR AND MOTOR

1. THET9 =  $\theta$  = TARGET L.O.S. FROM INERTIAL REFERENCE
2. DSTR1 =  $\delta^*$  = ACTUAL DRIVE ANGLE OF MIRROR
3. MTH1 =  $\theta_M$  = STRUCTURAL VIBRATION OF THE MOTOR MOUNT
4.  $2\delta^* + \theta_M$  = OX' (BENT OPTICAL AXIS) FROM INERTIAL REFERENCE
5.  $\phi = \theta - \theta_M - 2\delta^*$  = OPTICAL MISPOINT



CASE 1 TIME HISTORY  
(PAGE 1 OF 3)

CASE 1 TIME HISTORY

STATIONARY TARGET AT ZERO ----- INITIAL MISPOINT DSTR = 15  $\mu$ rad

INITIAL LINES-OF-SIGHT STATIONARY

Solid - Optical Mispoint of Flat Mirror

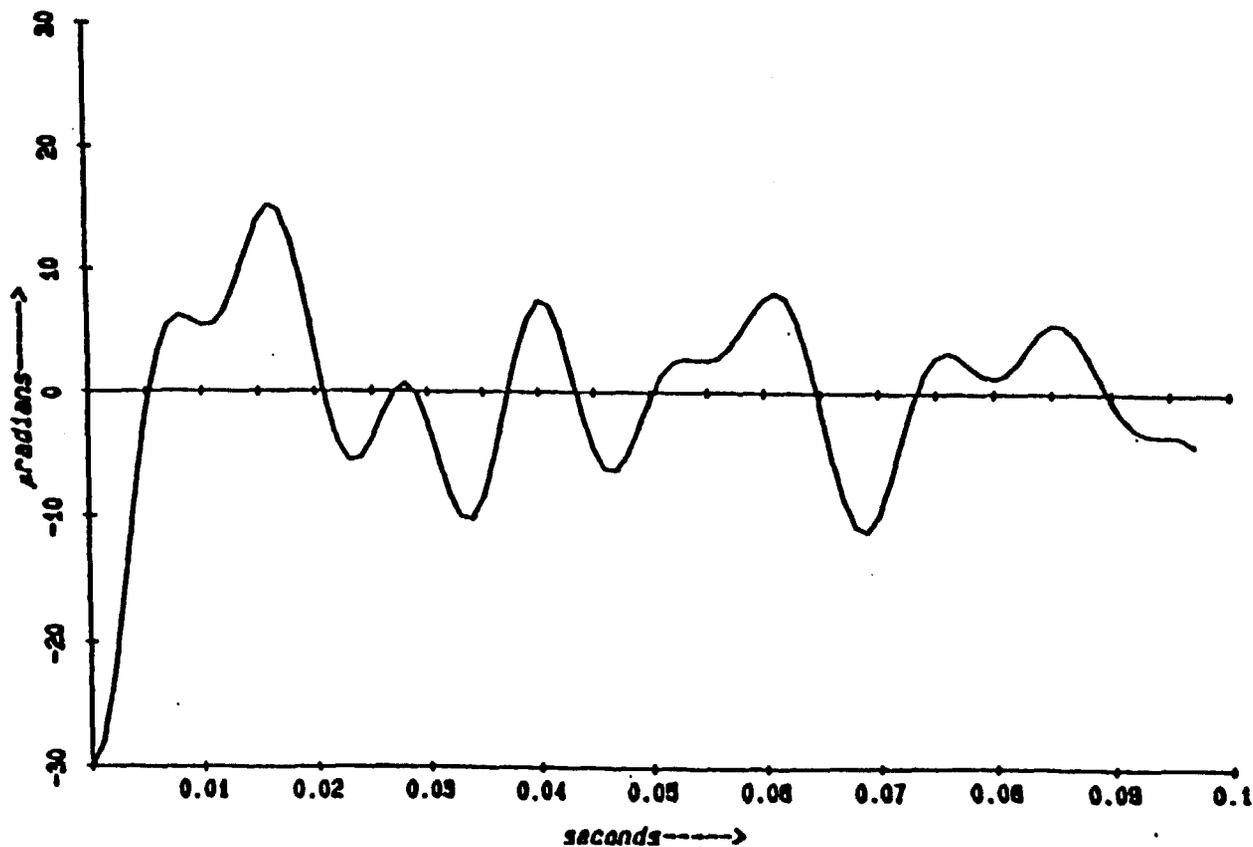
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CONTROLS:

Displ. Gain = Max Avail.

Vel. Gain = Max Avail.

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CASE B TIME HISTORY

*KAMAN*

ACCELERATING TARGET -----INITIAL MISPOINT DSTR = 15  $\mu$ rad

INITIAL LINES-OF-SIGHT APPROACHING

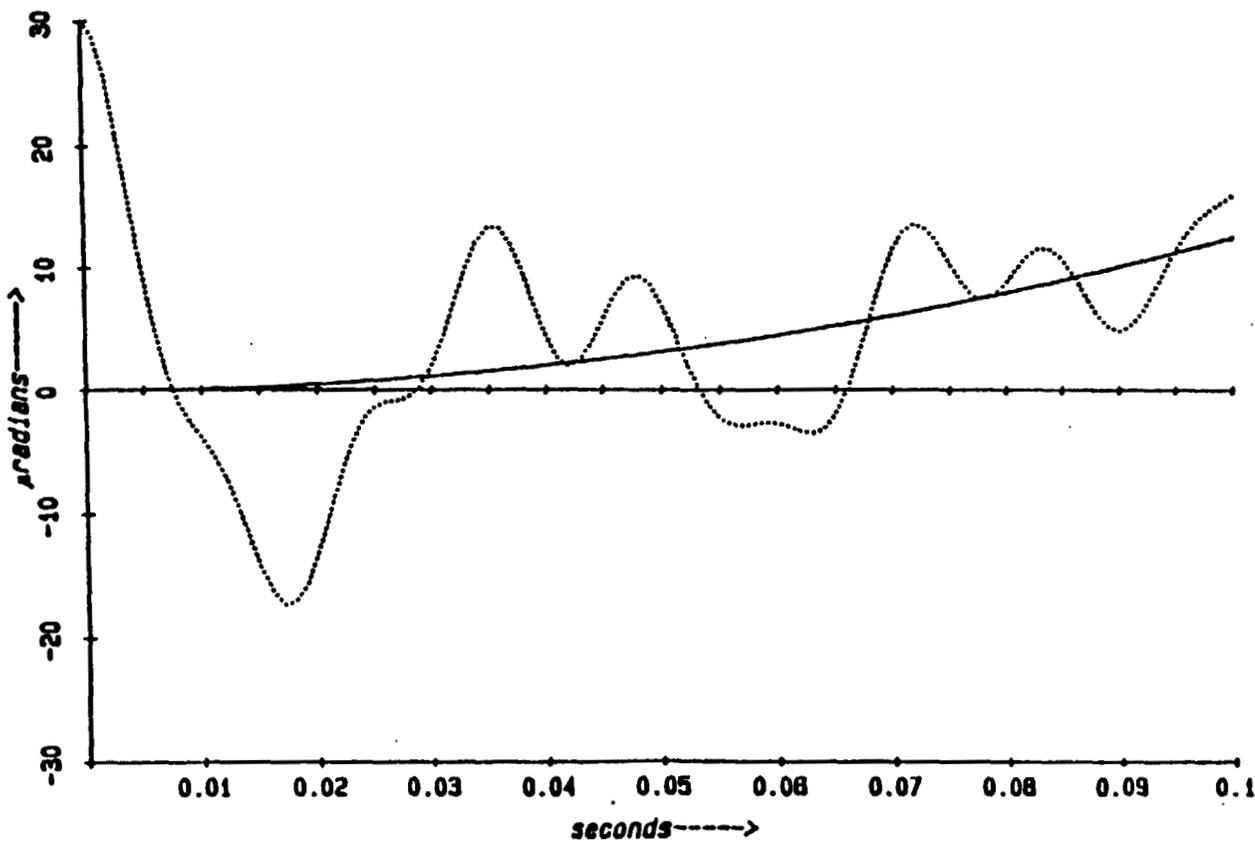
Solid - Target

Dotted - Pointing Axis L.O.S.

CONTROLS:

Displ. Gain = .65Max

Vel. Gain = .81 Max



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**CASE B TIME HISTORY**

ACCELERATING TARGET -----INITIAL MISPOINT DSTR = 15  $\mu$ rad

INITIAL LINES-OF-SIGHT APPROACHING

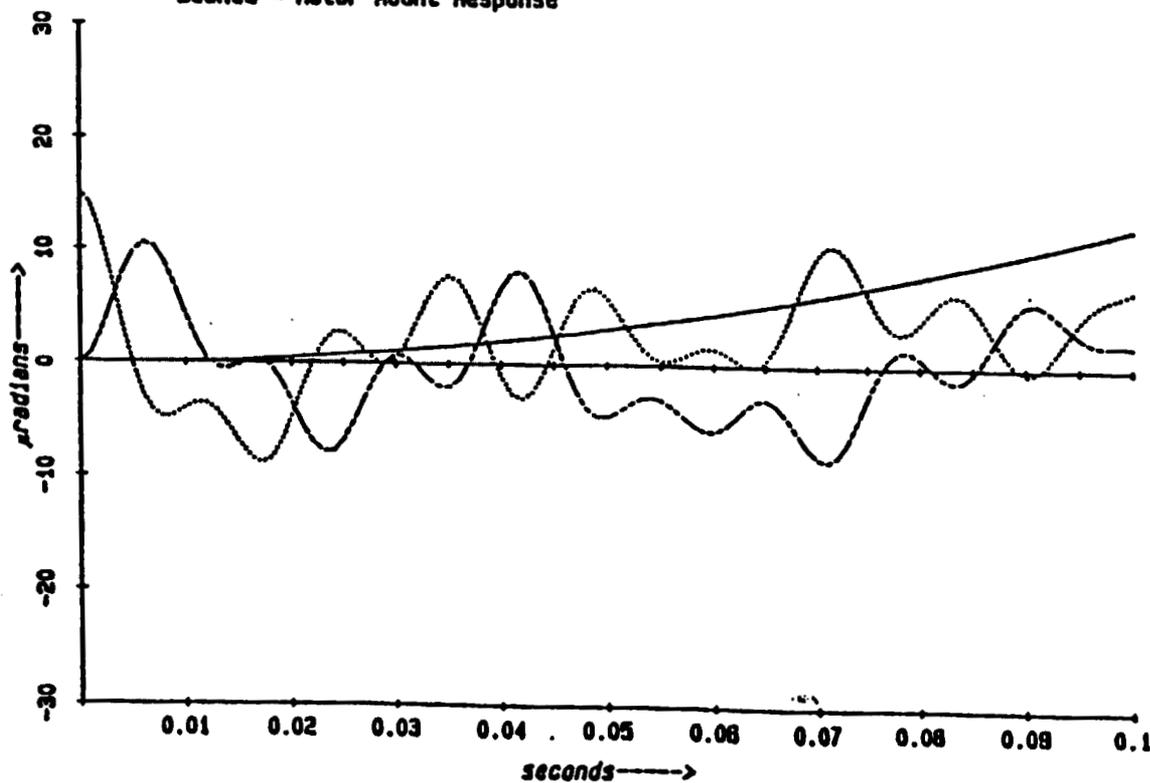
- Solid = Target
- Dotted = Driven Mirror Angle
- Dashed = Motor Mount Response

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CONTROLS:

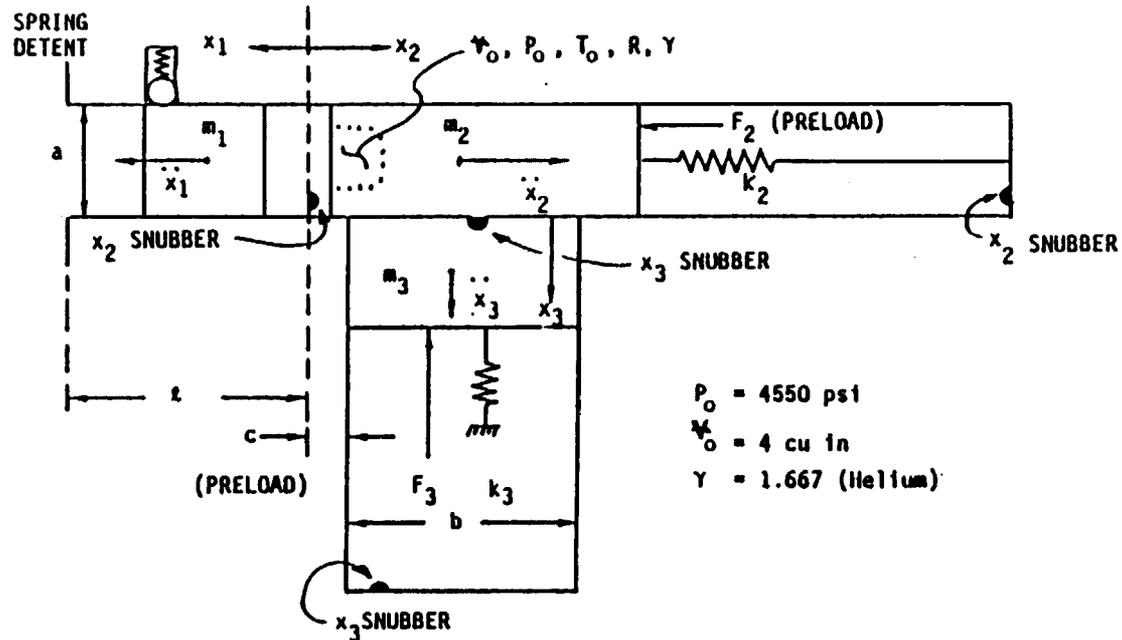
Displ. Gain = .85 Max

Vel. Gain = .81 Max



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**RAIL GUN PNEUMATIC PRE-ACCELERATOR**



$m_1 = \text{PROJECTILE}$      $m_2 = \text{BREECH BOLT}$      $m_3 = \text{MAGAZINE STACK}$

EQUATION OF MOTION -

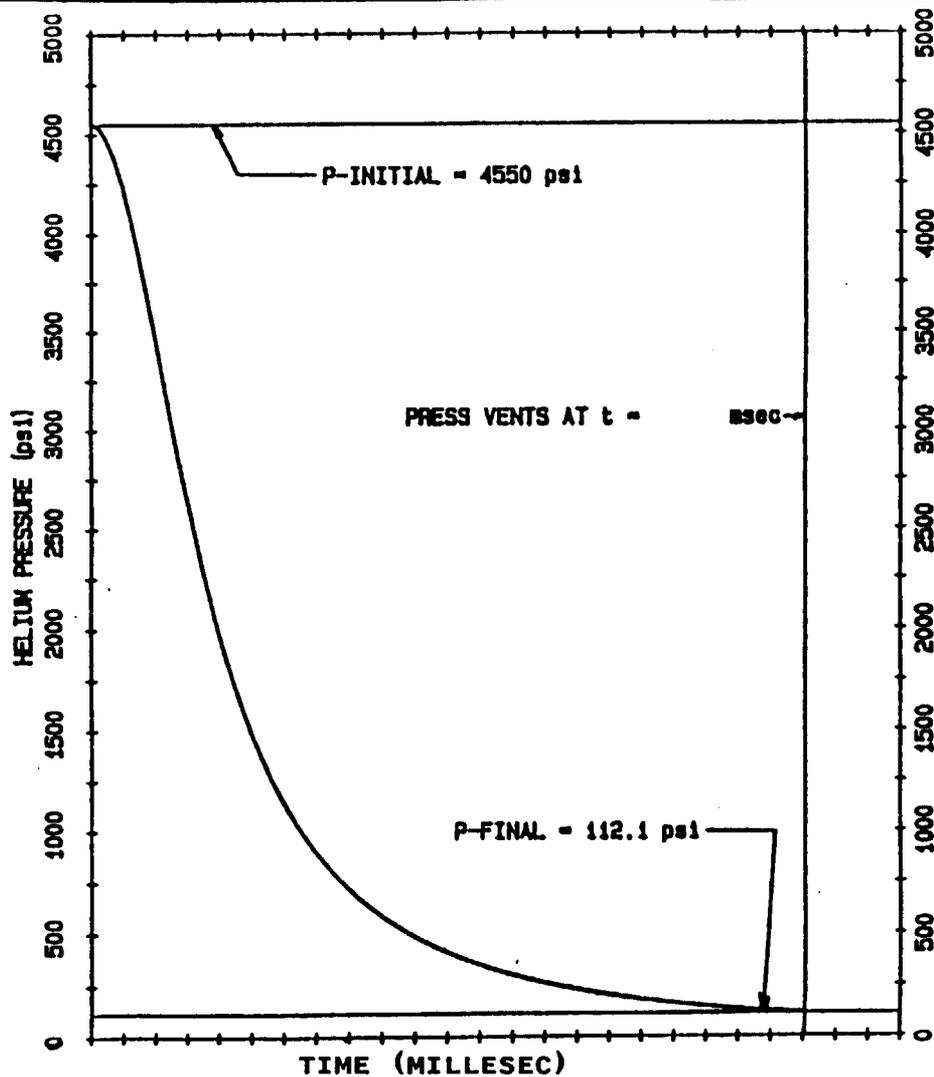
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + [0] \begin{Bmatrix} \dot{x} \end{Bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & k_2 \\ 0 & 0 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} A_1 P(t) \\ A_2 P(t) - F_2 \\ A_3 P(t) - F_3 \end{Bmatrix}$$

$A_3 = 0 \text{ UNLESS}$   
 $x_2 > c \text{ BEFORE}$   
 $\text{PRESSURE VENTS}$

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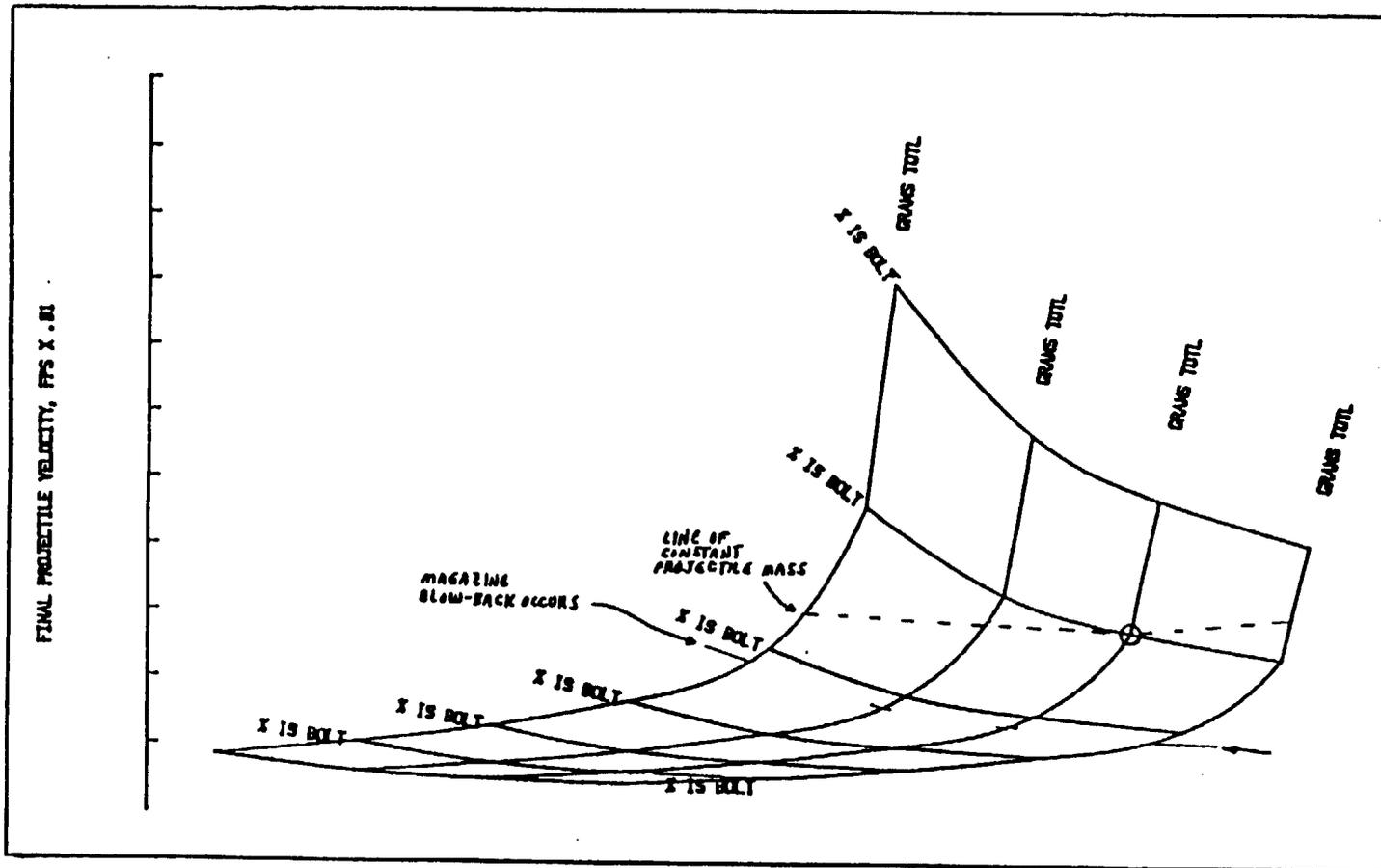
ET PRE-ACCELERATOR  
PRESSURE CASCADE

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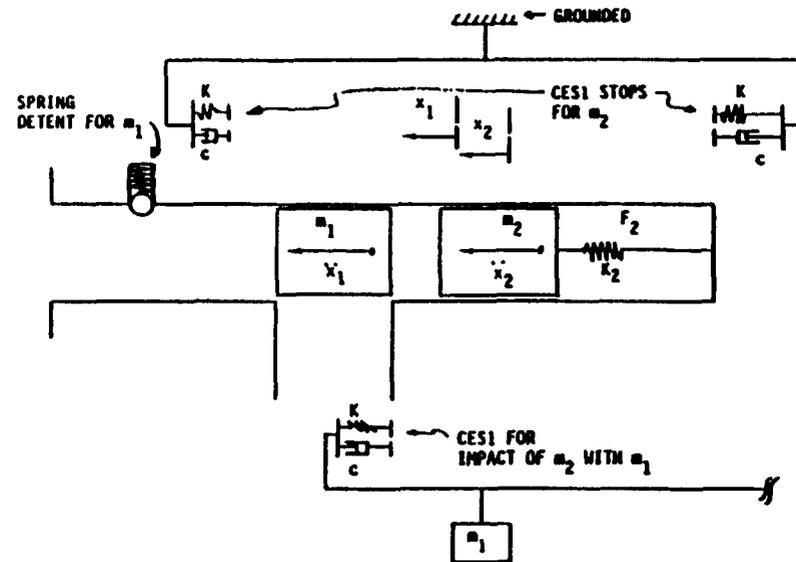
TRADES STUDIES FOR PRE-ACCELERATOR

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ET PREACCELERATOR, BEHAVIOR VS. TOTAL MASS AND PERCENT ALLOTTED TO BOLT

MODEL FOR HARMONIC BOLT RETURN AND IMPACT WITH  
FRESHLY LOADED PROJECTILE



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VARIOUS COEFFICIENTS OF RESTITUTION CAN BE  
MODELLED BY JUDICIOUS CHOICE OF K AND C

\*\*\*\*\* MODEL IMPACT 2 \*\*\*\*\*

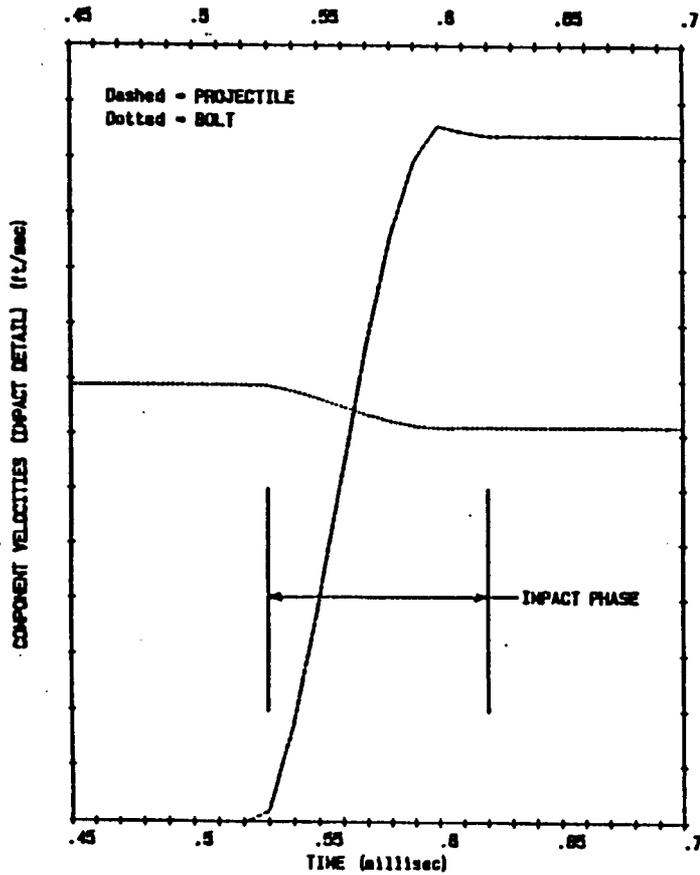
IMPACT MODEL PLUS BARREL FIT/MAGAZINE FRICTION AND BALL SPRING DETENT

INDEX	COMP	NO.	DATA SET	FORCE	DATA SET
1	CSF1		BR1DYN	NONE	
2	CES1		BOLT2.0	NONE	
3	CSD1		BALL2	NONE	
4	CDF1		IMPMU	NONE	

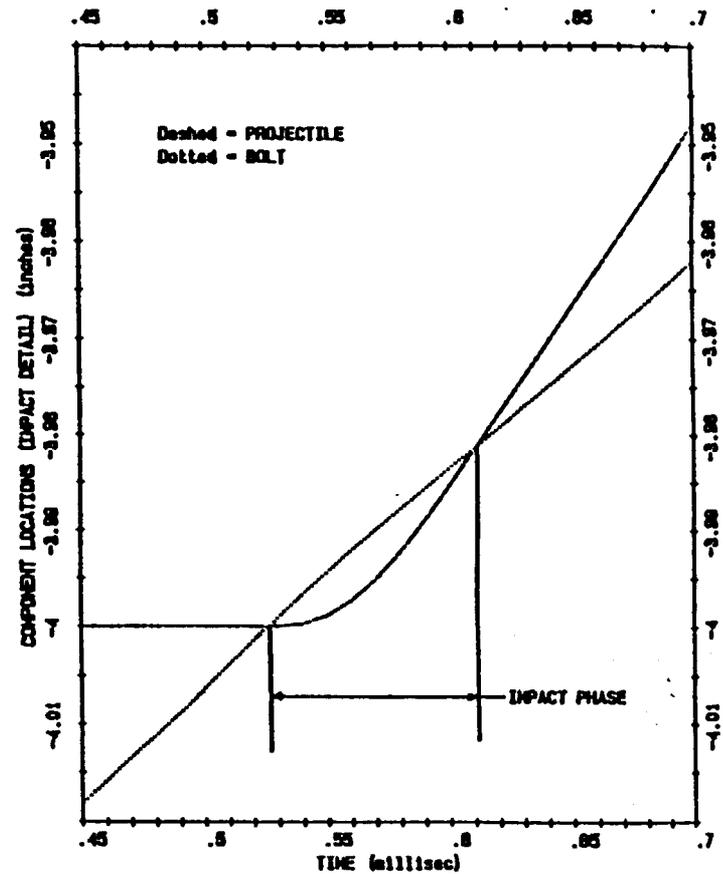
\*\*\*\*\*

IMPACT DETAIL  
DISPLACEMENT AND VELOCITY

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ET PRE-ACCELERATOR  
TIME DOMAIN VELOCITIES



ET PRE-ACCELERATOR  
TIME DOMAIN DISPLACEMENTS

N90-10090

**MODELING AND CONTROL SYSTEM DESIGN AND ANALYSIS TOOLS FOR FLEXIBLE STRUCTURES**

By

Amir A. Anissipour  
Russell A. Benson  
Edward E. Coleman

The Boeing Company  
Seattle, Washington

**ABSTRACT**

Design and analysis of control systems for flexible structures require accurate math models of flexible structures and software design with the analysis tools capable of handling these models while maintaining numerical accuracy. Since aeroelastic models of flexible structures tend to be large (e.g., 100 states), the availability of tools to handle such large models is crucial. Initial model development is based on aerodynamic mathematical models, wind tunnel data, mathematical structural models, and ground shake test results. Eventually, flight test data are used to update and refine the model. This paper describes Boeing software tools used for the development of control laws of flexible structures.

The Boeing Company has developed a software tool called Modern Control Software Package (MPAC). MPAC provides the environment necessary for linear model development, analysis, and controller design for large models of flexible structures. There are two features of MPAC which are particularly appropriate for use with large models: (1) numerical accuracy and (2) label-driven nature. With the first feature MPAC uses double precision arithmetic for all numerical operations and relies on EISPAC and LINPACK for the numerical foundation. With the second feature, all MPAC model inputs, outputs, and states are referenced by user-defined labels. This feature allows model modification while maintaining the same state, input, and output names. In addition, there is no need for the user to keep track of a model variable's matrix row and column locations.

There is a wide range of model manipulation, analysis, and design features within the numerically robust and flexible environment provided by MPAC. Models can be built or modified using either state space or transfer function representations. Existing models can be combined via parallel, series, and feedback connections; and loops of a closed-loop model may be broken for analysis. Analysis tools available include: eigenvalue/eigenvector, controllability matrix, observability matrix, transfer function generation, frequency response and singular value plots, covariance response to white noise or atmospheric turbulence models, model simulation using step, sinusoidal, random, or user-defined inputs. Control system design tools include: root locus, LQG full state feedback gain matrix computation, LQG full-order estimator design, and robust low order controller (SANDY) design as developed by Dr. Uy-Lot Ly at Stanford.

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The existing Boeing Company structural analysis and design software package, ATLAS, has been extended in order to form a state-space model for input to MPAC. The new capability, a module named DYFORM, is an outgrowth of earlier work under a NASA contract for Integrated Application of Active Controls. The structural and theoretical aerodynamic mathematical model originates within ATLAS in exactly the same fashion as for conventional flutter and dynamic loads analyses. The DYFORM module is then used to construct the state-variable model as required by MPAC. Its capabilities include (1) control surfaces and/or gust vector as inputs, (2) sensors and/or loads quantities as outputs, (3) formulation in body-fixed or inertial axes, (4) modification of the theoretical aerodynamics using wind tunnel/flight test data from rigid or flexible-model tests, and (5) use of S-plane rational airloads expressions to formulate the state model including augmented states to represent unsteady aerodynamic effects.

MPAC has been used for yaw damper design (including active flexible mode suppression) of the Boeing 767 and 747 airplanes. The flexible structural models of these planes, as large as 100 states, have been handled by MPAC without loss of numerical accuracy.

The Boeing Company plans for the development of a system identification and parameter estimation (SIPE) software tool. The system identification algorithms employ a multiple stepwise regression technique to determine the structure of the system. The parameter estimation algorithms update the current model using maximum likelihood estimation. The SIPE routines will be compatible with MPAC and RF\_DATA (a data correction and reformatting program also developed by Boeing). The SIPE routines will be flexible, allowing the user to select gradient methods, integration algorithms, and Riccati solution algorithms. The MPAC compatible model structure slated for the SIPE package will be applicable to any dynamic system. Aerodynamic, aeroelastic, ground effects, and sensor noise modeling will all be possible.

# **INTRODUCTION TO MPAC:**

## **A Control Law Design Tool Well Suited for Flexible Structure Applications**

**Edward E. Coleman**

**The Boeing Company  
Boeing Commercial Airplanes  
P.O. Box 3707  
Seattle, Washington 98124-2207**

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

### Tool Requirements for Models of Flexible Structures:

- o Large model capacity (more than 100 states)
- o Efficient user interface for handling large models
- o Numeric robustness
- o Model reduction techniques

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

**MPAC** - Multivariable control design and analysis PACkage

- Programmable "calculator" for synthesis, manipulation, and analysis of continuous and discrete linear dynamic system models
  
- MPAC supports:
  - Model development
  - Dynamic system analysis
  - Controller synthesis
  
- MPAC was originally developed as a batch process tool. An interactive interface is currently being developed for MPAC to improve its ease of use and efficiency.

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# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

### MPAC Features:

#### Label Driven Model Format:

- User defined state, input, and output labels of up to 8 characters.

#### Numeric Robustness:

- Built on Eispac, LINPACK, and ORCALS
- Double precision computation throughout
- Handles models up to 256 elements (states, inputs, and outputs)

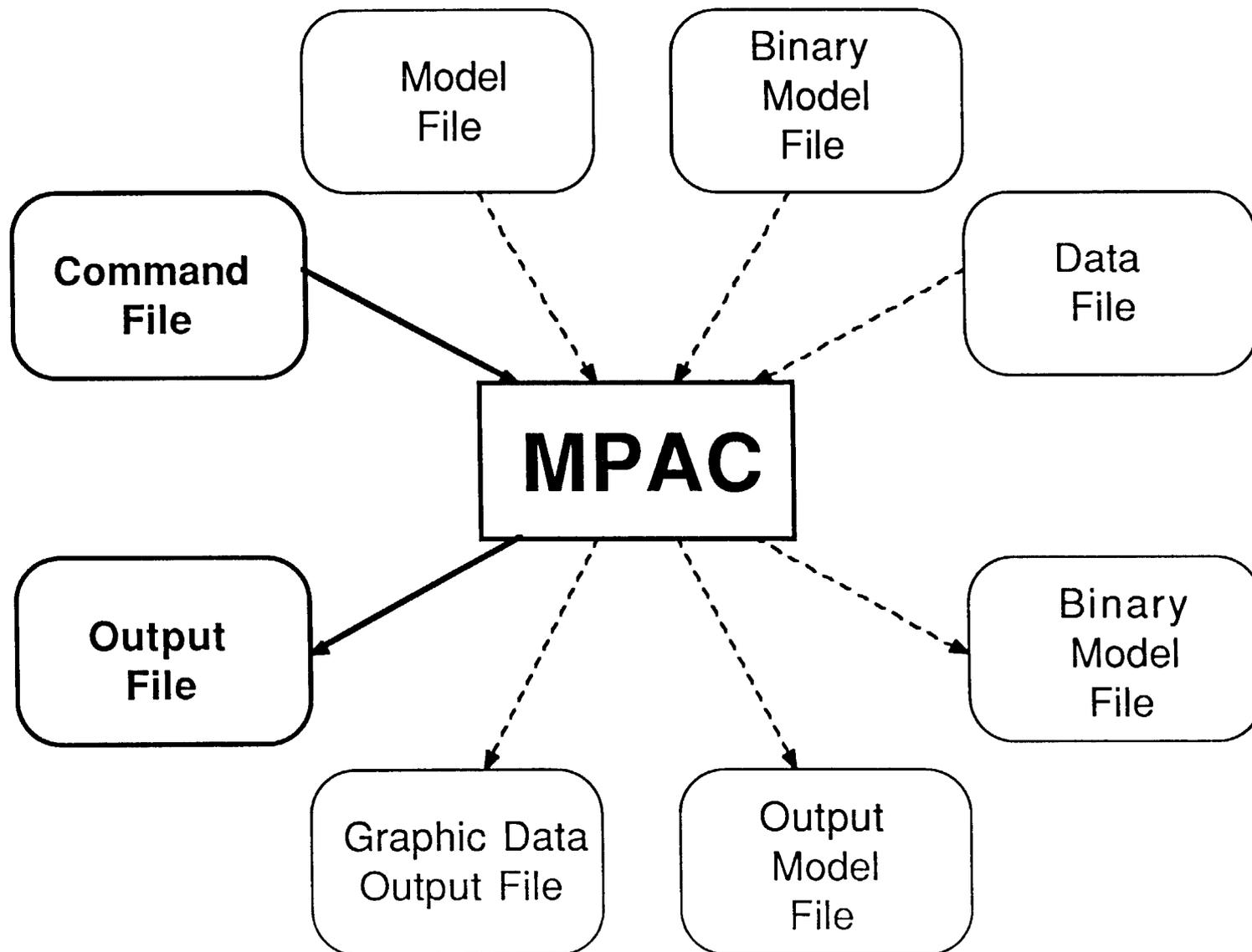
#### Modular Structure:

- Each command is a separate subroutine
- User need learn only those comands he/she wants to use
- Wide range of available commands
- Provisson for customized, user defined commands

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---



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ORIGINAL PAGE IS  
OF POOR QUALITY

Model Input Output

Model Properties

Model Modification

Model Reduction

Model Transformation

Connection

MATLAB

EASY

Time Response

Frequency Response

Covariance Response

Create Controller

Eigenstructure

Discrete Design

Continuous Design

User Commands

All commands

Last command

Execution mode : BUILD\_CMD\_FILE

AUTO-EXEC ? Yes

AUTO-VIEW ? Yes

Reorder MDI file

	No.	Comments
↑	1	-> ▲
	2	->
	3	->
	4	->
↓	5	->

Task Menu - Command Definition Level

- Enter command data / Write command
- View results
- Print results
- Plot results
- View CMD file
- Cancel (new command)
- Up (to File Definition Level)

All MPAC Commands

Model Input Output

Model Properties

Model Modification

Model Reduction

Model Transformation

Connection

MATLAB

EASY

No.	Comments
1	->
2	->
3	->
4	->
5	->

- COMPUTE THE ASYMPTOTIC POLES OF FULL STATE FEEDBACK GAIN FOR THE reg
- COMPUTE THE ASYMPTOTIC POLES OF KALMAN FILTER GAIN FOR THE reg
- COMPUTE THE COMPANION MODAL MODEL OF THE reg, replacement\_option
- COMPUTE THE CONJUGATE MODAL MODEL OF THE reg, replacement\_option
- COMPUTE THE CONTROLLABILITY MATRIX OF THE reg
- COMPUTE THE COVARIANCE RESPONSE OF THE reg TO turbulence\_model AND covariance\_print\_option
- COMPUTE THE COVARIANCE RESPONSE OF THE reg TO WHITE NOISE AND covariance\_print\_option
- COMPUTE THE DISCRETE COVARIANCE RESPONSE OF THE reg TO WHITE NOISE AND covariance\_print\_option
- COMPUTE THE EIGENVALUES OF THE reg
- COMPUTE THE EIGENVECTORS OF THE reg
- COMPUTE THE FREQUENCY RESPONSE OF THE reg
- COMPUTE THE OBSERVABILITY MATRIX OF THE reg
- COMPUTE THE RESIDUES OF THE reg
- COMPUTE THE ROOT LOCUS OF THE reg
- COMPUTE THE SETPOINT MATRICES FOR THE reg
- COMPUTE THE SIMILARITY TRANSFORM OF THE STATE MODEL FOR THE reg
- COMPUTE THE SINGULAR VALUES OF THE reg
- COMPUTE THE TRANSFER FUNCTION(S) OF THE reg
- COMPUTE THE TRANSMISSION ZEROS OF THE reg
- CONSTRUCT THE DRYDEN TURBULENCE MODEL FOR THE reg
- CONSTRUCT THE EQUATIONS OF MOTION FOR THE reg
- COPY THE STATE MODEL OF THE reg TO THE destination\_reg
- CREATE STATE ESTIMATOR FOR THE reg
- CREATE controller\_type FOR THE reg
- CREATE controller\_type FOR THE reg WITH MODEL FOLLOWING
- DESIGN THE DISCRETE FSF (FULL STATE FEEDBACK) GAIN MATRIX FOR THE reg
- DESIGN THE DISCRETE STATE ESTIMATOR (KALMAN FILTER) GAIN MATRIX FOR THE reg
- DESIGN THE EMF (EXPLICIT MODEL FOLLOWING) GAIN MATRIX FOR THE reg
- DESIGN THE FSF (FULL STATE FEEDBACK) GAIN MATRIX FOR THE reg
- DESIGN THE LTS (LINEAR TRACKING SYSTEM) GAIN MATRIX FOR THE reg
- DESIGN THE STATE ESTIMATOR (KALMAN FILTER) GAIN MATRIX FOR THE reg
- DESIGN ROBUST LOW-ORDER CONTROLLER

Cancel

All MPAC Commands

---

Model Input Output

Model Properties

Model Modification

Model Reduction

Model Transformation

Connection

MATLAB

EASY

	No.	Comments
↑	1	->
	2	->
	3	->
↓	4	->
	5	->

DESIGN THE STATE ESTIMATOR (KALMAN FILTER) GAIN MATRIX FOR THE reg

DESIGN ROBUST LOW-ORDER CONTROLLER

DISCONNECT THE CLOSED LOOP OF THE reg

FEEDBACK CONNECTION: a\_reg IS CONNECTED TO THE b\_reg AND IS PLACED INTO THE destination\_reg

FORM THE CLOSED LOOP SYSTEM: CONNECT THE controller\_option TO THE cls\_reg

FORM THE pim\_reg WITH THE IDEAL MODEL (AND PLACE IT INTO THE PLANT + IDEAL MODEL)

FORM THE pse\_reg WITH THE estimator\_option (AND PLACE IT INTO THE PLANT + STATE ESTIMATOR)

LOAD THE G (FULL STATE FEEDBACK GAIN) MATRIX

LOAD THE S (ESTIMATOR GAIN) MATRIX

LOAD THE STATE MODEL OF THE reg

LOAD transfer\_fn\_type TRANSFER FUNCTION INTO THE reg

MODIFY THE G MATRIX BY modification\_method

MODIFY THE S MATRIX BY modification\_method

MODIFY THE STATE MODEL OF THE reg

PARALLEL CONNECTION: a\_reg IS CONNECTED TO THE b\_reg AND IS PLACED INTO THE destination\_reg

PRINT THE G (FULL STATE FEEDBACK GAIN) MATRIX

PRINT THE S (ESTIMATOR GAIN) MATRIX

PRINT THE STATE MODEL OF THE reg

READ THE STATE MODEL FOR THE reg (IN MATLAB FORMAT)

READ THE STATE MODEL FOR THE reg (MPAC FORMAT)

REDUCE THE ORDER OF THE reduce\_reg

RESTORE THE ORIGINAL G MATRIX (AFTER MODIFY COMMAND)

RESTORE THE ORIGINAL S MATRIX (AFTER MODIFY COMMAND)

RUN MATLAB

SERIES CONNECTION: a\_reg IS CONNECTED TO THE b\_reg AND IS PLACED INTO THE destination\_reg

SIMULATE THE LINEAR STATE MODEL OF THE reg

SIMULATE THE LINEAR STATE MODEL OF THE reg WITH A lts\_controller\_reg (LINEAR TRACKING SYSTEM)

TRANSFORM reg USING transform\_option

WRITE THE STATE MODEL FOR THE reg (IN MATLAB FORMAT)

WRITE THE STATE MODEL FOR THE reg (IN EASY5 FORMAT)

WRITE THE STATE MODEL FOR THE reg (MPAC FORMAT)

XXX nopt

Cancel

COMMAND:

\*COMPUTE THE OBSERVABILITY MATRIX OF THE **reg**

Model Input Output

Time Response

Model Properties

Frequency Response

Model Modification

Covariance Response

Model Reduction

Create Controller

Model Transformation

Eigenstructure

Connection

Discrete Design

Continuous Design

MATLAB

User Commands

EASY

reg

PLANT  
REDUCED PLANT  
CONTROLLER  
REDUCED CONTROLLER  
FEEDFORWARD CONTROLLER  
LTS CONTROLLER  
STATE ESTIMATOR  
REDUCED STATE ESTIMATOR  
IDEAL MODEL  
PLANT + IDEAL MODEL  
PLANT + STATE ESTIMATOR  
CLOSED LOOP SYSTEM  
BROKEN LOOP SYSTEM  
MODEL 1  
MODEL 2  
MODEL 3

Cancel

Execution mode : BUILD\_CMD\_FILE

AUTO-EXEC ? Yes

AUTO-VIEW ? Yes

Position MDL file

No.	Comments
1	->
2	->
3	->
4	->
5	->

Task Menu - Command Definition Level

Enter command data / Write command

View results  
Print results  
Plot results  
View CMD file  
Cancel (new command)  
Up (to File Definition Level)

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

This is an example MPAC command file. The output file generated using this command file is given on the following pages.

```
*MPAC READ PLANT
LAT2.MDL

*COMPUTE EIGENVALUES OF PLANT

*DEFINE PLANT
DELETE STATE PSI
CREATE STATE BETA_INT.dot: 1. BETA
CREATE OUTPUT PHI_CRIT: 1. PHI.dot 5.0 PHI
CREATE OUTPUT BETA_CRIT: 1. BETA.dot 3.2 BETA 4. BETA_INT
END

*DESIGN GAIN MATRIX FOR PLANT
.001
2, 2
'AIL' 1.
'RUD' 2.
'PHI_CRIT' 4.
'BETA_CRIT' 1.
'WLOCUS' 'RHO' 1., 1., 1
$ FIRST CUT LATERAL GAIN LOCUS
$ AIL=1. RUD=2.
$ PHI_CRIT=4. BETA_CRIT=1.

*CREATE CONTROLLER FOR PLANT
'NODIRECT'

*FORM PLANT + CONTROLLER

*PRINT CLOSED-LOOP SYSTEM

*COMPUTE EIGENVALUES OF CLOSED-LOOP SYSTEM

*MPAC WRITE CLOSED-LOOP SYSTEM
CLOSED_LOOP.MDL
```

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

MPAC output file example. Output file generated using command file on previous

MPAC RELEASE VERSION 4.00 05 MAY 1987

(CONFIGURATION CONTROL I

```
*****
*=====
*          MPAC INPUT/OUTPUT FILE DESCRIPTION          *
*=====
*****
```

```
COMMAND FILE ----- example.cmd
MODEL FILE -----
INPUT BINARY FILE -----
OUTPUT BINARY FILE -----
MPAC OUTPUT FILE ----- example.out
MPAC GGP PLOT FILE ----- example.ggp
MPAC USER DATA FILE NO.1 -----
MPAC USER DATA FILE NO.2 -----
MPAC USER-DEFINED UBIN FILE --
TIME OF MPAC JOB EXECUTION --- Tuesday, July 5, 1988  3:50:36 pm (PST)
```

```
*****
```

```
*****
*****
*****
*****
***** MODERN CONTROL THEORY ANALYSIS/SYNTHESIS SOFTWARE PACKAGE *****
*****
***** APOLLO-VERSION: MPAC 4.00 ON APOLLO FORTRAN 8.40 *****
*****
*****
*****
*****
*****
```

```
*****
* 07/05/88 *
* 15:50:54 *
*****
```

```
*****
***** TASK 1 *****
*****
***** *MPAC READ PLANT *****
*****
*****
***** TASK 1 *****
*****
```

\*\*\* MODEL READ FROM FILE: LAT2.MDL \*\*\*

ELAPSED TIME (SEC): 0.24

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

```

*****
***** TASK 2 *****
*****
***** *COMPUTE EIGENVALUES OF PLANT *****
*****
***** TASK 2 *****
*****

```

SAMPLING TIME : DELTA = 0.0000

\*\*\*\*\* EIGENVALUES OF PLANT \*\*\*\*\*

COUNT	REAL PART	IMAG PART	DAMPING	FREQ (RAD/S)	FREQ (HZ)
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	-1.2989E-02	0.0000	1.000	1.2989E-02	2.0673E-03
3	-0.1403	1.676	8.3426E-02	1.682	0.2677
4	-0.1403	-1.676	8.3426E-02	1.682	0.2677
5	-1.946	0.0000	1.000	1.946	0.3097

ELAPSED TIME (SEC): 0.14

```

*****
***** TASK 3 *****
*****
***** *DEFINE PLANT *****
*****
***** TASK 3 *****
*****

```

```

*****
DELETED STATE PSI
CREATED STATE BETA INT.: 1.000 BETA
CREATED OUTPUT PHI_CRIT: 1.000 PHI 5.000 PHI
CREATED OUTPUT BETA_CRIT: 1.000 BETA. 3.200 BETA
*****

```

ELAPSED TIME (SEC): 0.10

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

```

*****
***** TASK 4 *****
*****
*****          *****
***** *DESIGN GAIN MATRIX FOR PLANT *****
*****          *****
*****
***** TASK 4 *****
*****

```

DESIGN PARAMETERS:

ALPHA = 1.00000E-03

CONTROL VARIABLE	CONTROL WEIGHT (R)
AIL	1.0000
RUD	2.0000

CRITERIA VARIABLE	CRITERIA WEIGHT (Q)
PHI_CRIT	4.0000
BETA_CRIT	1.0000

=====

===== STEADY STATE RICCATI SOLUTION =====

=====

	1	2	3	4	5
1	15.95	-2.100	-6.876	-5.891	9.802
2	-2.100	1.105	4.050	0.7099	-0.3128
3	-6.876	4.050	15.73	2.397	-2.262
4	-5.891	0.7099	2.397	3.222	-3.365
5	9.802	-0.3128	-2.262	-3.365	14.39

SAMPLING TIME : DELTA = 0.0000

```

***** EIGENVALUES OF A + B*G *****
***** OPTIMAL CL EIGENVALUES *****

```

COUNT	REAL PART	IMAG PART	DAMPING	FREQ (RAD/S)	FREQ (HZ)
1	-0.9492	0.0000	1.000	0.9492	0.1511
2	-1.574	1.453	0.7346	2.142	0.3409
3	-1.574	-1.453	0.7346	2.142	0.3409
4	-4.442	2.532	0.8688	5.113	0.8138
5	-4.442	-2.532	0.8688	5.113	0.8138

=====

===== FEEDBACK GAIN MATRIX =====

=====

	BETA	P	PHI	R	BETA_INT
AIL	5.470	-2.605	-9.524	-2.000	1.093
RUD	-3.065	-0.5127	-2.059	2.056	-2.722

ELAPSED TIME (SEC): 1.17

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

```
*****
***** TASK 5 *****
*****
***** *CREATE CONTROLLER FOR PLANT *****
*****
***** TASK 5 *****
*****
```

```
*****
***** FULL STATE FEEDBACK CONTROLLER *****
***** NO MODEL FOLLOWING *****
***** DIRECT F.B. STATES TO PLANT *****
BETA          P          PHI          R          BETA_INT
*****
```

ELAPSED TIME (SEC): 0.50

```
*****
***** TASK 6 *****
*****
***** *FORM PLANT + CONTROLLER *****
*****
***** TASK 6 *****
*****
```

```
*****
***** FULL STATE FEEDBACK CONTROLLER *****
***** NO MODEL FOLLOWING *****
*****
```

ELAPSED TIME (SEC): 2.16

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

```

*****
***** TASK 7 *****
*****
***** *PRINT CLOSED-LOOP SYSTEM *****
*****
***** TASK 7 *****
*****

```

```

*****
***** CLOSED LOOP SYSTEM *****
*****

```

SAMPLING TIME : DELTA = 0.0000

```

=====
===== A =====
=====

```

	BETA	P	PHI	R	BETA_INT
BETA	-0.2401	5.9648E-02	-4.8458E-02	-0.9132	-0.1075
P	-4.579	-8.932	-25.99	2.8649E-02	-3.112
PHI	0.0000	1.000	0.0000	8.7813E-02	0.0000
R	7.526	0.4025	2.215	-3.809	4.611
BETA_INT	1.000	0.0000	0.0000	0.0000	0.0000

```

=====
===== B =====
=====

```

	AIL	RUD
BETA	2.5320E-03	4.0504E-02
P	2.284	2.060
PHI	0.0000	0.0000
R	0.1228	-1.645
BETA_INT	0.0000	0.0000

```

=====
===== C =====
=====

```

	BETA	P	PHI	R	BETA_INT
PHI_CRIT	0.0000	1.000	5.000	8.7813E-02	0.0000
BETA_CRIT	2.960	5.9648E-02	-4.8458E-02	-0.9132	3.892
AIL+	5.470	-2.605	-9.524	-2.000	1.093
RUD+	-3.065	-0.5127	-2.059	2.056	-2.722
AIL=-	5.470	-2.605	-9.524	-2.000	1.093
RUD=-	-3.065	-0.5127	-2.059	2.056	-2.722

```

=====
===== D =====
=====

```

	AIL	RUD
PHI_CRIT	0.0000	0.0000
BETA_CRIT	2.5320E-03	4.0504E-02
AIL+	0.0000	0.0000
RUD+	0.0000	0.0000
AIL=-	1.000	0.0000
RUD=-	0.0000	1.000

ELAPSED TIME (SEC): 0.12

# BOEING COMMERCIAL AIRPLANES

## GUIDANCE AND CONTROL RESEARCH

---

```

*****
***** TASK 8 *****
*****
*****
***** *COMPUTE EIGENVALUES OF CLOSED-LOOP SYSTEM *****
*****
***** TASK 8 *****
*****

```

SAMPLING TIME : DELTA = 0.0000

\*\*\*\*\* EIGENVALUES OF CLOSED-LOOP SYSTEM \*\*\*\*\*

COUNT	REAL PART	IMAG PART	DAMPING	FREQ (RAD/S)	FREQ (HZ)
1	-0.9492	0.0000	1.000	0.9492	0.1511
2	-1.574	1.453	0.7346	2.142	0.3409
3	-1.574	-1.453	0.7346	2.142	0.3409
4	-4.442	2.532	0.8688	5.113	0.8138
5	-4.442	-2.532	0.8688	5.113	0.8138

ELAPSED TIME (SEC): 0.18

```

*****
***** TASK 9 *****
*****
*****
***** *MPAC WRITE CLOSED-LOOP SYSTEM *****
*****
***** TASK 9 *****
*****

```

\*\*\* MODEL WRITTEN TO FILE: CLOSED\_LOOP.MDL \*\*\*

ELAPSED TIME (SEC): 0.32

TOTAL JOB ELAPSED TIME (SEC): 10.24

# **PROPOSED SIPE TOOLBOX**

## **A Graphic/Engineering Software Concept for Modeling**

**Amir A. Anissipour**

**The Boeing Company**

**Boeing Commercial Airplanes**

**P.O. Box 3707**

**Seattle, Washington 98124-2207**

# CONCEPT OBJECTIVES

---

CREATE A SUPERIOR COMPUTATIONAL FRAMEWORK FOR MODELING

- SUPPORT LINEAR AND NON-LINEAR SYSTEM MODELS
- HANDLE HIGH-ORDER MODELS
- BASIS FOR FUTURE ENHANCEMENTS
- USER FRIENDLY ANALYSIS
- CONSOLIDATE NASA DRYDEN AND NASA LARC METHODS
- INTERACTIVE GRAPHICS ENVIRONMENT FOR HIGH PRODUCTIVITY AND VISIBILITY



N90-10091

**LUMPED MASS FORMULATIONS  
FOR  
MODELING FLEXIBLE BODY SYSTEMS**

**Rajiv Rampalli**

*Director, Product Technology Group  
Mechanical Dynamics, Inc.  
3055, Plymouth Road  
Ann Arbor, Mi 48105-3203*

**ABSTRACT**

*This paper presents the efforts of Mechanical Dynamics, Inc. in obtaining a general formulation for flexible bodies in a multibody setting. The efforts being supported by MDI, both in house and externally are summarized. The feasibility of using lumped mass approaches to modeling flexibility in a multibody dynamics context is examined. The kinematics and kinetics for a simple system consisting of two rigid bodies connected together by an elastic beam are developed in detail. Accuracy, efficiency and ease of use using this approach are some of the issues that are then looked at.*

*The formulation is then generalized to a "superelement" containing several nodes and connecting several bodies. Superelement kinematics and kinetics equations are developed.*

*The feasibility and effectiveness of the method is illustrated by the use of some examples illustrating phenomena common in the context of spacecraft motions.*

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## **SCOPE OF THE PRESENTATION**

- **PROFILE OF MECHANICAL DYNAMICS, INC.**
- **MDI EFFORTS TO MODEL FLEXIBILITY**
- **LUMPED MASS APPROACHES TO FLEXIBILITY**
- **EXAMPLES**

## **PROFILE OF MECHANICAL DYNAMICS, INC.**

- **COMPANY BACKGROUND**

- **HISTORY**
- **PRODUCTS & SERVICES**
- **CUSTOMERS**

- **CURRENT PRODUCTS**

- **ADAMS**
- **ADAMS / MODAL**
- **POST PROCESSORS**

- **SERVICES**

- **CONSULTING**
- **TRAINING**
- **HOTLINE**

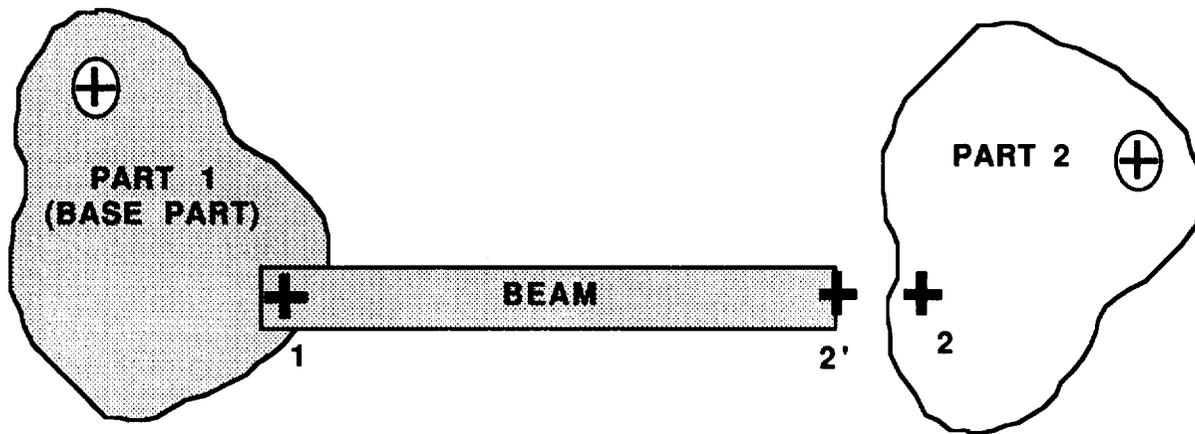
- **AVAILABILITY OF PRODUCTS**

## **MDI EFFORTS IN FLEXIBILITY**

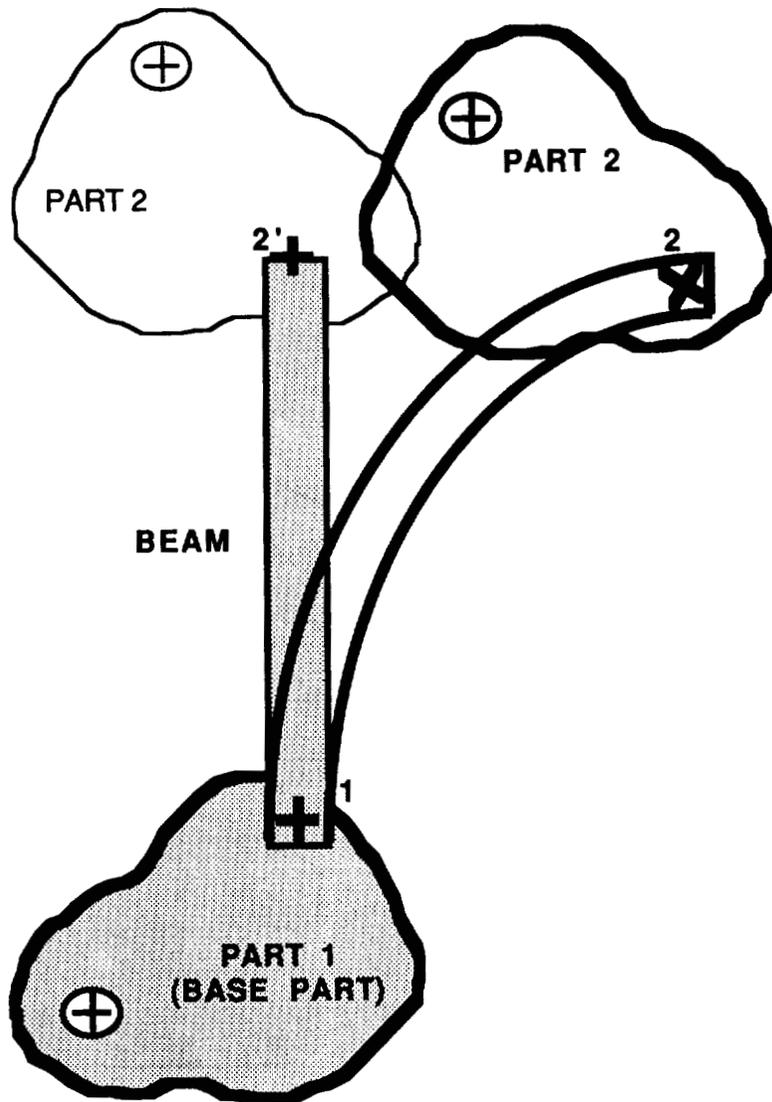
- **INTERNAL R&D**
  - **LUMPED MASS APPROACHES TO FLEXIBILITY**
  
- **EXTERNAL R&D**
  - **UNIVERSITY FUNDED RESEARCH IN MODAL APPROACHES**
  
- **INTERFACE TO FEA PROGRAMS**
  - **NASTRAN**
  - **ANSYS**

- MECHANICAL DYNAMICS, INC. -

**LUMPED MASS APPROACH TO FLEXIBILITY**

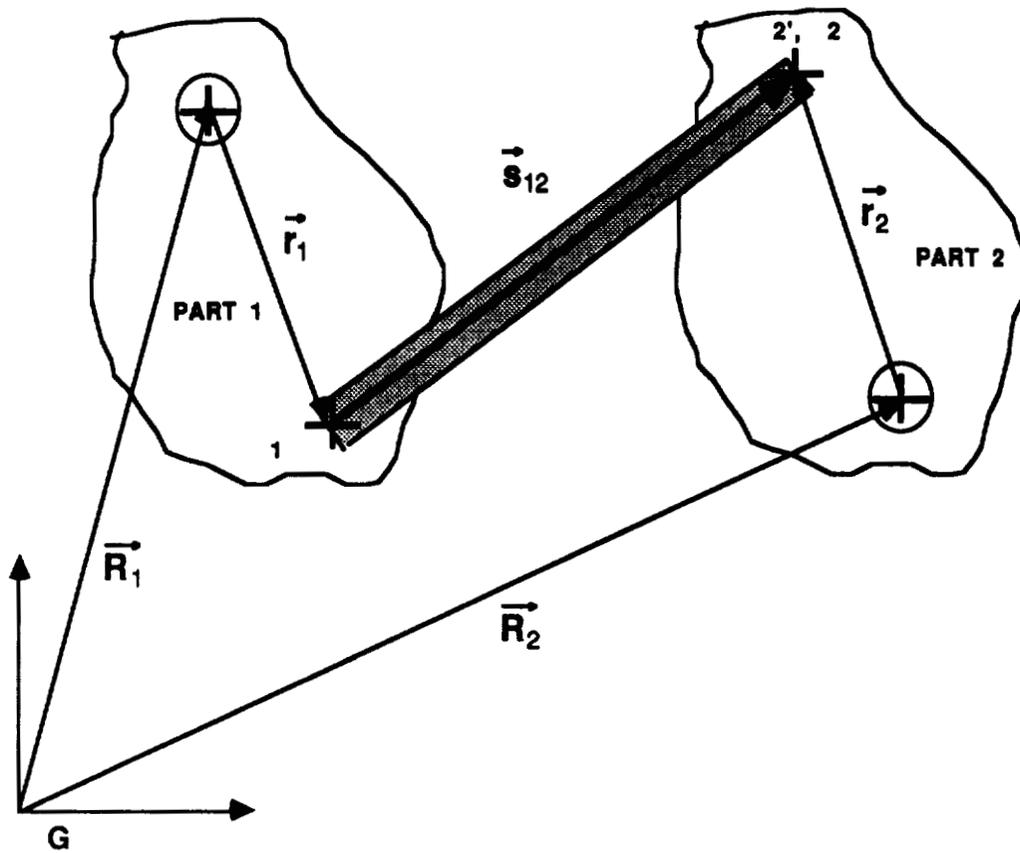


**EXPLODED VIEW OF INITIAL CONFIGURATION**



**BEAM DEFORMATION DURING MOTION**

TRANSLATIONAL DISPLACEMENT COMPUTATION



$$\vec{S}_{12} = \vec{R}_2 + \vec{r}_2 - \vec{R}_1 - \vec{r}_1$$

$${}^1\vec{S}_{12} = \mathbf{A}^{1G} [ \mathbf{R}_2 + \mathbf{A}^{2G} \mathbf{r}_2 - \mathbf{R}_1 - \mathbf{A}^{1G} \mathbf{r}_1 ]$$

$${}^1\Delta_{2'2} = {}^1\vec{S}_{12} \cdot {}^1\mathbf{L}$$

$${}^1\mathbf{L} = \{ L \ 0 \ 0 \}^T$$

ANGULAR DISPLACEMENT COMPUTATION

SPACE 1-2-3 ANGLES ARE USED FOR MEASURING ANGLES

$$\Delta^{2'2} = \begin{bmatrix} C_2 C_3 & S_1 S_2 C_3 - S_3 C_1 & C_1 S_2 C_3 + S_3 S_1 \\ C_2 S_3 & S_1 S_2 S_3 - C_3 S_1 & C_1 S_2 S_3 + C_3 S_1 \\ -S_2 & S_1 C_2 & C_1 C_2 \end{bmatrix}$$

$$\beta_2 = \text{SIN}^{-1}(-a_{31})$$

IF  $\beta_2 \neq \pi/2$  THEN

$$\beta_1 = \text{ARCTAN2}(a_{32}, a_{33})$$

$$\beta_3 = \text{ARCTAN2}(a_{12} + a_{13}, a_{13} \cdot a_{22}) - \beta_1$$

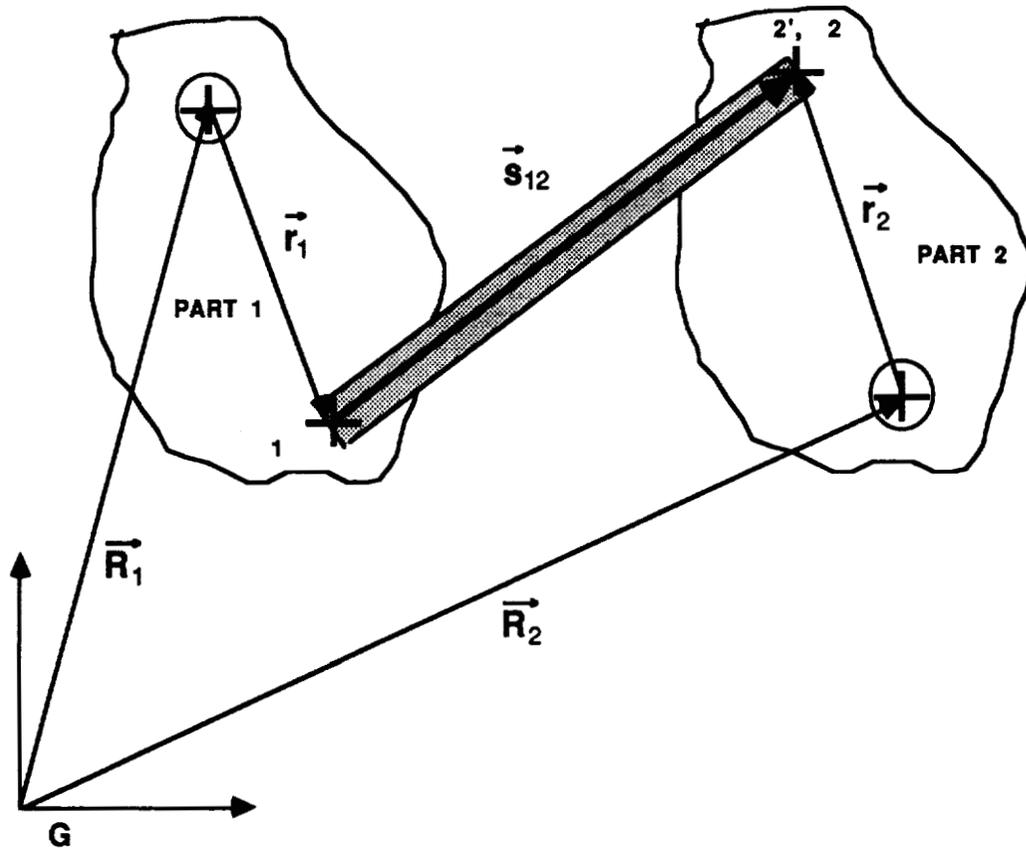
ELSE IF  $\beta_2 = \pi/2$  THEN

$$\beta_3 = \text{ARCTAN2}(a_{21}, a_{11})$$

$$\beta_3 = \text{ARCTAN2}(a_{12}, a_{13}) + \beta_3$$

$$\mathbf{R}^{2'2} = [\beta_1 \ \beta_2 \ \beta_3]^T$$

VELOCITY COMPUTATION



$${}^{(1)}\vec{V}_{2'2} = {}^{(G)}\vec{V}_{21} - \vec{\omega}_1 \times \vec{S}_{21}$$

$${}^1\mathbf{V}_{2'2} = \mathbf{A}^{IJ} [ \mathbf{A}^{IG} (\mathbf{R}_I - \mathbf{R}_J) - \mathbf{A}^{JI} \omega_{\sim I} \mathbf{L}_I - \omega_{\sim J} (\mathbf{A}^{JI} \mathbf{L}_I + \mathbf{A}^{JG} \mathbf{R}_I - \mathbf{A}^{JG} \mathbf{R}_I) ]$$

$$\vec{\omega}_{12} = \vec{\omega}_2 - \vec{\omega}_1$$

$${}^1\omega_{12} = \mathbf{A}^{IJ} [ \mathbf{A}^{JI} \omega_I - \omega_J ]$$

FORCE COMPUTATION

FORCES AT ORIGIN OF COORDINATE SYSTEM ON REF. FRAME 1

$$\begin{aligned} {}_1\mathbf{F}_2 = & - [ \mathbf{K}_{11} \Delta_{2'2} + \mathbf{K}_{12} \mathbf{B}_{12} ] \\ & - [ \mathbf{C}_{11} \dot{\Delta}_{2'2} + \mathbf{C}_{12} \dot{\mathbf{B}}_{12} ] \end{aligned}$$

$$\begin{aligned} {}_1\mathbf{T}_2 = & - [ \mathbf{K}_{21} \Delta_{2'2} + \mathbf{K}_{22} \mathbf{B}_{12} ] \\ & - [ \mathbf{C}_{21} \dot{\Delta}_{2'2} + \mathbf{C}_{22} \dot{\mathbf{B}}_{12} ] \end{aligned}$$

$\mathbf{K}$  IS THE STANDARD MATRIX FOUND IN ANY STRUCTURAL ANALYSIS TEXT

FORCES AT ORIGIN OF COORDINATE SYSTEM ON REF. FRAME 2

SINCE THE BEAM IS MASSLESS, APPLYING LAWS OF EQUILIBRIUM :

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \vec{\mathbf{0}}$$

$${}_1\mathbf{F}_1 = - {}_1\mathbf{F}_2$$

$$\vec{\mathbf{T}}_1 + \vec{\mathbf{T}}_2 + \vec{\mathbf{S}}_{12} \times \vec{\mathbf{F}}_2 = \vec{\mathbf{0}}$$

$${}_1\mathbf{T}_1 = - [ {}_1\mathbf{T}_2 + {}_1\tilde{\mathbf{S}}_{12} {}_1\mathbf{F}_2 ]$$

**ACCURACY OF METHOD**

- **DIRECTLY RELATED TO DEGREE OF DISCRETIZATION**
- **METHOD DOES NOT YIELD WRONG ANSWERS**
- **DEGREE OF DISCRETIZATION DEPENDENT ON FREQUENCY  
CONTENT DESIRED. ADAMS/MODAL WILL COMPUTE  
EIGENVALUES AND EIGENVECTORS FOR ANY ADAMS MODEL.  
CAN ANIMATE LINEAR MODEL USING SELECTED SET OF MODE  
SHAPES AND FREQUENCIES.**

**FLEXIBLE BODY DYNAMICS**  
*Benchmark Problem #1: Uniform Cantilever Beam Spin-up*

The inset of Fig. 1 shows a uniform, homogeneous, cantilever beam supported by a circular hub of radius  $r$ . At time  $t = 0$ , the system is at rest in a Newtonian reference frame and the beam is undeformed. Subsequent to this initial time, the hub is made to rotate about a vertical axis  $X - X$ , passing through the center of the hub, in such a way that  $\Omega$ , the angular speed of the hub, is given by

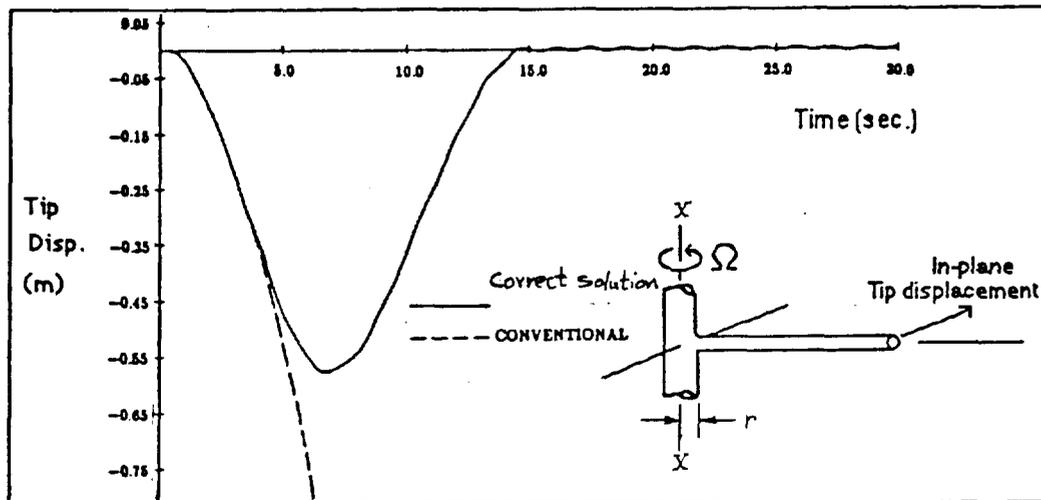
$$\Omega(t) = \begin{cases} (2/5) \left[ t - (7.5/\pi) \sin(\pi t/7.5) \right] \text{ rad/sec} & 0 \leq t \leq 15 \text{ sec} \\ 6 \text{ rad/sec} & t > 15 \text{ sec} \end{cases}$$

which represents a smooth transition from zero hub motion to a constant angular speed of 6 rad/sec. The beam has a length  $L$ , Young's modulus  $E$ , shear modulus  $G$ , mass per unit length  $\rho$ , and a circular cross-section of area  $A$  and area moment of inertia  $I$ .

The solid line in the figure below shows the time history of the displacement of the beam tip, in the plane of rotation, relative to a line fixed in the hub and originally parallel to the centroidal axis of the beam. This result was obtained using the theory and algorithm presented in Refs.[1] and [2] with three assumed modes and the following parameter values

$$\begin{aligned} r &= 0 \text{ m} & \rho &= 1.2 \text{ kg/m} \\ L &= 10 \text{ m} & A &= 4 \times 10^{-4} \text{ m}^2 \\ E &= 7 \times 10^{10} \text{ N/m}^2 & G &= 3 \times 10^{10} \text{ N/m}^2 & I &= 2 \times 10^{-7} \text{ m}^4 \end{aligned}$$

All external forces were neglected and the assumed modal functions were chosen to be equal to the first three eigenfunctions of an identical uniform cantilever beam with its root fixed in a Newtonian reference frame. The numerical integration was carried out using a 4<sup>th</sup> - 5<sup>th</sup> order, variable step-size, Runge-Kutta-Merson method with a print step and initial time step of .03 seconds and an error tolerance of  $1 \times 10^{-6}$ . The dashed line result was produced with an algorithm based on the assumed-mode formulation utilized in most flexible multibody programs. This result was verified by Fidelis Eke [(818) 354-2916] at Jet Propulsion Labs using DISCOS.



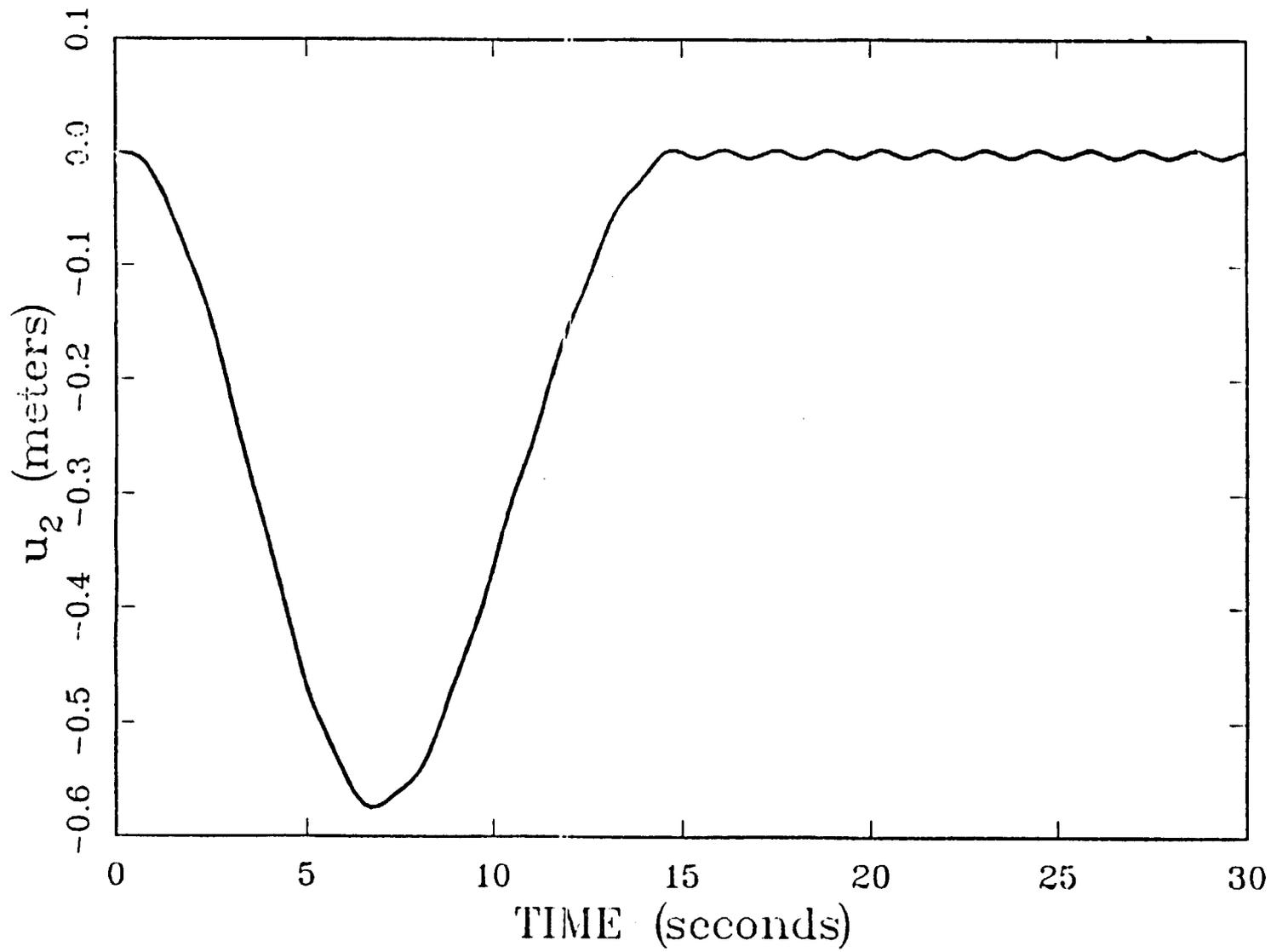
**Fig. 1 Spin-up of Homogeneous Uniform Cantilever Beam**

- [1] Ryan, R.R., "Flexibility Modeling Methods in Multibody Dynamics," Ph.D. dissertation, Stanford University, Available from University Microfilms, Ann Arbor, Michigan, 1986.
- [2] Kane, T.R., Ryan, R.R., and Banerjee, A.K., "Dynamics of a Cantilever Beam Attached to a Moving Base," *Journal of Guidance, Control, and Dynamics*, Vol.10, No.2, March-April, 1987.

# UNIFORM BEAM SPIN-UP PROBLEM

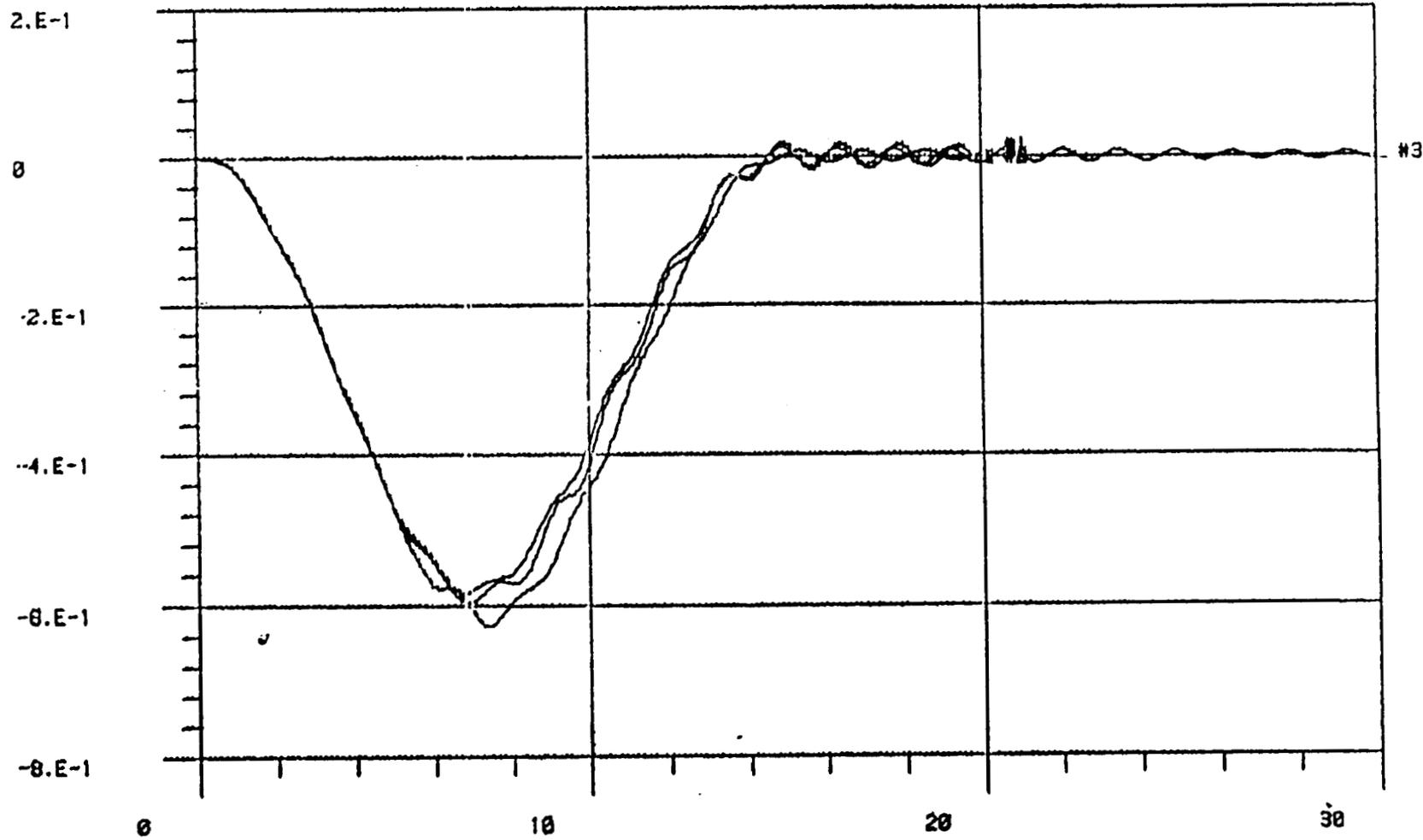
## BEAM TIP DEFLECTION VS. TIME

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# UNIFORM BEAM SPIN - UP PROBLEM : ADAMS APPROACH

## BEAM TIP DEFLECTION Vs. TIME

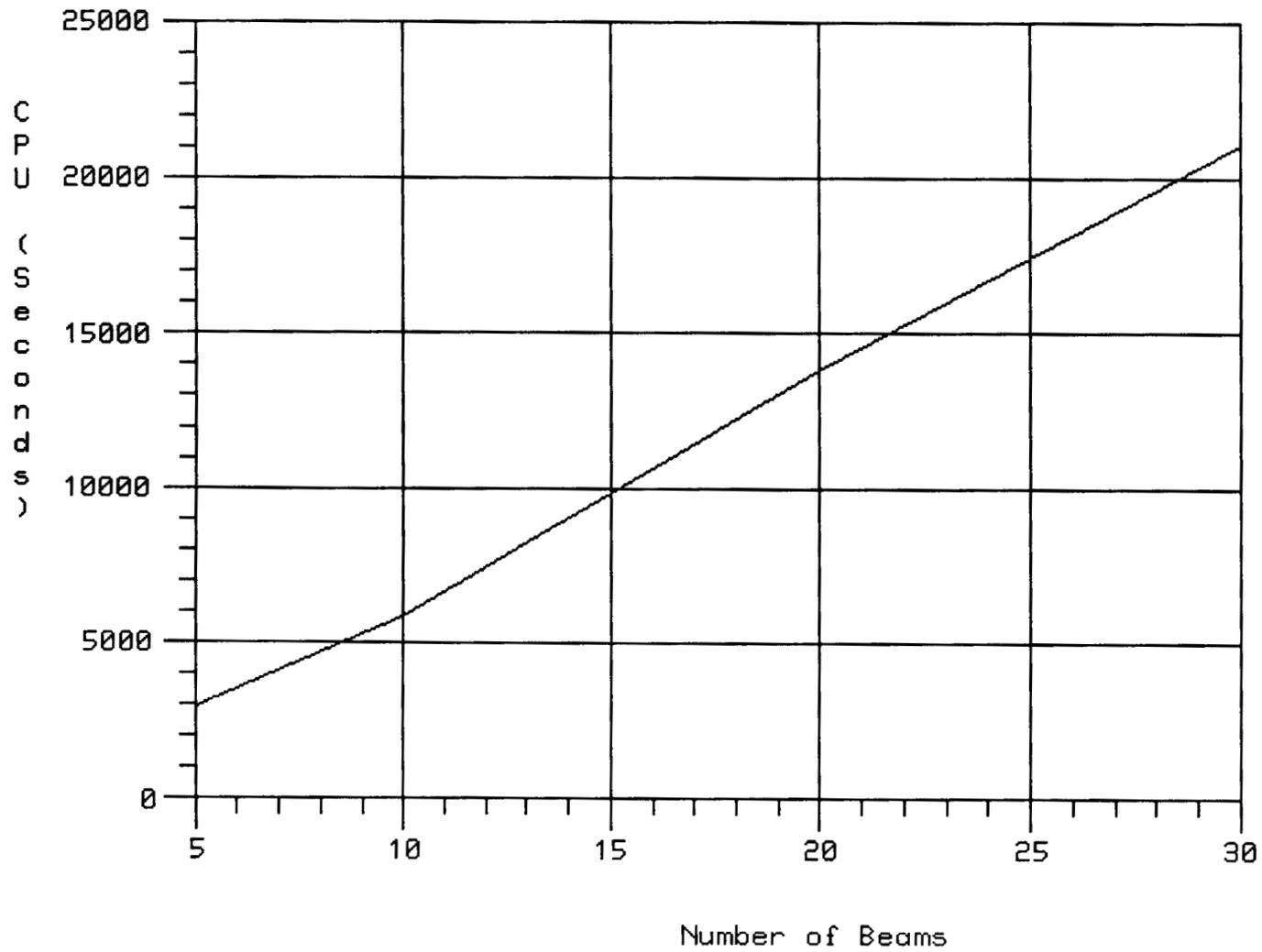


257

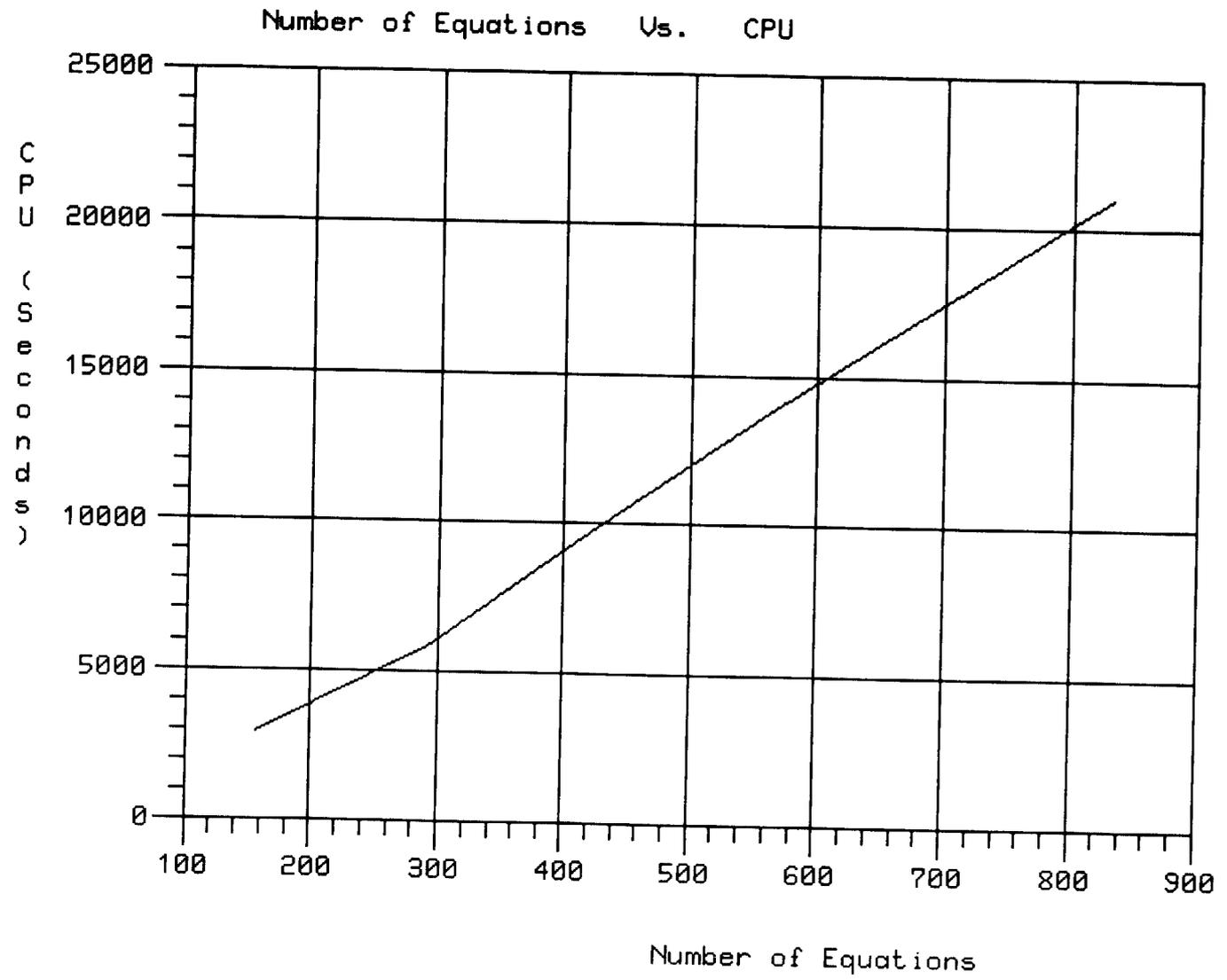
**EFFICIENCY OF METHOD**

- **THIS METHOD IS USABLE FOR SMALL TO MEDIUM SIZE PROBLEMS  
( MEDIUM = 300 RIGID AND FLEXIBLE DOF )**
  
- **FOR LARGER PROBLEMS IT MAY PROVE TO BE MORE CPU INTENSIVE  
THAN DESIRABLE.**
  
- **THE CPU TIME TAKEN FOR A SIMULATION IS **LINEARLY**  
PROPORTIONAL TO THE NUMBER OF FLEXIBLE BEAMS IN  
THE SYSTEM**

Number of Beams Vs. CPU Time



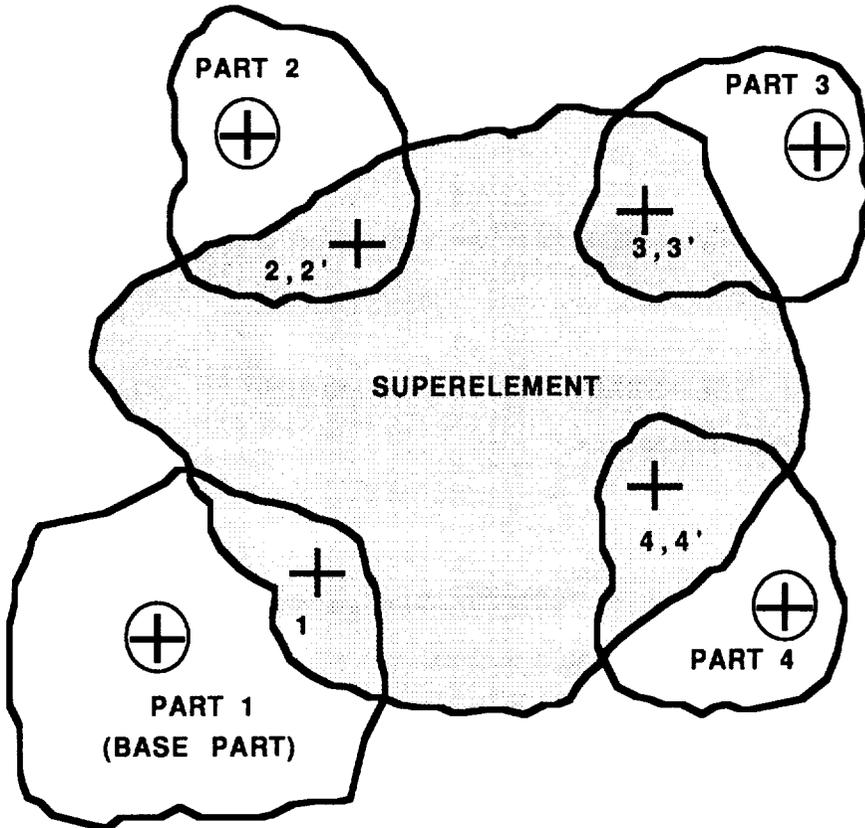
259



**EASE OF USE**

- **THE RESULTING PROGRAM IS EXTREMELY EASY TO USE.**
- **USERS DO NOT NEED STRONG FEA BACKGROUND TO CREATE MODELS OF STRUCTURES**
- **RECOGNITION AND SELECTION OF PROPER MODES IN AN ART. THE RESULTS ARE ONLY AS GOOD AS THE SELECTED MODES. DIFFICULTY ALLEVIATED IN THIS APPROACH.**

## GENERALIZATION TO SUPERELEMENTS



### NODES

- LOCATION OF NODES 2, 3, 4 WRT. TO A KNOWN REFERENCE FRAME
- MASS AND INERTIA PROPERTIES FOR EACH NODE OBTAINED FROM MASS MATRIX.

### LOCATION AND ORIENTATION OF COORDINATE SYSTEMS

2 ON PART 2

2' ON PART 1

3 ON PART 3

3' ON PART 1

4 ON PART 4

4' ON PART 1

**ASSEMBLY**

- **PRE-TENSION AND INITIAL DISPLACEMENTS  
AT CONNECTION POINTS 2-2', 3-3', 4-4'**

**FLEXIBILITY PROPERTIES**

- **STIFFNESS MATRIX**
- **DAMPING MATRIX**

**DISPLACEMENT COORDINATES**

$${}^1\Delta = [ {}^1\Delta_{2'2} \quad {}^1\Delta_{3'3} \quad {}^1\Delta_{4'4} ]^T$$

$$\mathbb{B} = [ \mathbb{B}_{2'2} \quad \mathbb{B}_{3'3} \quad \mathbb{B}_{4'4} ]^T$$

**VELOCITY COORDINATES**

$${}^1\underline{Y} = [ {}^1\underline{Y}_{2'2} \quad {}^1\underline{Y}_{3'3} \quad {}^1\underline{Y}_{4'4} ]^T$$

$${}^1\Omega = [ {}^1\Omega_{2'2} \quad {}^1\Omega_{3'3} \quad {}^1\Omega_{4'4} ]^T$$

**FORCE DEFINITION AT COORDINATE SYSTEMS 2, 3, 4**

$${}^1\mathbf{E} = [ {}^1\mathbf{E}_2 \quad {}^1\mathbf{E}_3 \quad {}^1\mathbf{E}_4 ]^T$$

$${}^1\mathbf{T} = [ {}^1\mathbf{T}_2 \quad {}^1\mathbf{T}_3 \quad {}^1\mathbf{T}_4 ]^T$$

FORCE COMPUTATION AT COORDINATE SYSTEMS 2, 3, 4

$$\begin{aligned} {}_1\mathbf{E} = & -[ \mathbf{K}_{11} {}_1\Delta + \mathbf{K}_{12} \mathbf{B} ] \\ & -[ \mathbf{C}_{11} {}_1\mathbf{Y} + \mathbf{C}_{12} {}_1\omega ] + \mathbf{E}_0 \end{aligned}$$

$$\begin{aligned} {}_1\mathbf{T} = & -[ \mathbf{K}_{21} {}_1\Delta + \mathbf{K}_{22} \mathbf{B} ] \\ & -[ \mathbf{C}_{21} {}_1\mathbf{Y} + \mathbf{C}_{22} {}_1\omega ] + \mathbf{T}_0 \end{aligned}$$

FORCE AT COORDINATE SYSTEM 1

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = \vec{\mathbf{0}}$$

$${}_1\mathbf{E}_1 = - [ {}_1\mathbf{E}_2 + {}_1\mathbf{E}_3 + {}_1\mathbf{E}_4 ]$$

$$\vec{\mathbf{T}}_1 + \vec{\mathbf{T}}_2 + \vec{\mathbf{T}}_3 + \vec{\mathbf{T}}_4 + \vec{\mathbf{S}}_{12} \times \vec{\mathbf{F}}_2 + \vec{\mathbf{s}}_{13} \times \vec{\mathbf{F}}_3 + \vec{\mathbf{s}}_{14} \times \vec{\mathbf{F}}_4 = \vec{\mathbf{0}}$$

$${}_1\mathbf{T}_1 = - [ {}_1\mathbf{E}_2 + {}_1\mathbf{E}_3 + {}_1\mathbf{E}_4 ]$$

$$- [ {}_1\tilde{\mathbf{s}}_{12} {}_1\mathbf{E}_2 + {}_1\tilde{\mathbf{s}}_{13} {}_1\mathbf{E}_3 + {}_1\tilde{\mathbf{s}}_{14} {}_1\mathbf{E}_4 ]$$

**N90-10092**

**A Comparison of Software for the Modeling and  
Control of Flexible Systems**

**Lawrence W. Taylor, Jr.  
NASA Langley Research Center**

**Computational Aspects Workshop  
July 12-14, 1988 Williamsburg, VA**

Memorandum

To: Whom it may concern

From: 161/Chief Scientist, GCD

Subject: Development of Software for the Control of Flexible Systems

*A memorandum prepared*

I propose a cooperative effort among specialists who use or develop software for simulating and analyzing the control of flexible, aerospace systems. A comparison of existing software for modeling control systems and flexible structures, applied to several example problems would be quite valuable. The comparison would indicate computational efficiency and capabilities with respect to handling nonlinearities and graphical output.

Because of the diversity of applications of such software, <sup>*no secrets*</sup> I believe that the proposed cooperative effort can transcend projects involving specific applications. Comparisons of software capability and efficiency can be made and gaps can be identified. In this way the results of the cooperative effort can provide guidance for individual projects.

Enclosed are several charts which outline the objectives and approach of the proposed cooperative effort. I would appreciate your suggestions and expressions of interest in this matter.

*See your file*  
Sincerely,

*Lawrence W. Taylor, Jr.*

Lawrence W. Taylor, Jr.  
Mail Stop 161  
NASA Langley Research Center  
Hampton, VA 23665  
(804)-865-4591

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## **OBJECTIVE**

**"To Evaluate Software for the Control, Analysis, Simulation and Design of Flexible Aerospace Systems....**

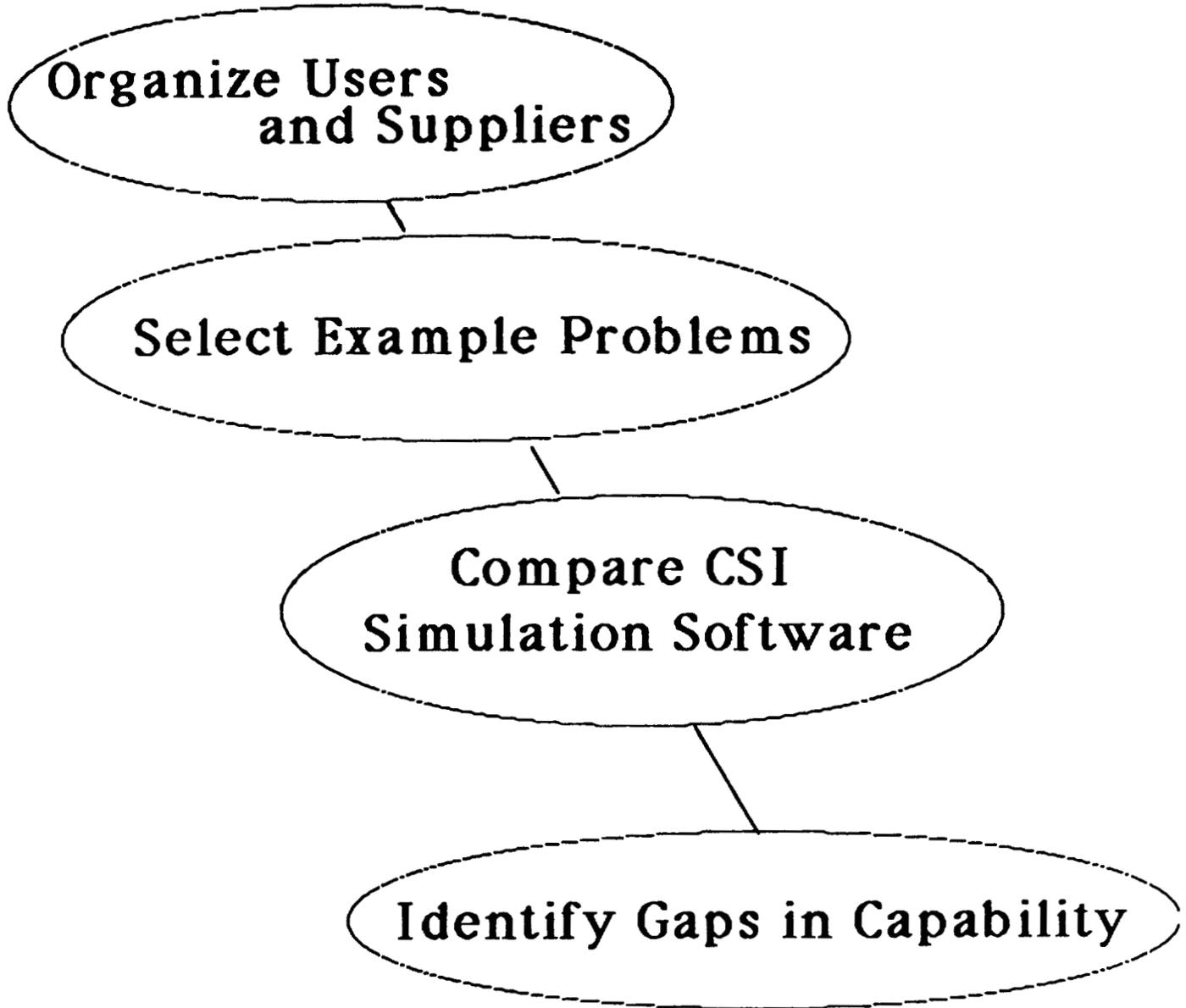
**Which includes:**

- **Control Law Dynamics**
- **Actuator/Sensor Dynamics**
- **Structural Dynamics**

**And Which is Efficient and Accurate**

**And Which is Easy to Use."**

## **APPROACH**



## **Example Problems**

- **Uniform Beam - Pinned-Pinned**
- **SCOLE (L. Taylor-LaRC)**
- **Translation/Rotation/Flex (Juang-LaRC)**
- **SAFE(70% Deployment)(L. Taylor-LaRC)**
- **Pinhole Occulter (Henry Waites-MSFC)**
- **Manned Space Station**
  - a. **Reboost**
  - b. **Solar Dynamic Pointing**
  - c. **MRMS Operation**
  - d. **Docking**

**Others?**

## Pinned-Pinned Beam



$$m\ddot{u} + c|\dot{u}''|\dot{u}'' + EIu'''' = 0$$

$$u(x,0) = A \sin(\pi x/L)$$

$$A = 1.3 \text{ Ft.}$$

$$L = 130 \text{ Ft.}$$

$$EI = 40,000,000 \text{ Lb/Ft}^2$$

$$m = .09556 \text{ Slugs/Ft}$$

$$c = 280.32$$

$$f_{50}(s) = \frac{K\dot{u}(50,s)}{(1 + Ts)}$$

$$K = -.5$$

$$T = .2 \text{ Sec}$$

### Problem:

1. Calculate Time History of  $u(65,t)$   $0 < t < 5.26$
2. Plot Time History
3. Calculate Modal Characteristics  $1 < k < 10$
4. Express Final Shape in Modal Coordinates

## CSI Simulation Software

- NASTRAN
- DISCOS
- TREETOPS(CONTOPS)
- EAL
- LATDYN
- DADS
- Multi-MACS
- ORACLS
- EISPAC
- LINPAC
- Matrix<sub>x</sub>
- CTRL-C
- SYSPAC

# Information Sheet

**NAME of SOFTWARE:** DISCOS

**RESPONSIBLE PERSON:** Harry Frisch  
NASA Goddard Space Center  
Bldg. 11, Rm. S221A  
Greenbelt, MD 20771

## **CAPABILITIES:**

- |  |   |  |       |
|--|---|--|-------|
| 1. Nonlinear Kinematics?                     | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 2. Finite Element Modeling?                  | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 3. All Rotational and Translational D.O.F.?  | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 4. Linked to Distributed Parameters?         | Yes <input type="checkbox"/>            | No <input type="checkbox"/>            | _____ |
| 5. Large Amplitudes?                         | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 6. Nonlinear Damping?                        | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |
| 7. Control Law Dynamics?                     | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |
| 8. Sensors and Actuator Dynamics?            | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |
| 9. Nonlinear Joints?                         | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |
| 10. Distributed Parameter System?            | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |
| 11. Optimal Control Synthesis?               | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |
| 12. Sensitivity Functions for P.E. & Design? | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |

**DOCUMENTATION:** Complete.

**SIZE:** 500K

**LANGUAGE:** Fortran 77 +

**INTERACTIVE:** No.

**GRAPHICS:** Plots

**AVAILABILITY:** Free, Nonproprietary

# Information Sheet

**NAME of SOFTWARE:** LATDYN

**RESPONSIBLE PERSON:** Jerry Housner  
Mail Stop - 230  
NASA Langley Research Center  
Hampton, VA 23665

## **CAPABILITIES:**

- |  |   |  |                             |
|--|---|--|-----------------------------|
| 1. Nonlinear Kinematics?                     | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>2-D.O.F.</u>             |
| 2. Finite Element Modeling?                  | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>2-D.O.F.</u>             |
| 3. All Rotational and Translational D.O.F.?  | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>2-D.O.F.</u>             |
| 4. Linked to Distributed Parameters?         | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>2-D.O.F.</u>             |
| 5. Large Amplitudes?                         | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>2-D.O.F.</u>             |
| 6. Nonlinear Damping?                        | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>Add Code</u>             |
| 7. Control Law Dynamics?                     | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>Add Code</u>             |
| 8. Sensors and Actuator Dynamics?            | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>Add Code</u>             |
| 9. Nonlinear Joints?                         | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>Add Code</u>             |
| 10. Distributed Parameter System?            | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | <u>2-D.O.F.</u>             |
| 11. Optimal Control Synthesis?               | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | <u>                    </u> |
| 12. Sensitivity Functions for P.E. & Design? | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | <u>                    </u> |

**DOCUMENTATION:** 2-D.O.F. Written, 3-D.O.F. Under Development

**SIZE:** 400K

**LANGUAGE:** Fortran 77

**INTERACTIVE:** Yes

**GRAPHICS:** Time Histories, Line Drawing, PSD, Movies

**AVAILABILITY:** Free, Nonproprietary

# Information Sheet

**NAME:** Multibody Analysis & Control Synthesis (MACS)

**RESPONSIBLE PERSON:** Lawrence W. Taylor  
NASA Langley Research Center  
Hampton, VA 23665  
(804)-865-4591

## **CAPABILITIES:**

- |   |   |  |         |
|---|---|--|---------|
| 1. Nonlinear Kinematics?                    | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____   |
| 2. Finite Element Modeling?                 | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____   |
| 3. All Rotational and Translational D.O.F.? | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____   |
| 4. Linked to Distributed Parameters?        | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____   |
| 5. Large Amplitudes?                        | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____   |
| 6. Nonlinear Damping?                       | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | Not Yet |
| 7. Control Law Dynamics?                    | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____   |
| 8. Sensors and Actuator Dynamics?           | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____   |
| 9. Nonlinear Joints?                        | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | Not Yet |
| 10. Distributed Parameter System?           | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____   |
| 11. Optimal Control Synthesis?              | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____   |

**DOCUMENTATION:** Incomplete

**SIZE:** 25K Core Memory

**LANGUAGE:** FORTRAN 77

**INTERACTIVE:** No

**GRAPHICS:** None

**AVAILABILITY:** No Charge

# Information Sheet

**NAME of SOFTWARE:** TREETOPS (CONTOPS)

**RESPONSIBLE PERSON:** Ramen Singh  
Dynacs Engineering Company  
2280 U.S. 19 No., Suite 111  
Clearwater, FL 34623

## **CAPABILITIES:**

- |  |   |  |       |
|--|---|--|-------|
| 1. Nonlinear Kinematics?                     | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 2. Finite Element Modeling?                  | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 3. All Rotational and Translational D.O.F.?  | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 4. Linked to Distributed Parameters?         | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 5. Large Amplitudes?                         | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 6. Nonlinear Damping?                        | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 7. Control Law Dynamics?                     | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 8. Sensors and Actuator Dynamics?            | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 9. Nonlinear Joints?                         | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 10. Distributed Parameter System?            | Yes <input checked="" type="checkbox"/> | No <input type="checkbox"/>            | _____ |
| 11. Optimal Control Synthesis?               | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |
| 12. Sensitivity Functions for P.E. & Design? | Yes <input type="checkbox"/>            | No <input checked="" type="checkbox"/> | _____ |

**DOCUMENTATION:** Complete. Course Available.

**SIZE:** 600K

**LANGUAGE:** Fortran 77 +

**INTERACTIVE:** Yes. Sun, MicroVAX, Masscomp

**GRAPHICS:** Plots, Windows, Movies

**AVAILABILITY:** Free, Nonpropriety

# Information Sheet

**NAME of SOFTWARE:** \_\_\_\_\_

**RESPONSIBLE PERSON:** \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

## **CAPABILITIES:**

- |  |                              |                             |       |
|--|------------------------------|-----------------------------|-------|
| 1. Nonlinear Kinematics?                     | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 2. Finite Element Modeling?                  | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 3. All Rotational and Translational D.O.F.?  | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 4. Linked to Distributed Parameters?         | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 5. Large Amplitudes?                         | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 6. Nonlinear Damping?                        | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 7. Control Law Dynamics?                     | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 8. Sensors and Actuator Dynamics?            | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 9. Nonlinear Joints?                         | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 10. Distributed Parameter System?            | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 11. Optimal Control Synthesis?               | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |
| 12. Sensitivity Functions for P.E. & Design? | Yes <input type="checkbox"/> | No <input type="checkbox"/> | _____ |

**DOCUMENTATION:** \_\_\_\_\_

**SIZE:** \_\_\_\_\_

**LANGUAGE:** \_\_\_\_\_

**INTERACTIVE:** \_\_\_\_\_

**GRAPHICS:** \_\_\_\_\_

**AVAILABILITY:** \_\_\_\_\_

## Solution Characteristics

- Time Required - CPU Sec.
- Accuracy
  - a. Modal Characteristics
  - b. Time Histories (PSD)
- Memory Required
- Input
- Output

## PANEL DISCUSSION

1. Should We Compare Software for  
Control/Modeling?
2. Should We Establish Example Problems?
3. Should This Workshop be Repeated?  
.....Changed?  
.....Merged?

**SESSION III - COMPUTATIONS EFFICIENCY AND CAPABILITY**

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N90-10093

**A FINITE ELEMENT APPROACH FOR LARGE MOTION DYNAMIC ANALYSIS OF  
MULTIBODY STRUCTURES IN SPACE**

By

Che-Wei Chang  
The COMTEK Company  
Grafton, Virginia

**ABSTRACT**

A three-dimensional finite element formulation for modeling the transient dynamics of constrained multibody space structures with truss-like configurations is presented. Convected coordinate systems are used to define rigid-body motion of individual elements in the system. These systems are located at one end of each element and are oriented such that one axis passes through the other end of the element. Deformation of each element, relative to its convected coordinate system, is defined by cubic flexural shape functions as used in finite element methods of structural analysis. The formulation is oriented toward joint dominated structures and places the generalized coordinates at the joint. A transformation matrix is derived to integrate joint degree-of-freedom into the equations of motion of the element. Based on the derivation, a general-purpose code LATDYN (Large Angle Transient DYNAMics) has been developed. Two examples are presented to illustrate the application of the code. For the spin-up of a flexible beam, results are compared with existing solutions available in the literature. For the deployment of one bay of a deployable space truss (the "Minimast"), results are verified by the geometric knowledge of the system and converged solution of a successively refined model.

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**LATDYN**

**Large Angle Transient DYNamics**

**(Finite-Element-Based)**

**A NASA Facility for Research**

**in**

**Applications and Analysis Techniques**

**for Space Structure Dynamics**

**Presented by**

**Che-Wei Chang**

**COMTEK**

**284**

## **TALK OUTLINE**

- \* Motivation**
- \* Capability**
- \* Theory**
- \* Modelling**
- \* Present LATDYN  
(verifications)**
- \* Future LATDYN**
- \* Conclusions**

## **CAPABILITIES**

- \* Three-Dimensional**
- \* Deformable Bodies**
- \* Multi-Connection Joints**
- \* Large Angular Motion**
- \* Variable Constraints**
- \* Impacts & Joint-Lock**
- \* Experimental Data**
- \* User's Control Strategy**

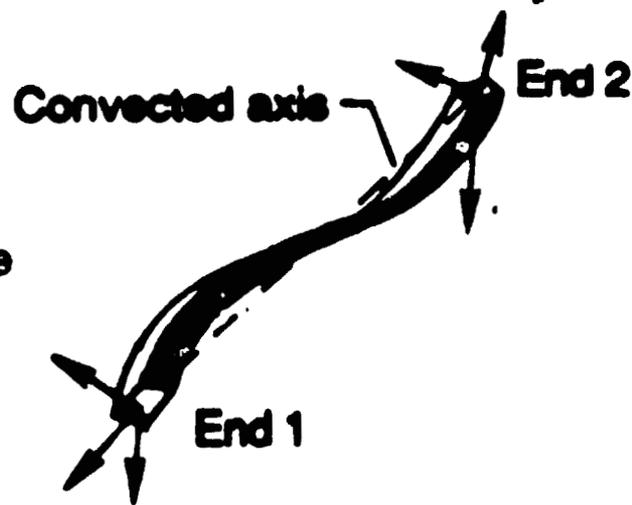
## **BACKGROUND THEORY**

- 1. Corotational Axes  
(convected system)**
- 2. F-E Connectivity through  
Joint Kinematics**
- 3. Numerical Integrations**

# DEFORMED FINITE ELEMENT AND ELEMENT COORDINATE SYSTEMS

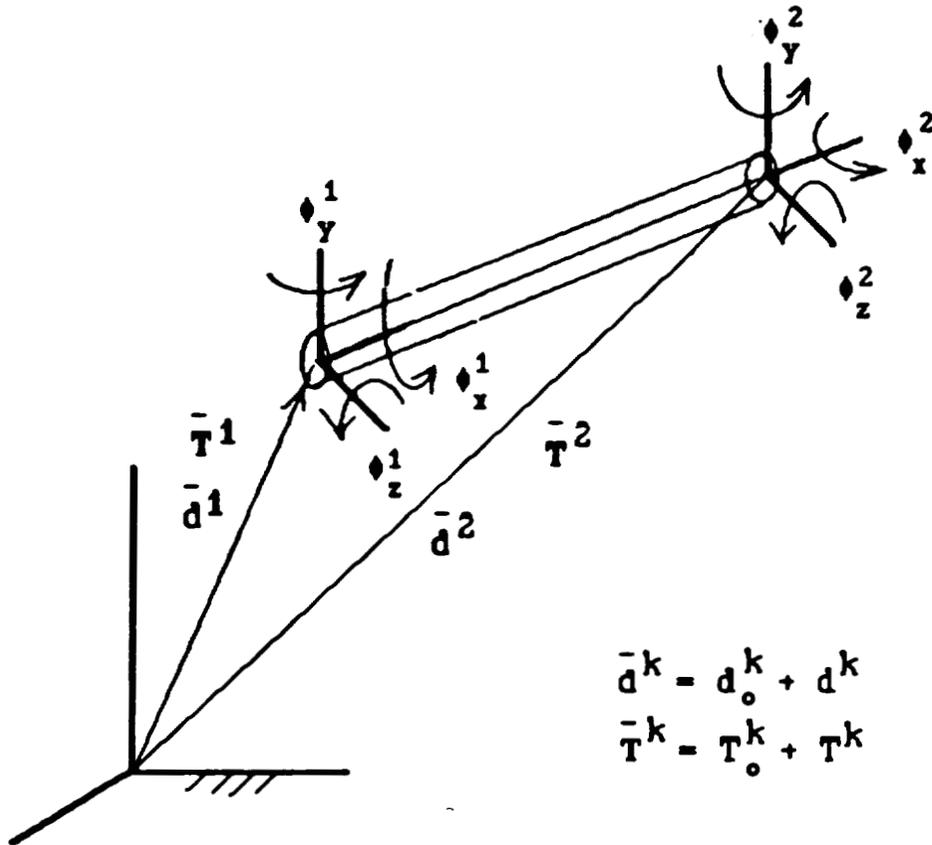
288

Element coordinates move  
with cross-section



Deformations are measured from convected axes

## Deformation : u



$$\begin{aligned} \bar{d}^k &= d_o^k + d^k \\ \bar{T}^k &= T_o^k + T^k \end{aligned} \quad ; k = 1 \text{ or } 2$$

$$\Phi = \left[ \begin{matrix} \phi_x^1, \phi_y^1, \phi_z^1, \phi_x^2, \phi_y^2, \phi_z^2 \\ u_x^1, u_y^1, u_z^1, u_x^2, u_y^2, u_z^2 \end{matrix} \right]^T$$

$$\mathbf{u} = \mathbf{N} \Phi$$

$$\mathbf{u} = [u_x, u_y, u_z]^T$$

$$\Phi = \Phi(d, T)$$

$$\mathbf{d} = [d^{1T}, d^{2T}]^T$$

$$\mathbf{T} = \mathbf{T}(\theta^1, \theta^2)$$

## Internal Force

because

$$\boldsymbol{\varepsilon} = \mathbf{D} \boldsymbol{\Phi} \quad \& \quad \boldsymbol{\sigma} = \mathbf{E} \boldsymbol{\varepsilon}$$

$\therefore$

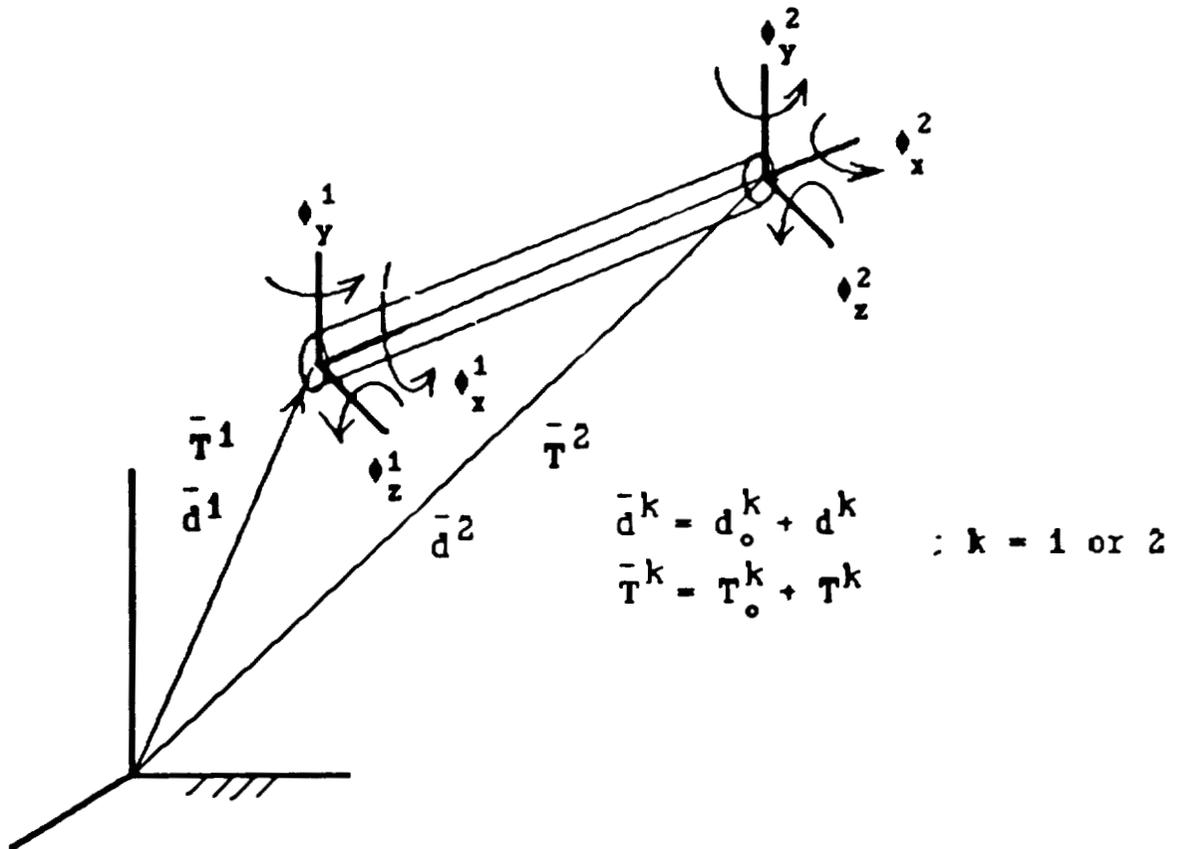
$$\boldsymbol{\sigma} = \mathbf{E} \mathbf{D} \boldsymbol{\Phi}$$

$$\delta \boldsymbol{\varepsilon} = \mathbf{D} \delta \boldsymbol{\Phi} = \mathbf{D} \mathbf{B} \delta \mathbf{q}$$

virtual work done by internal force

$$\begin{aligned} \delta W &= -\delta U \\ &= -\int_{dV} \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} \, dV \\ &= \delta \mathbf{q}^T \left\{ -\int_{dV} (\mathbf{D} \mathbf{B})^T \mathbf{D} \boldsymbol{\Phi} \, dV \right\} \end{aligned}$$

## Total displacement $\bar{u}$



$$\bar{u} = \bar{d}^1 + T_c u$$

$$\delta \bar{u} = C \delta q$$

$$\dot{\bar{u}} = C \dot{q}$$

$$\ddot{\bar{u}} = C \ddot{q} + \dot{C} \dot{q}$$

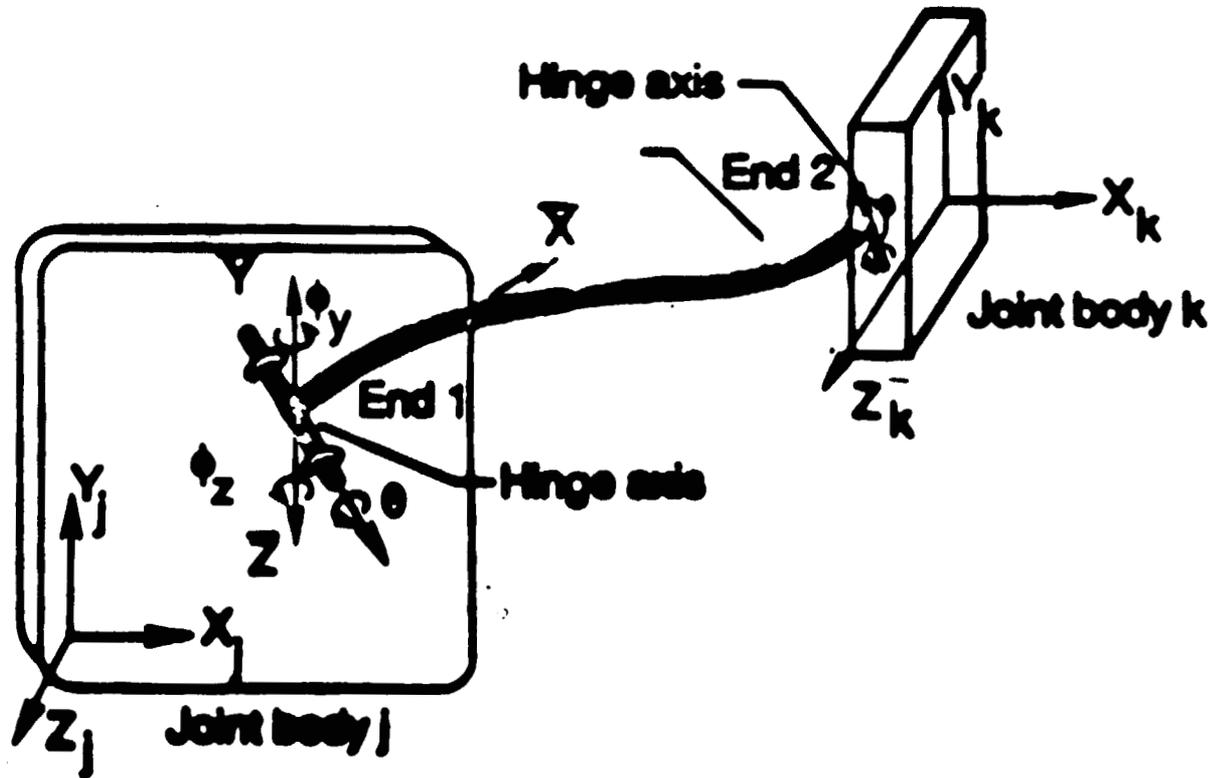
C-4

## Inertia

virtual work done by inertia force

$$\begin{aligned}\delta W &= \int_{dV} \{ \delta \bar{u}^T (-\rho \ddot{u}) \} dV \\ &= \delta q^T \left\{ - \left( \int_{dV} \rho C^T C dV \right) \ddot{q} \right\} \\ &\quad + \delta q^T \left\{ - \left( \int_{dV} \rho C^T \dot{C} dV \right) \dot{q} \right\}\end{aligned}$$

# TYPICAL INTERCONNECTION OF TWO JOINT BODIES THROUGH FLEXIBLE BEAM



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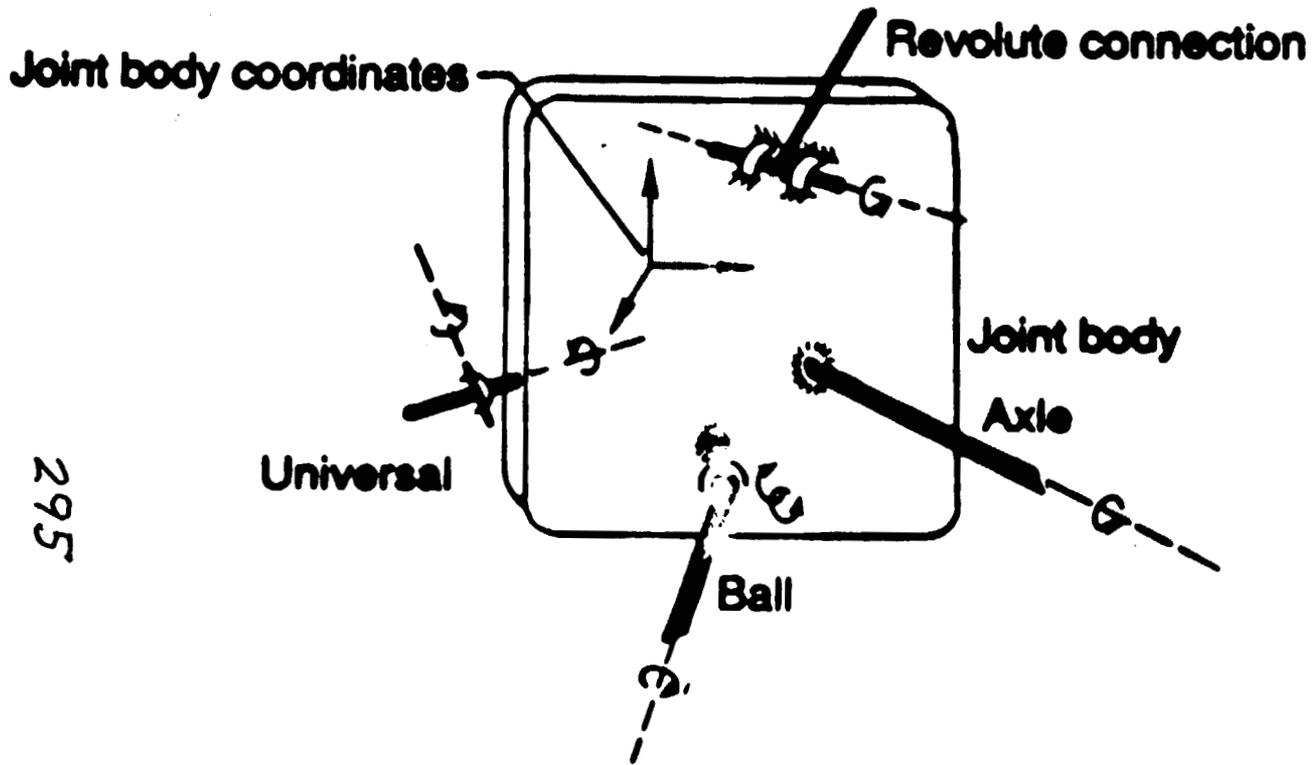
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**Element EQ's of Motion**

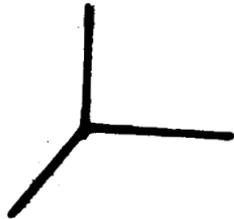
**in terms of nodal disp.**

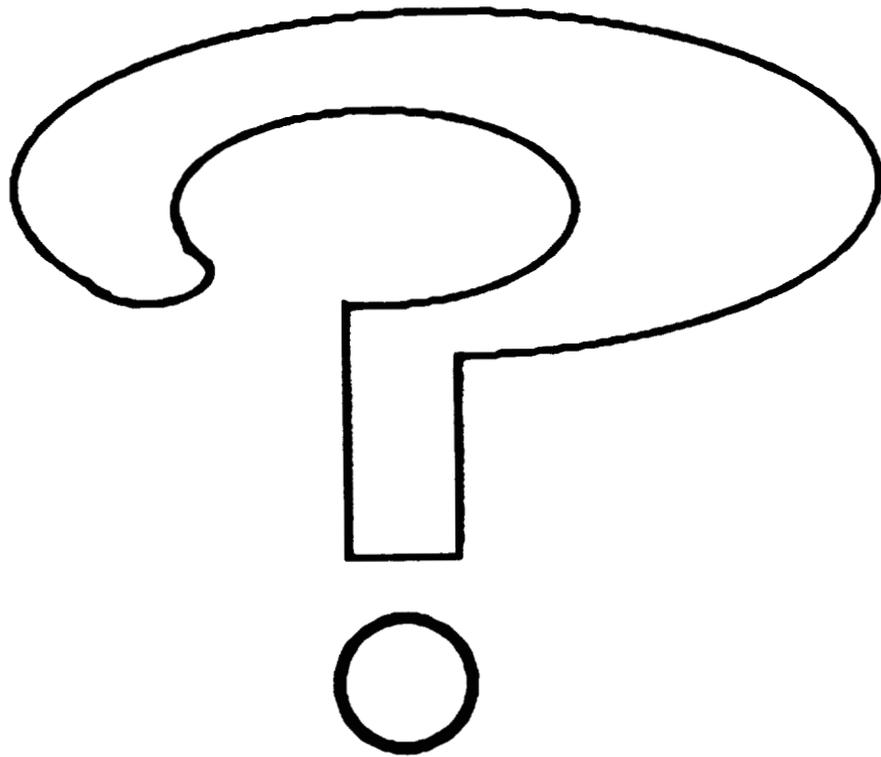
$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{F}^{\mathbf{L}} + \mathbf{F}^{\mathbf{I}} + \mathbf{g}$$

# GENERIC JOINT BODY WITH VARIOUS TYPES OF HINGE CONNECTIONS



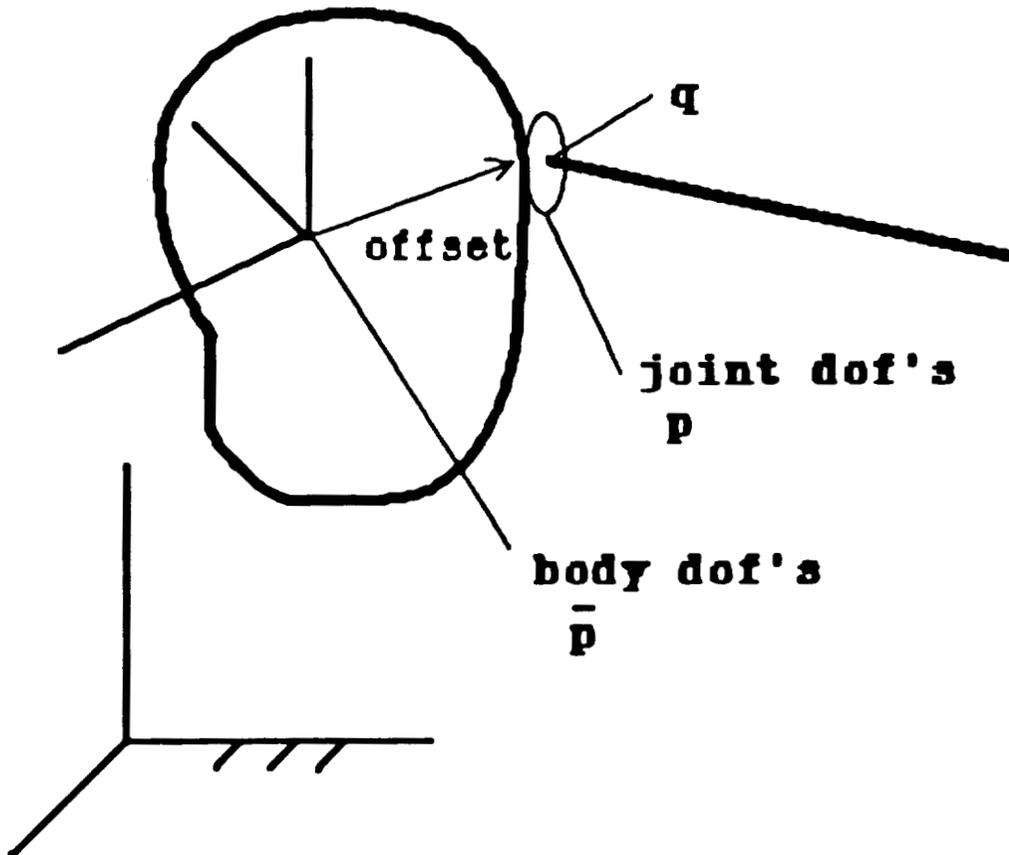
295





$$M\ddot{q} = F^E + F^I + g$$

## Joint Kinematics



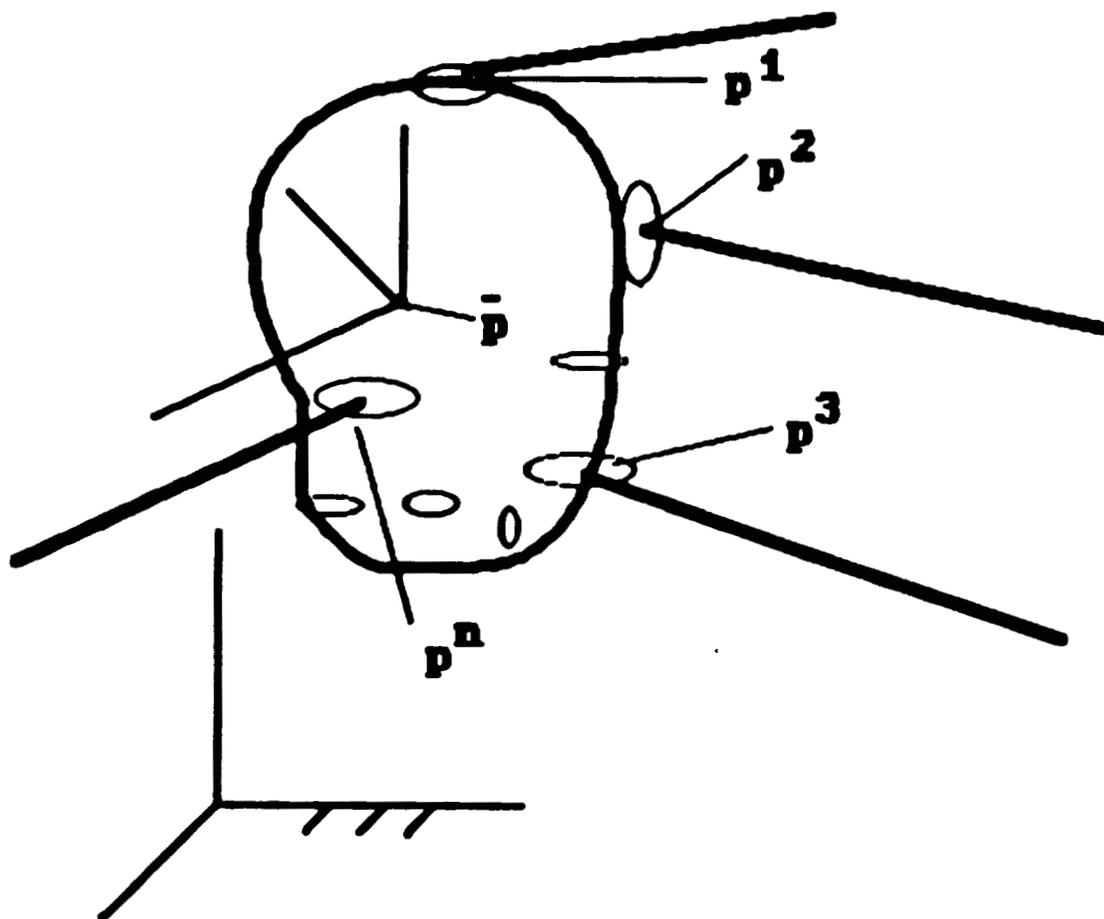
$$q = q(\bar{p}, p) \\ = q(\bar{q})$$

$$\delta q = H \delta \bar{q}$$

$$\dot{q} = H \dot{\bar{q}}$$

$$\ddot{q} = H \ddot{\bar{q}} \\ + \dot{H} \dot{\bar{q}}$$

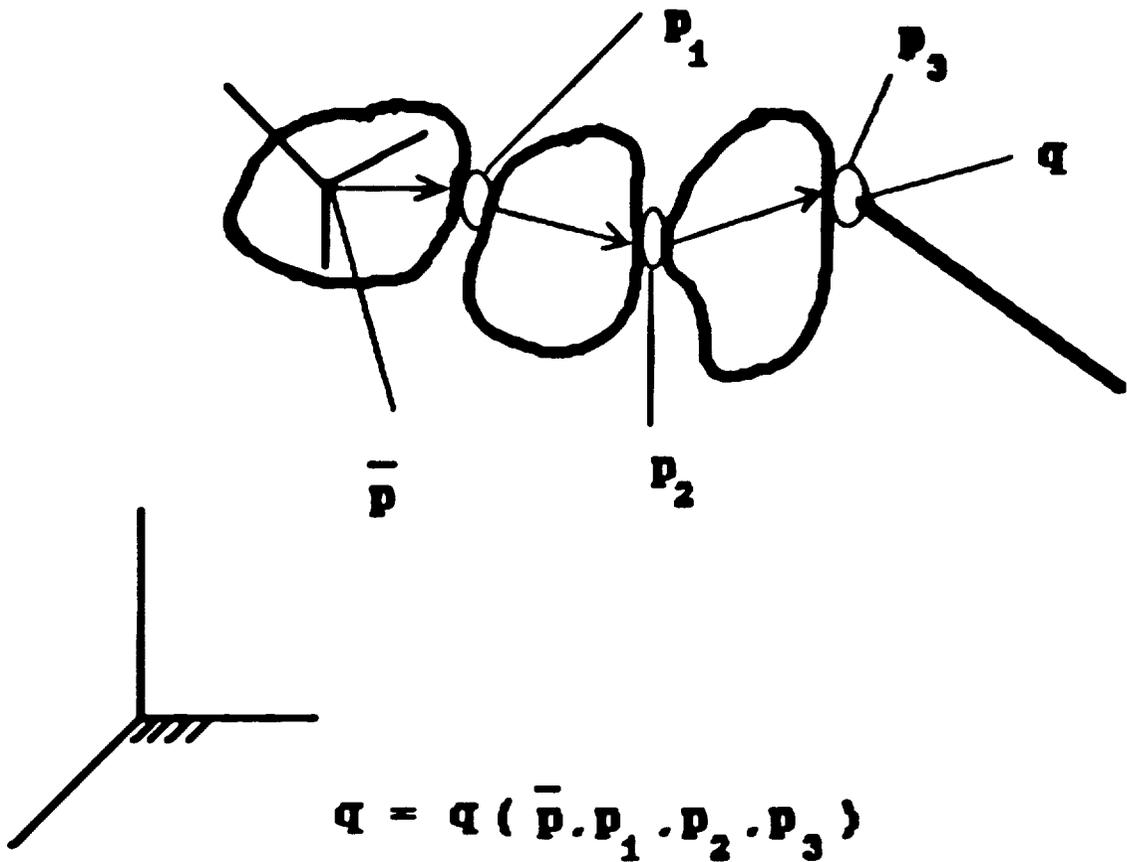
## Multi-Joint Body



generalized coords.

$$\bar{q} = [\bar{p}^T, p^{1T}, p^{2T}, \dots, p^{nT}]^T$$

## Rigid Chain



generalized coord.

$$\bar{q} = [\bar{p}^T, p_1^T, p_2^T, p_3^T]^T$$

## Generalized Coordinates

at each joint body:

3 translational disp.

3 orientational disp.

+ No. of relative(joint)

d-o-f's

## System EQ's of Motion

in terms of joint body  
and joint dof's

$$\bar{M} \ddot{\bar{q}} = \bar{F}^E + \bar{F}^I + \bar{g}$$

## Equations of Motion and Their Numerical Integration

At  $n^{\text{th}}$  time step,

$$M^n a^n + f'' + g^n = F^n$$

Newmark-Beta Integrator at  $k^{\text{th}}$  iteration:

$$a_k^n = a_{k-1}^n + \left[ M_{k-1}'' + \frac{h}{2} G_{k-1}^n + \beta h^2 K_{k-1}^n \right]^{-1} R_k^n \quad \text{:Update Accelerations}$$

$$R_k^n = \text{iterative residual} = F^n - f_{k-1}^n - M_{k-1}^n a_{k-1}^n$$

$$V_k^n = V^{n-1} + \left( \frac{h}{2} \right) (a^{n-1} + a_k^n) \quad \text{:Update Velocities}$$

## Equations of Motion and Their Numerical Integration (cont'd)

Split into translational and rotational d.o.f.

Translational displacements are

$$d_k^n = d_k^{n-1} + hv^{n-1} + \left(\frac{1}{2} - \beta\right) h^2 a_k^{n-1} + \beta h^2 a_k^n \quad \text{:Update Translational d.o.f.}$$

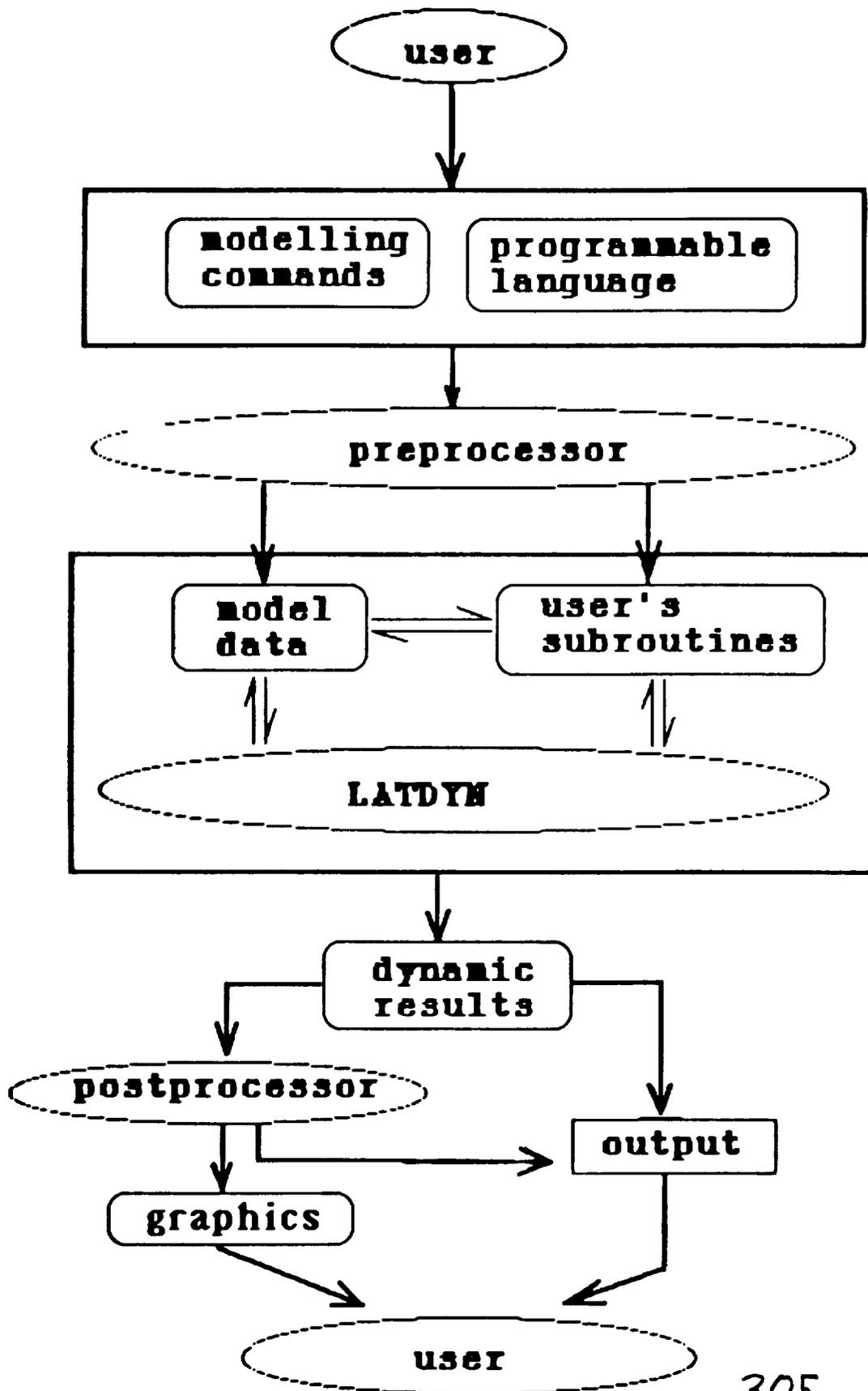
Rotational motions are given by transformation matrix:

$$T_k^n = \left[ 1 + h\bar{\omega}_k^n + \frac{1}{2}h(\bar{\omega}_k^n)^2 \right] T_k^{n-1} \quad \text{:Update hinge body transformation}$$

$$\bar{\omega}_k^n = \omega_k^n + \omega_k^{n-1}$$

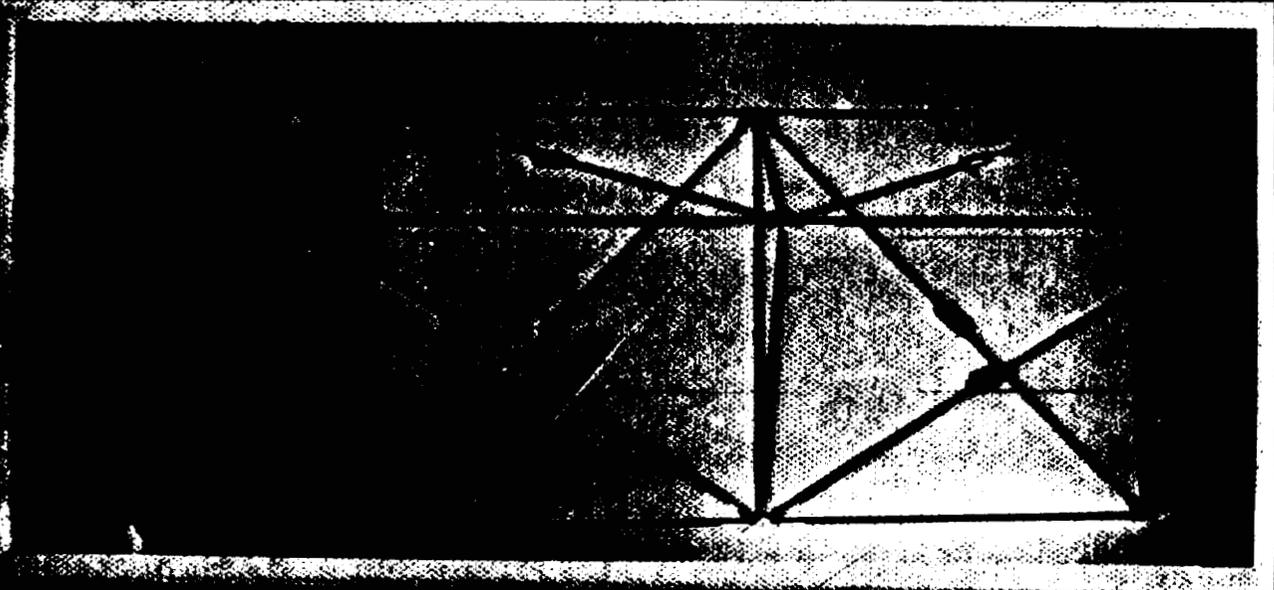
## Modelling Techniques

- \* How does user work with  
LATDYN ?
  
- \* How does program model  
a system ?



## Defining the Model

1. Numerical Control
2. Flexible Bodies
  - \* material properties
  - \* element properties
  - \* grid points
3. Rigid Connections
  - \* body geometry & mass
  - \* joint connections
4. Forcing Elements
  - \* Forcing functions
  - \* spring-damper-actuators
5. Initial Conditions
6. programmable language



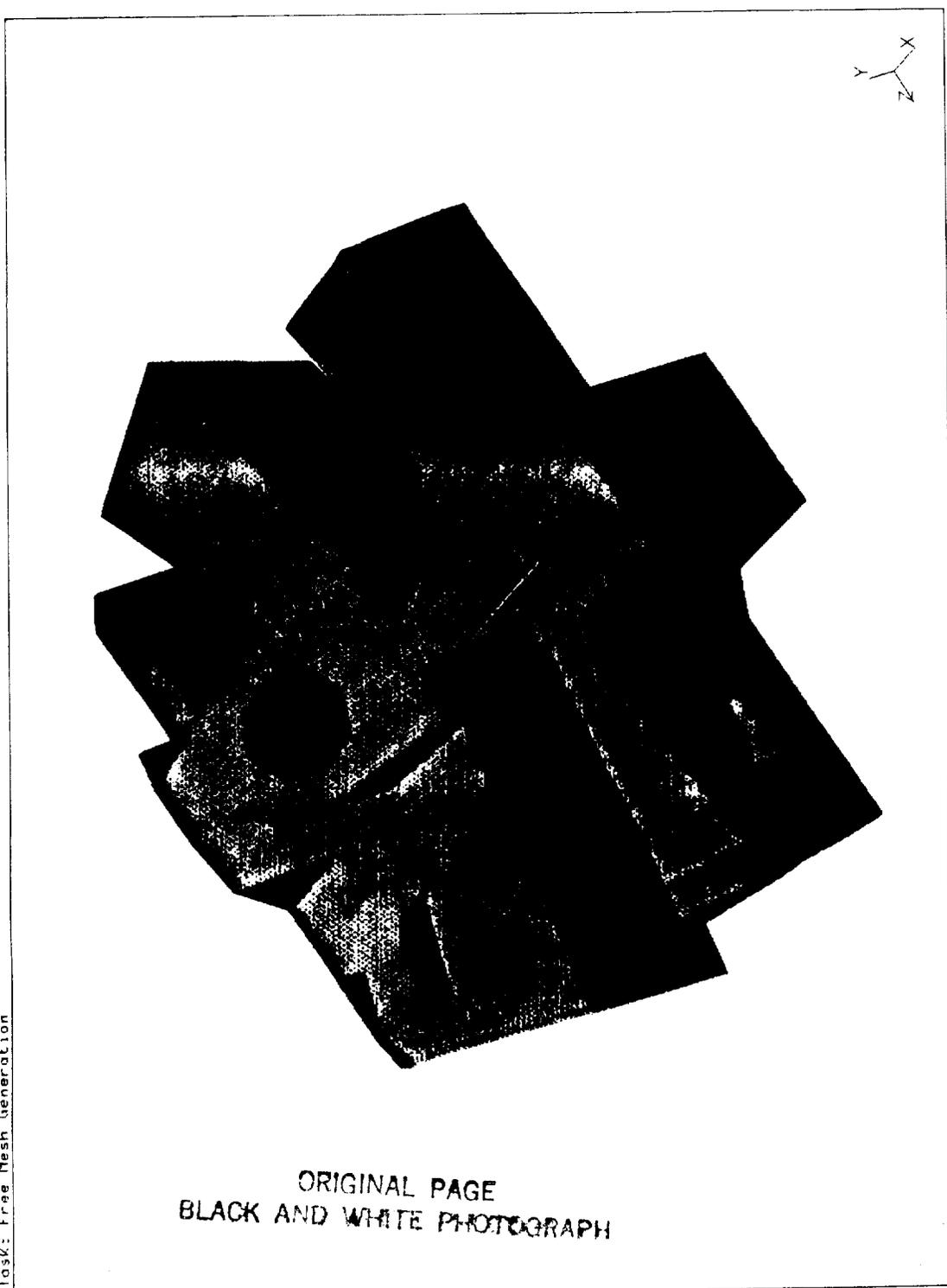
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UNITS = IN  
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SDRC I-DEAS 3.9: Pre/Post Processing

DATABASE: CORNERBODY B FOR MINIMAST  
VIEW: WHOLE (modified)  
Task: Free Mesh Generation

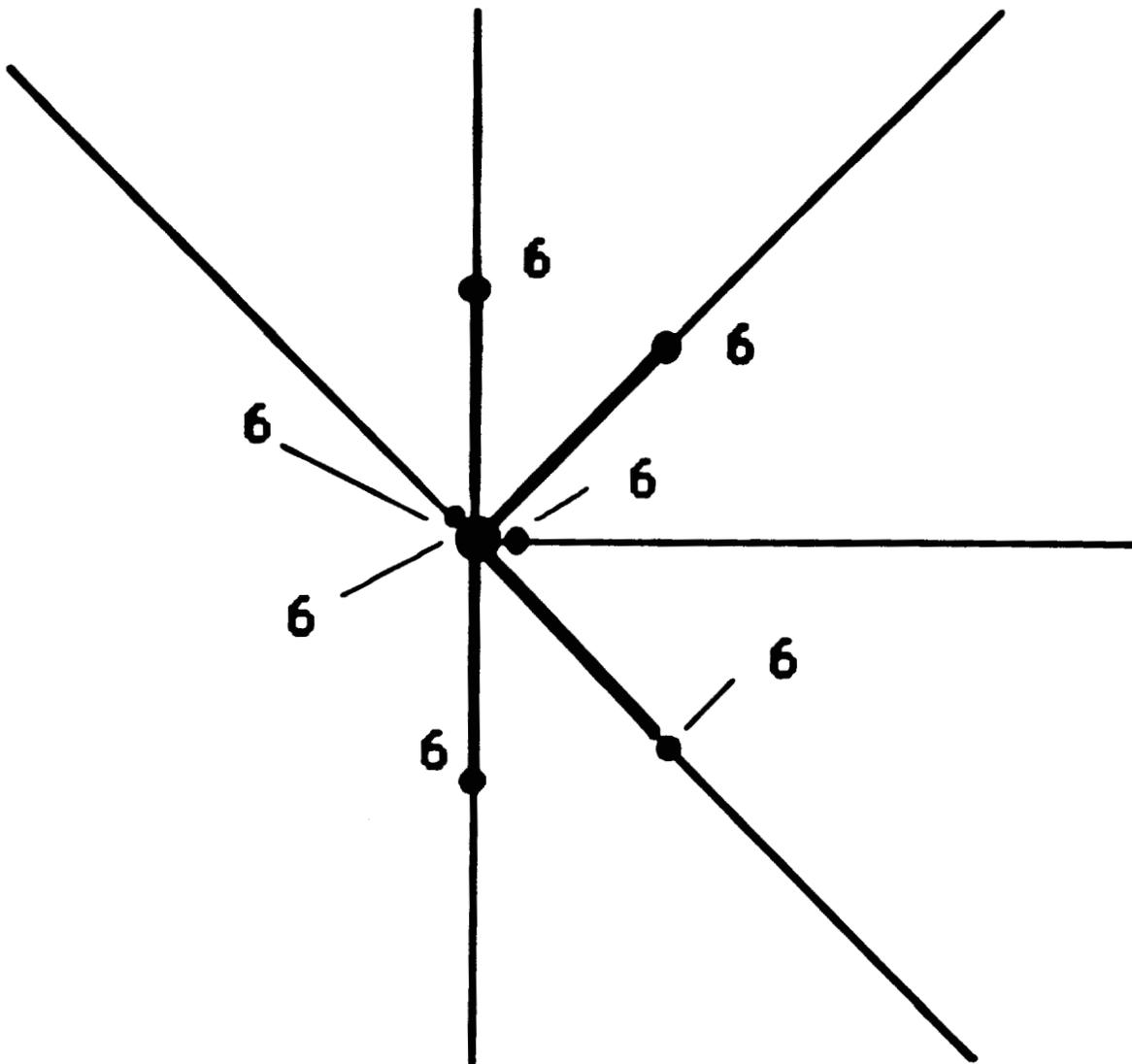


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**Conventional**

**F-E Model of corner body**

● grid (6-dof's)



**42-dof's**

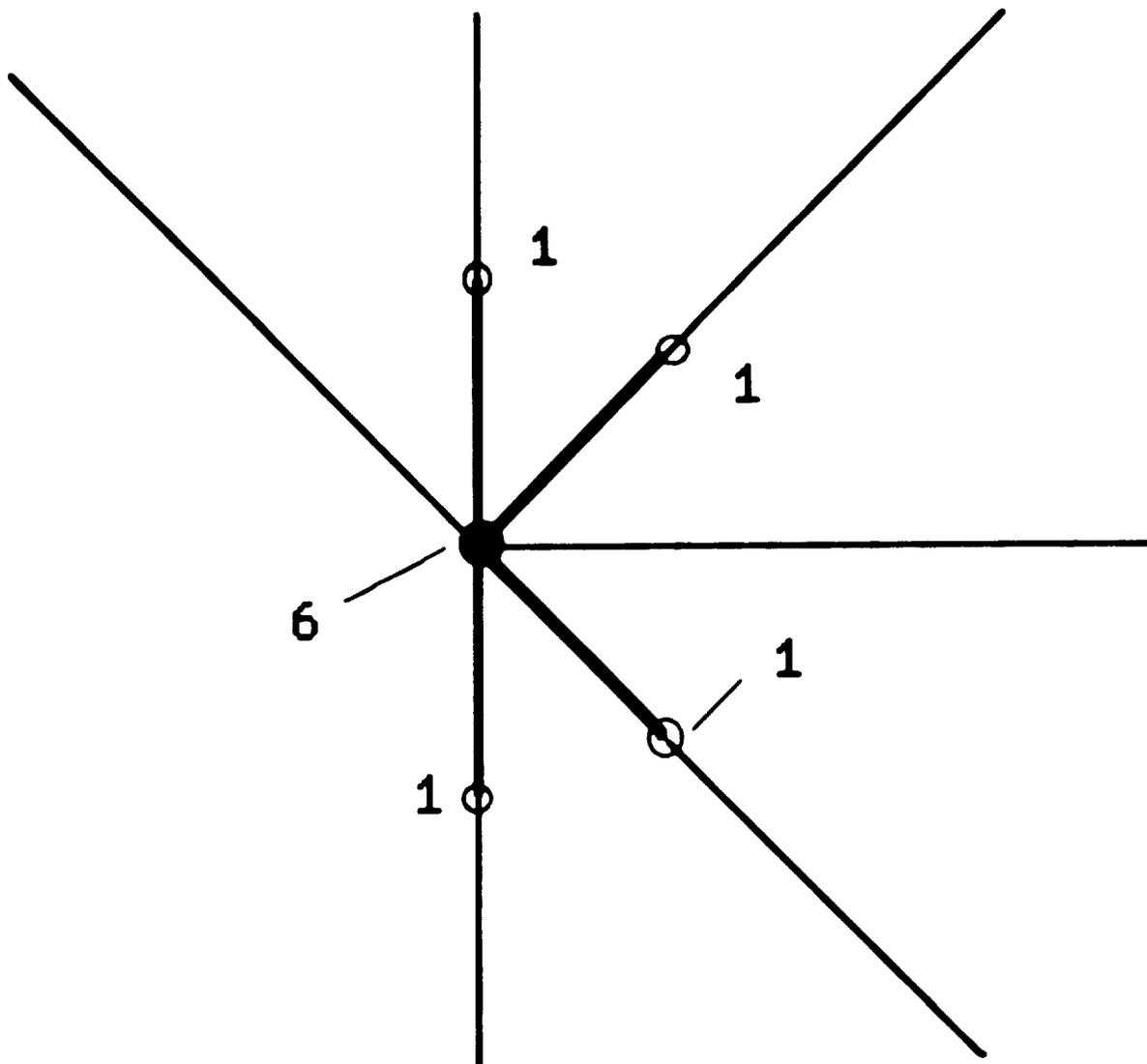
**32-constraints**

**LATDYN**

**F-E Model of corner body**

● grid (6-dof's)

○ hinge (1-dof)

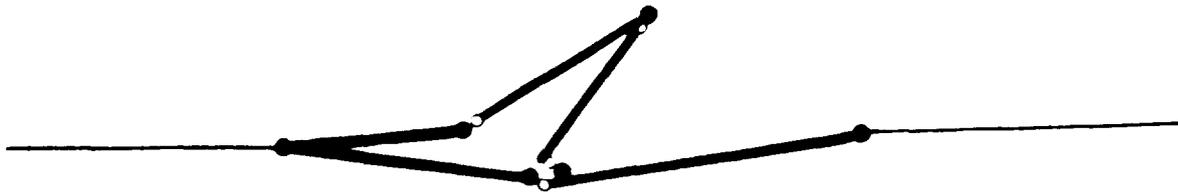


**10-dof's**

**0-constraint**

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## **3-D LATDYN Model of Mini-Mast Locking Joint**



**Note: closed loop**

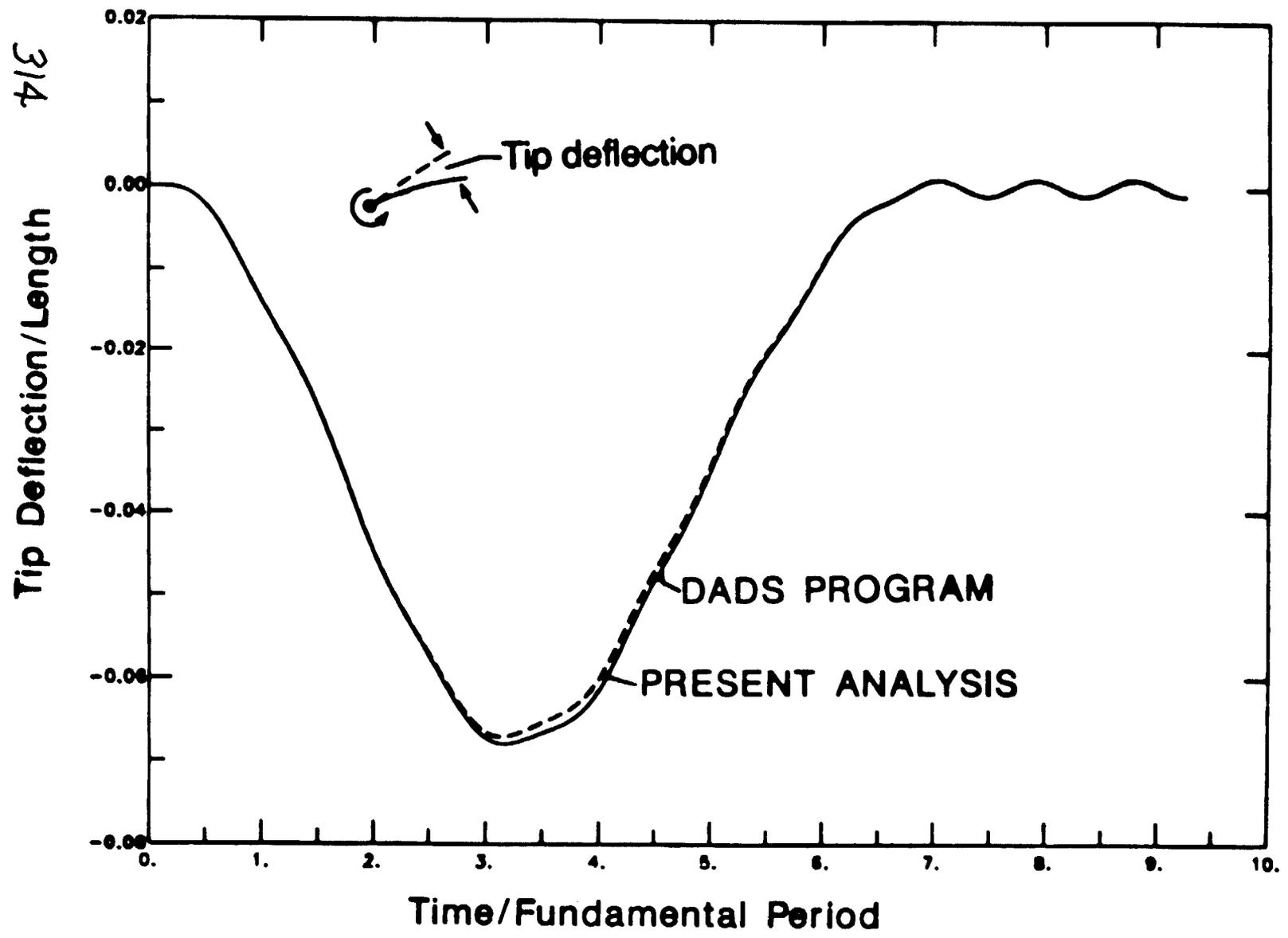
**Target is that user will not have to specify  
how rigid members are formulated.**

**Program will determine most efficient arrange-  
ment, and will cut closed loops and implement  
constraints automatically.**

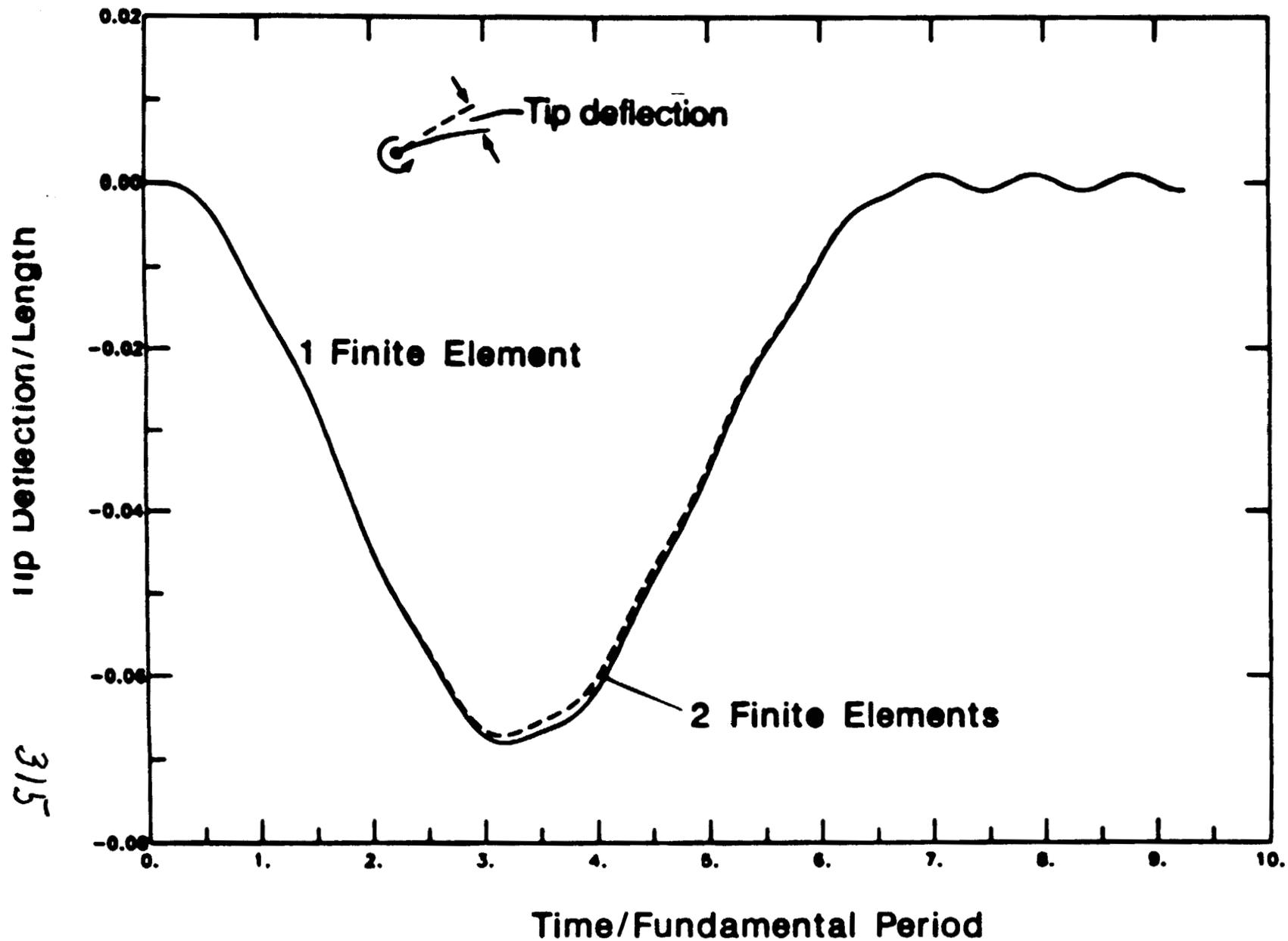
## Present LATDYN

- \* three-dimensional
- \* Euler-Bernoulli beam  
elements
- \* hinge connections
- \* Newmark- $\beta$  explicit &  
implicit methods
- \* constraints & joints
- \* external forcing function  
& spring-damper-actuator

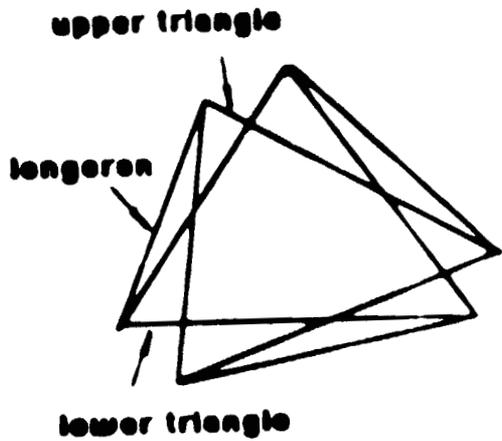
# COMPARISON OF RESULTS FOR FLEXIBLE BEAM SPIN-UP ON A PLANE



# CONVERGENCE OF SOLUTION FOR FLEXIBLE BEAM SPIN-UP ON A PLA

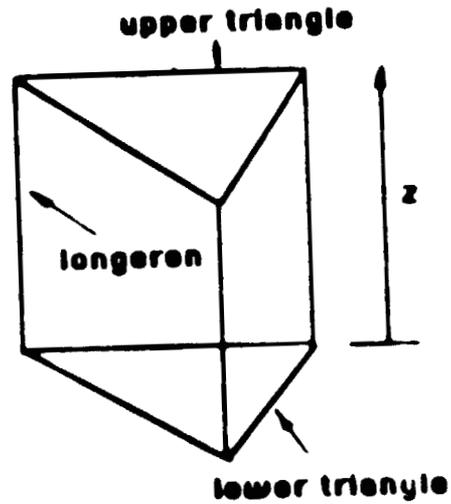


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Fully Packed

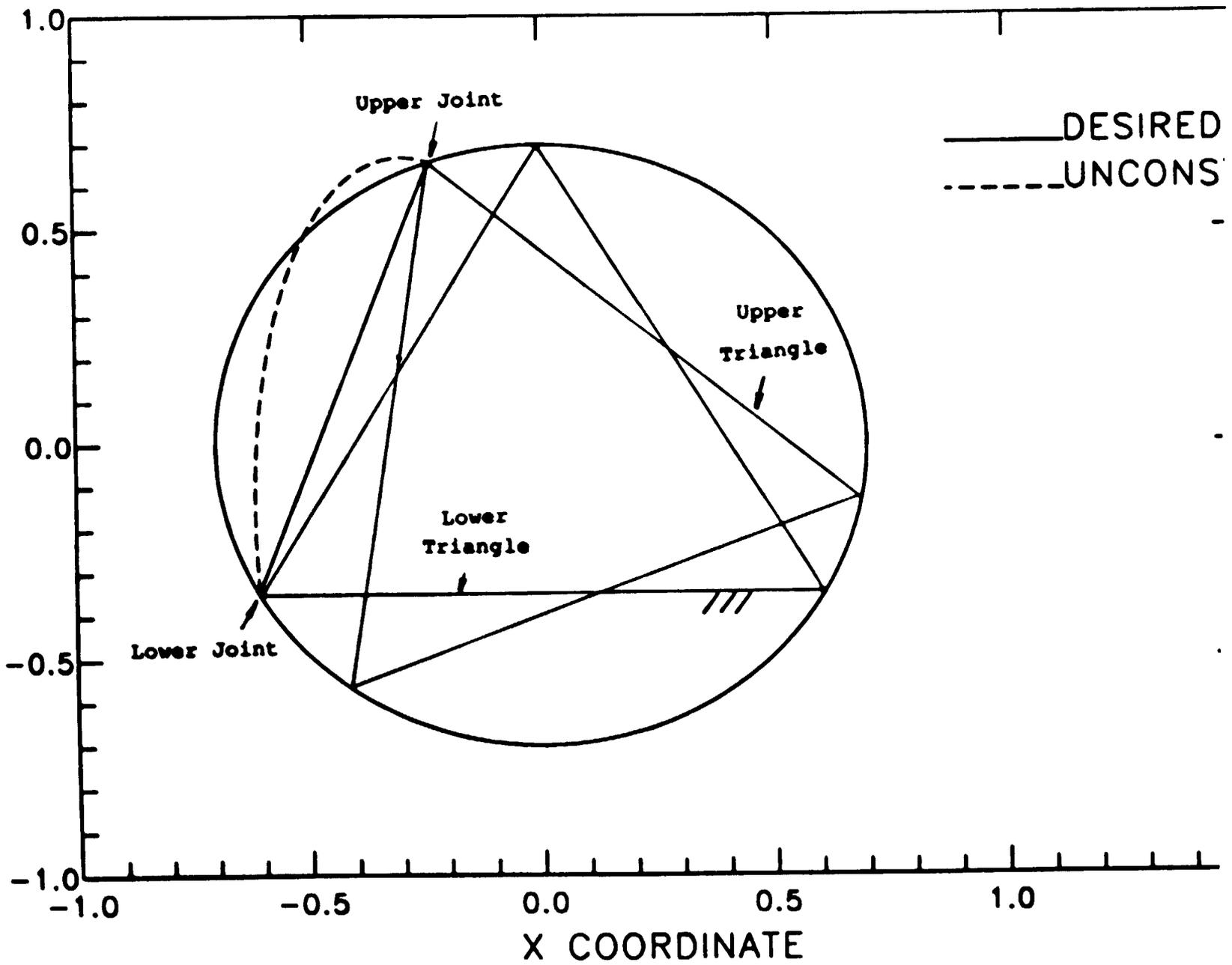
Apply  
Driving Function

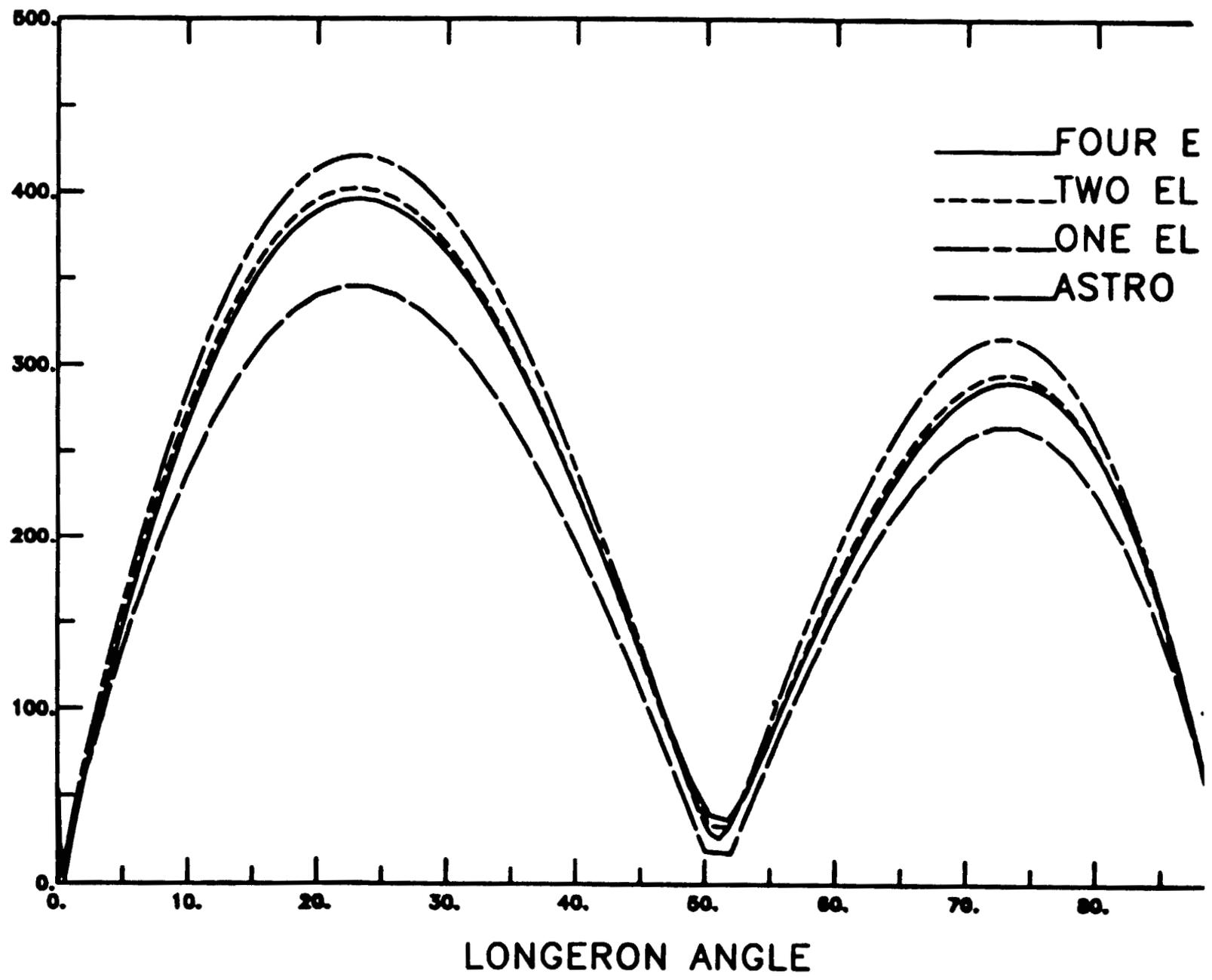


Fully Deployed

Y COORDINATE

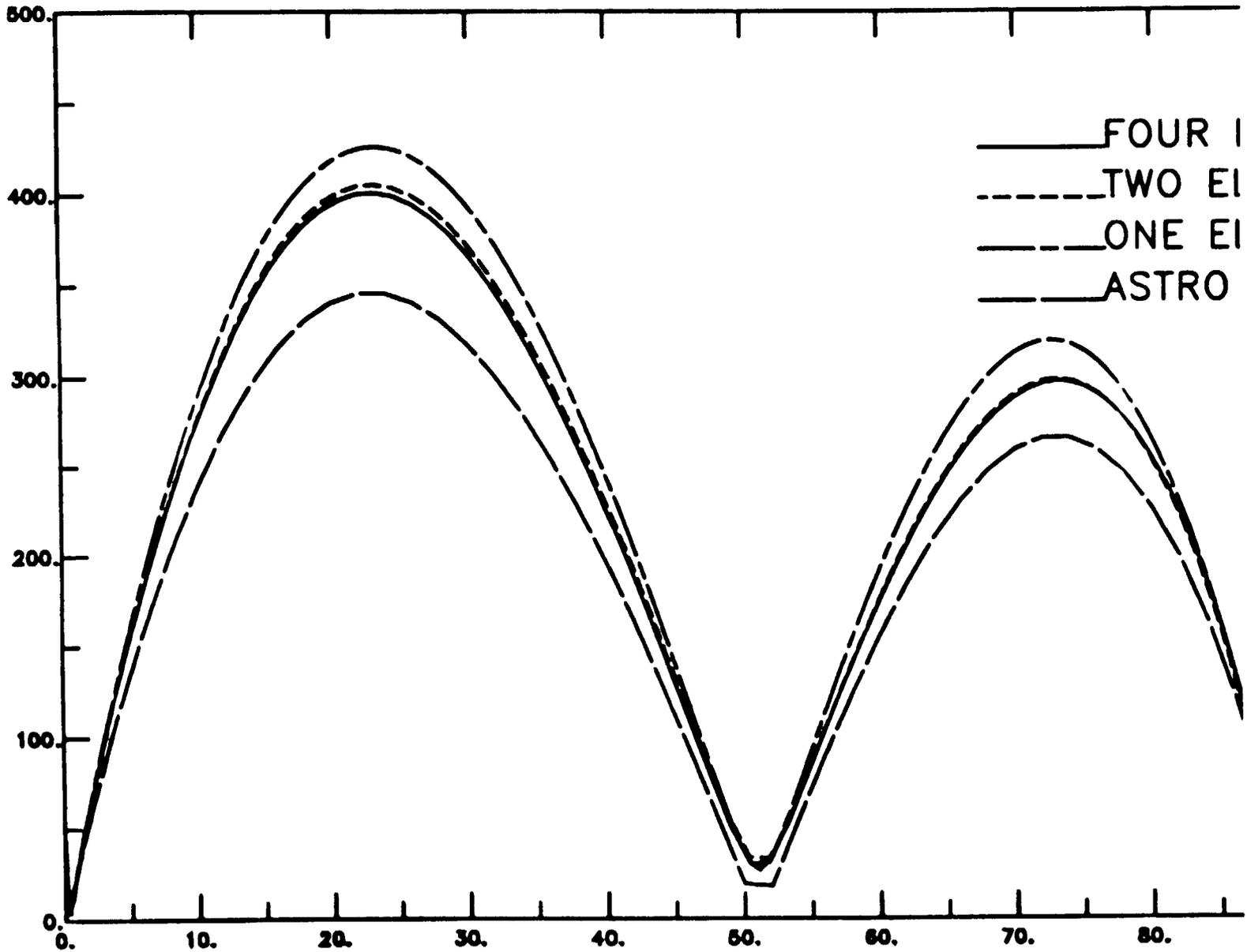
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# AT UPPER JOINT

RESULTANT BENDING MOMENT



LONGERON ANGLE

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## Future LATDYN

- \* various elements
- \* various joint connections
- \* various integrations  
(parallel version)
- \* control and structure  
interactions

## Conclusions

- \* A finite-element-based research code is developed.
- \* It provides a modelling, calculation, and analysis tool for researcher & Engr.
- \* To analyze complex space structures and/or mechanisms.
- \* In the simulation of Control design as well as structural dynamics.

N90-10094

**ENHANCED ELEMENT-SPECIFIC MODAL FORMULATIONS FOR  
FLEXIBLE MULTIBODY DYNAMICS**

By

Robert R. Ryan  
University of Michigan  
Ann Arbor, Michigan

**ABSTRACT**

The accuracy of current flexible multibody formalisms based on assumed modes is examined in the context of standard spacecraft motions involving structural components undergoing both slow and fast overall translational and rotational motions as well as small deformations. Limitations of current techniques in treating (1) element-specific coupling behavior of large motion and small deformation, and (2) motion-induced structural stiffness variations, are noted.

The roles of nonlinear and linear elastic structural theories in accurately predicting transient large-displacement dynamic behavior of flexible multibody systems are examined in detail. Coupling effects between deformation and overall motion are carefully scrutinized in the context of assumed-mode discretization techniques. Consistently linearized beam, plate, and shell formulations involving in-plane stretch variables are proposed and shown to yield very accurate simulation results and extremely fast modal convergence for most motions involving small strains. In some particular cases, however, in which membrane stiffness dominates bending stiffness, a nonlinear strain formulation is required in order to capture proper coupling between deformation and overall motion. Unfortunately, with standard component modes, algorithmic formalisms involving nonlinear strain-displacement expressions show very slow modal convergence. A procedure involving use of constraint modes is proposed to alleviate this problem.

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NASA - OAST WORKSHOP  
Computational Aspects in the Control of Flexible Systems

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ENHANCED ELEMENT-SPECIFIC MODAL FORMULATIONS  
FOR  
FLEXIBLE MULTIBODY DYNAMICS

July 12-14, 1988

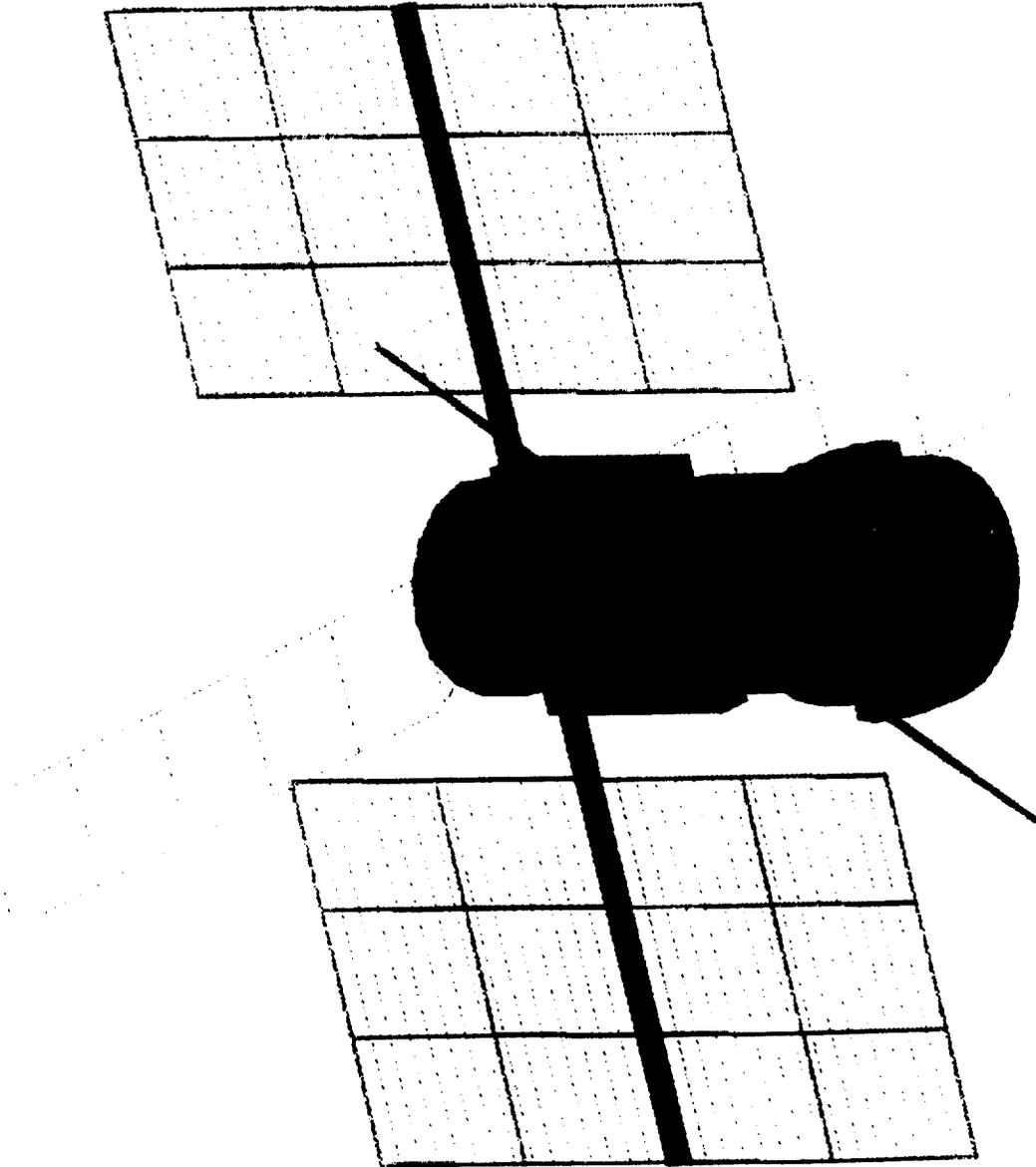
Robert R. Ryan  
University of Michigan

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////////// Outline \\\\\\\\\\\

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I. Limitations of Existing Flexible Multibody Formalisms

- Examples
- Verification

II. Linear and Nonlinear Element-Specific Formulations

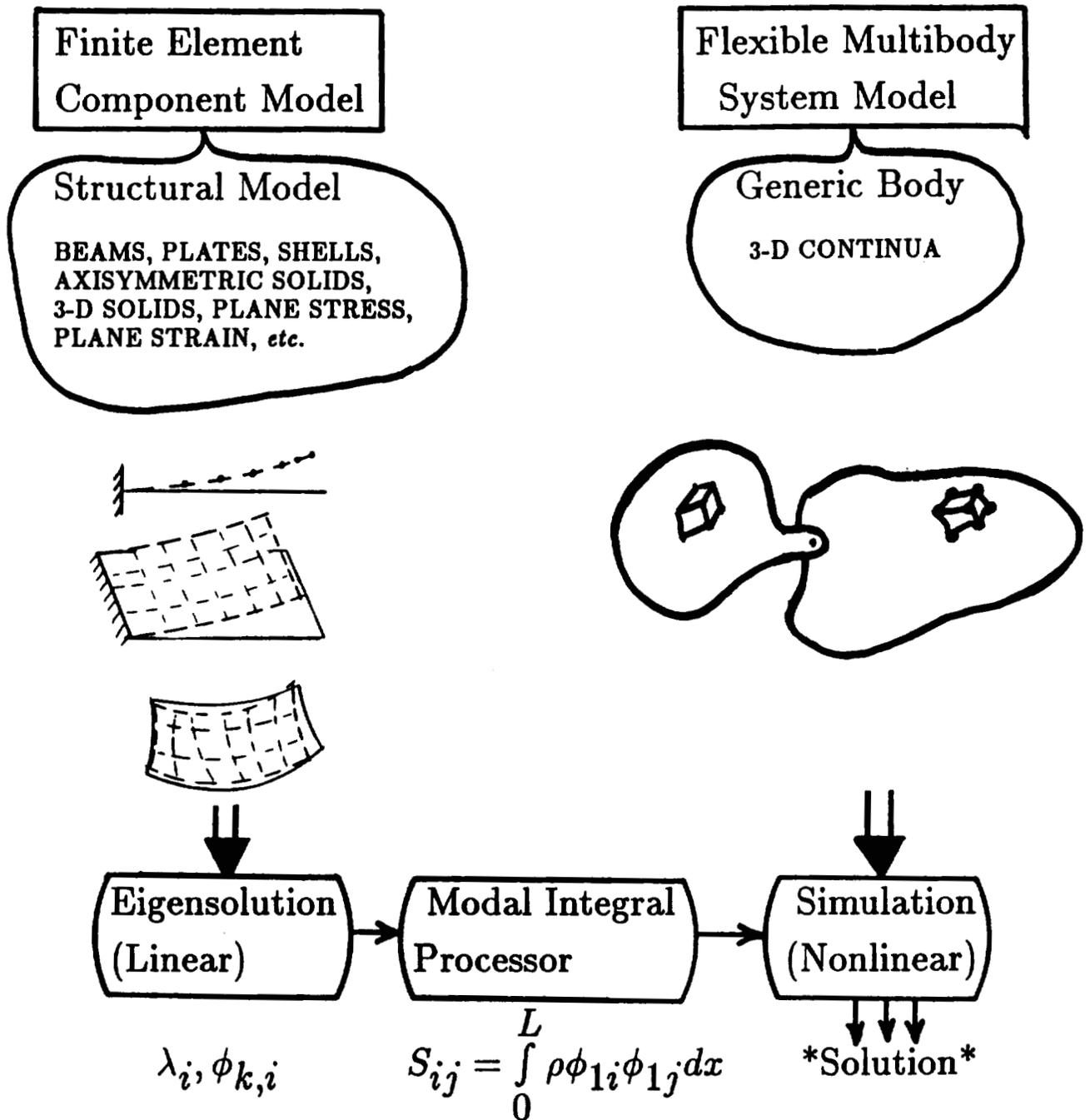
- Consistently-Linearized Beam, Plate, Shell Multibody Models
- Second-Order Beam, Plate Models

III. Simulation Results

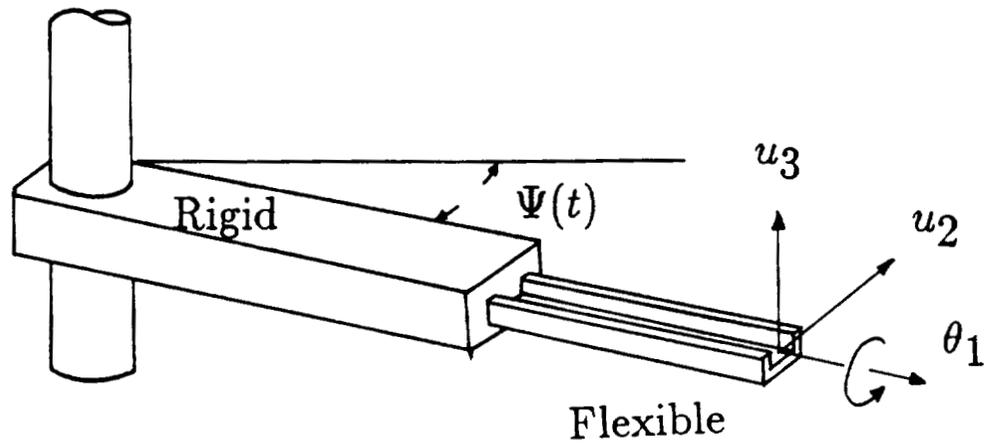
- Membrane/Bending Problems
- Convergence

## Current Flexible Multibody Formalisms - Modal Approach "Limitations"

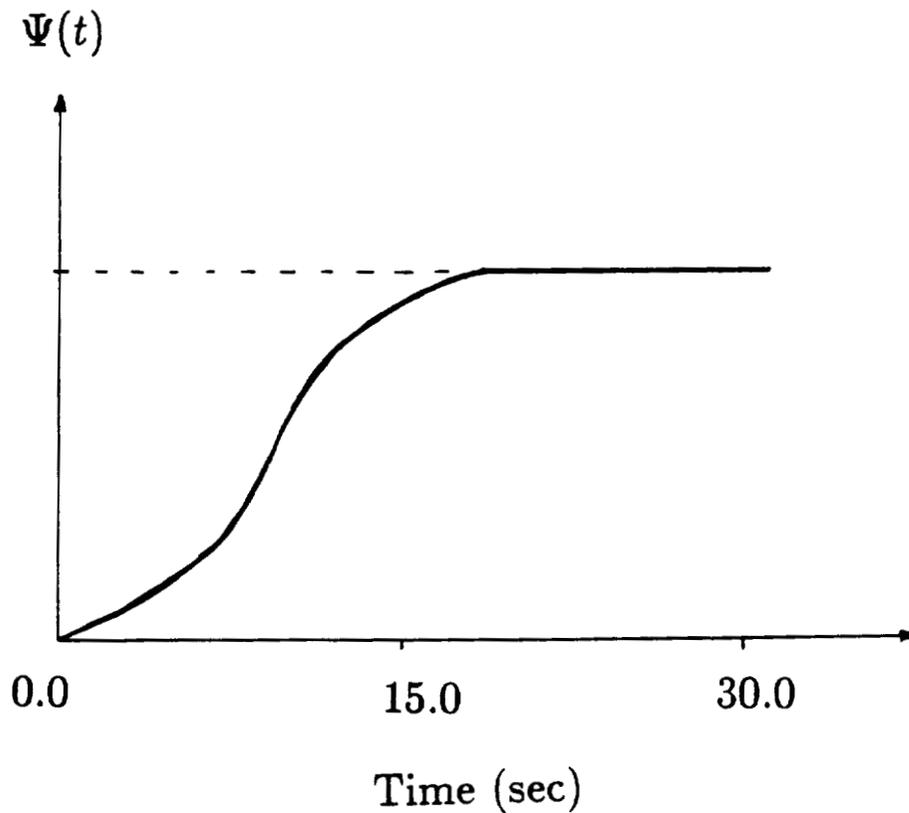
- Do Not Account For Large-Displacement Element-Specific Behavior
- Inadequate Account of Motion-Induced Stiffness Variations

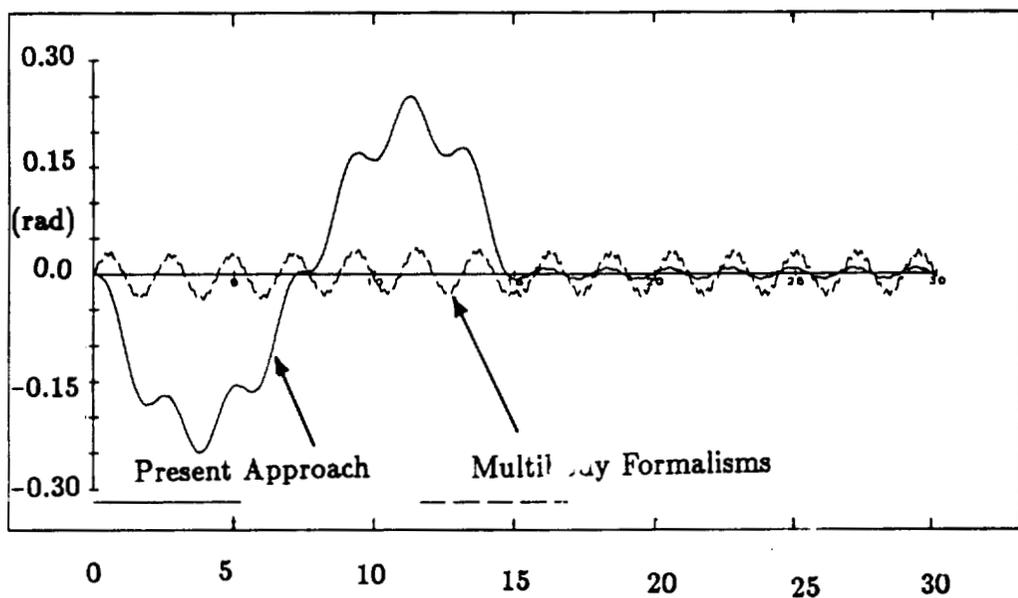
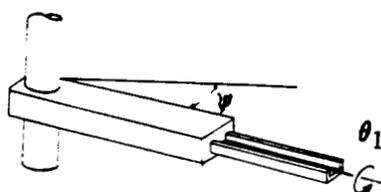
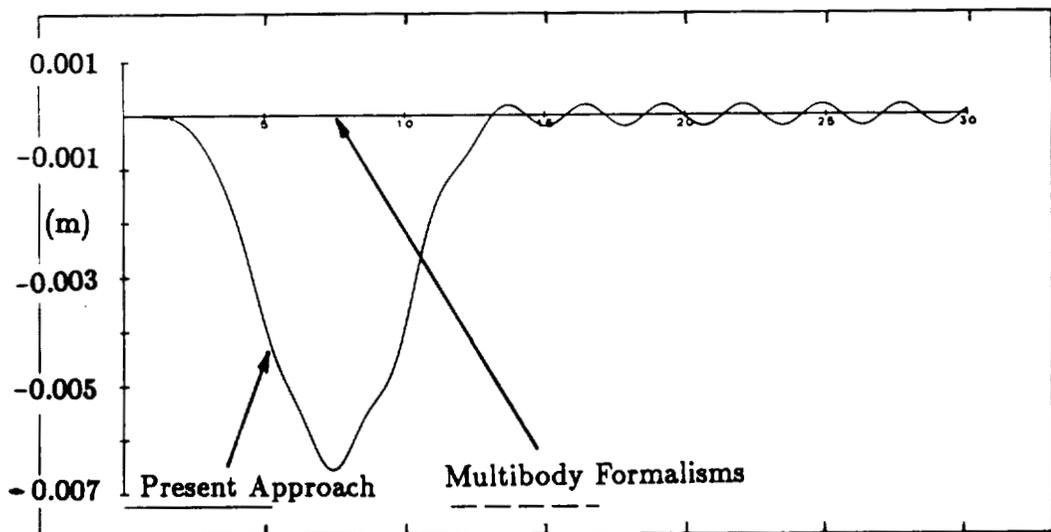
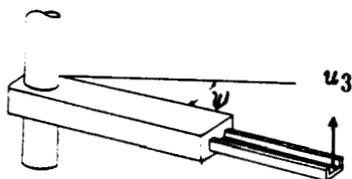
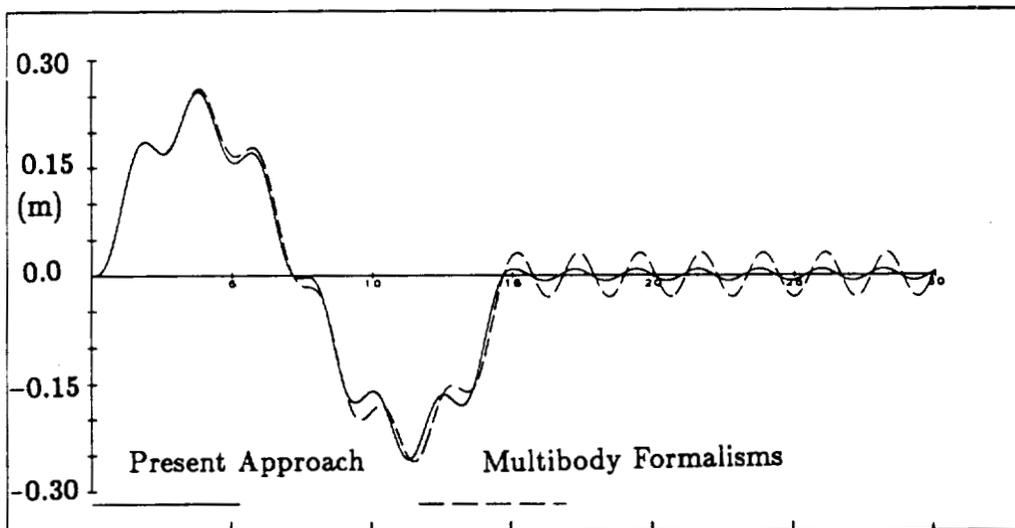
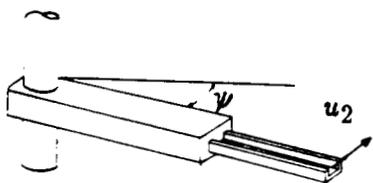


## Slow Repositional Maneuver of Channel Beam



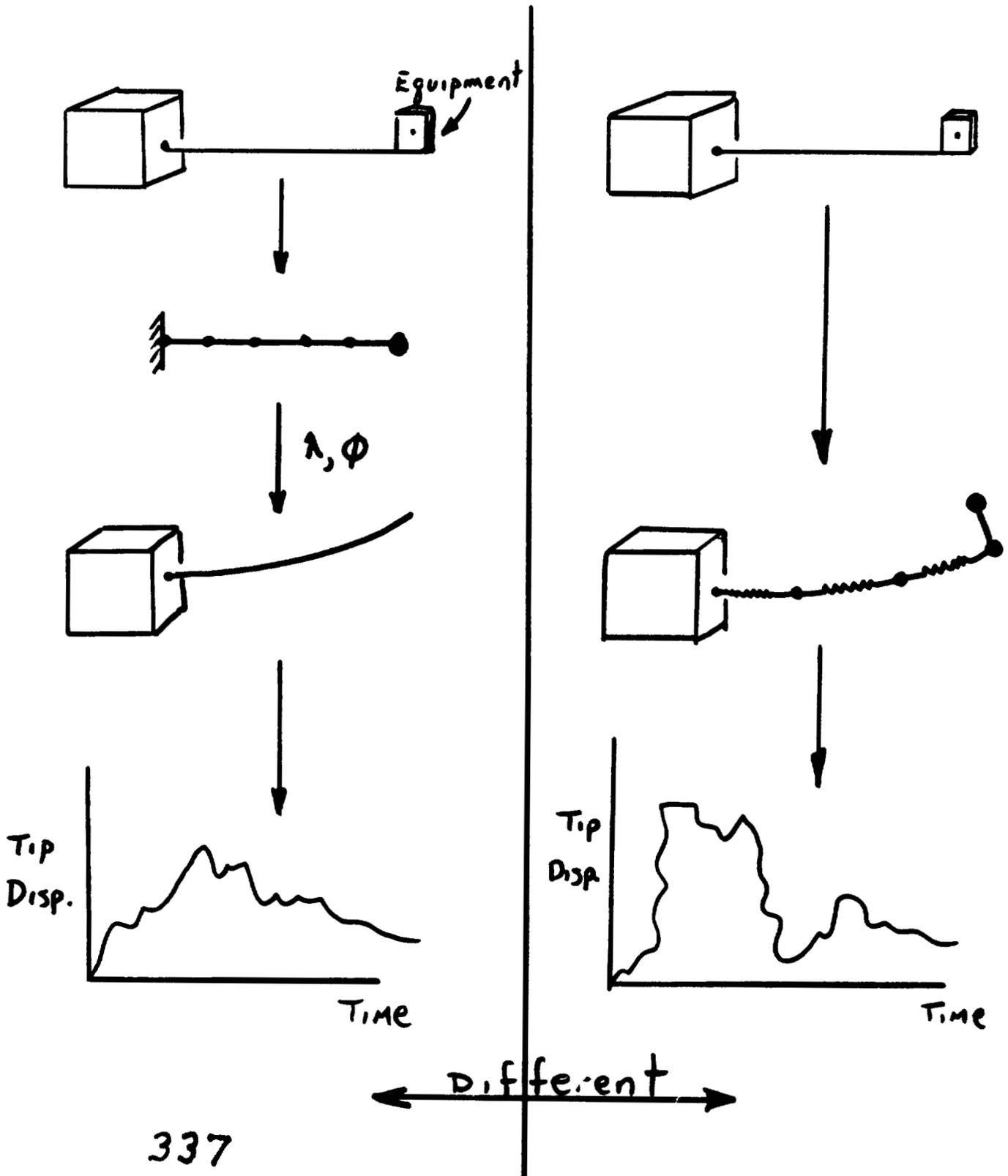
### Repositional Maneuver Angle





Time (sec)

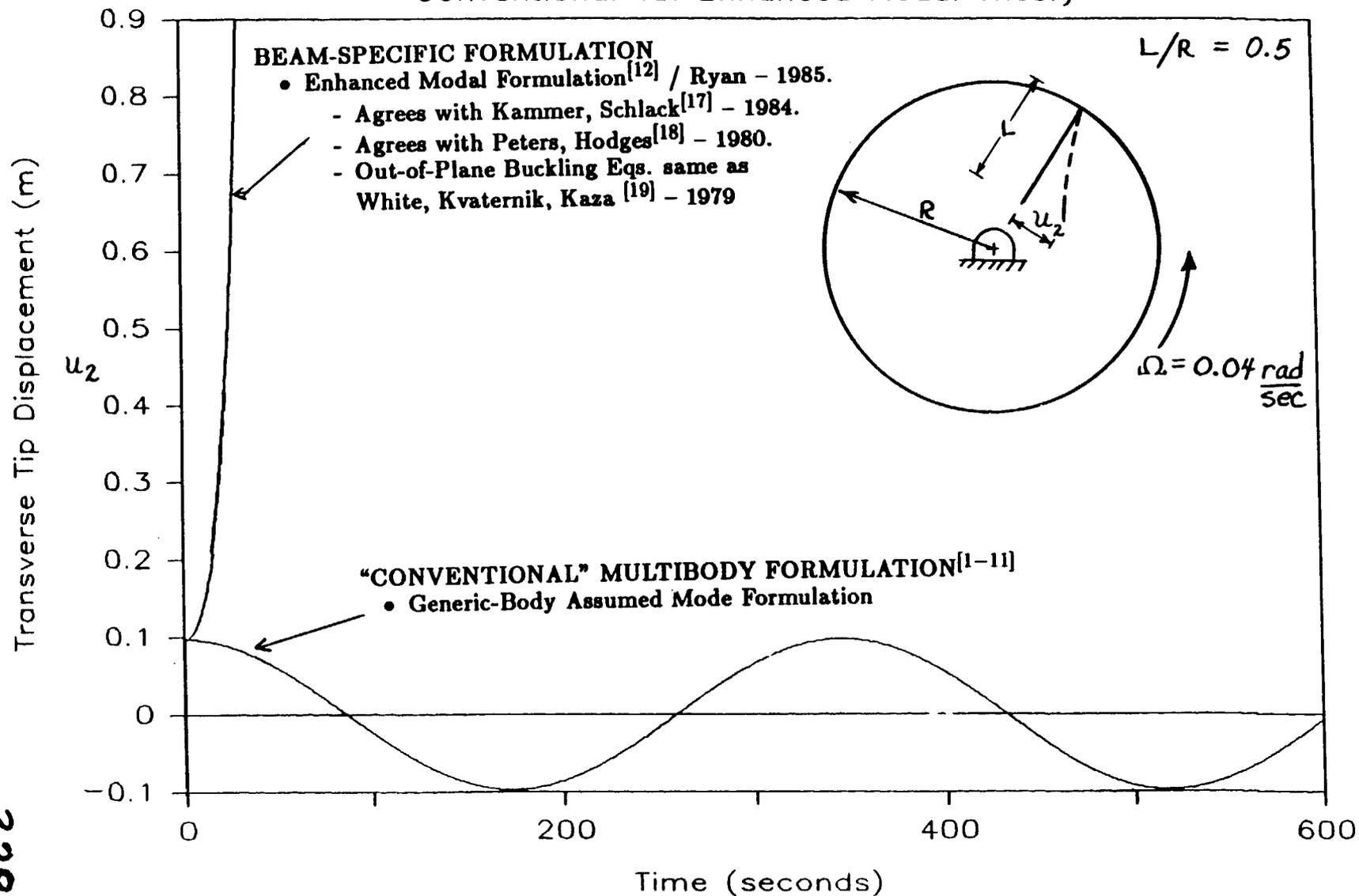
# BEAM WITH OFFSET TIP MASS



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# Constant Speed Spin – Buckling Analysis

Conventional Vs. Enhanced Modal Theory



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////////// Alternatives to Present \\\\\\\\\\\

////////// Flexible Multibody Dynamic Formalisms \\\\\\\\\\\

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- Discrete Representations
- Nonlinear Finite Element Methods
- Linear and Nonlinear  
Enhanced Modal Approaches

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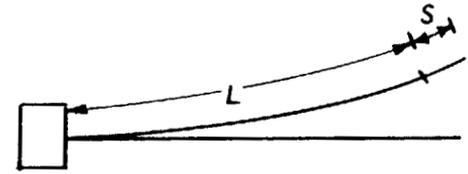
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## Consistently-Linearized Multibody Structural Theories

- Beam

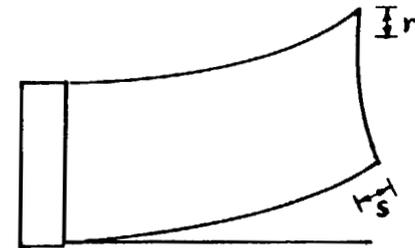
$$s \triangleq u_1 + \frac{1}{2} \int_0^x \left[ \left( \frac{\partial u_2}{\partial \sigma} \right)^2 + \left( \frac{\partial u_3}{\partial \sigma} \right)^2 \right] d\sigma$$



- Plate

$$r \triangleq u_2 + \frac{1}{2} \int_0^y \left( \frac{\partial u_3}{\partial \eta} \right)^2 d\eta$$

$$s \triangleq u_1 + \frac{1}{2} \int_0^x \left( \frac{\partial u_3}{\partial \xi} \right)^2 d\xi$$



$$M\ddot{q} + G\dot{q} + (K_I + K_L + K_g)q = F$$

### Advantages:

- Excellent Convergence
- Captures Motion-Induced Bending Stiffness Variation
- Ease of Modal Reduction/Controls
- Easily-Implemented – Linear in Deformation

### Disadvantages:

- Doesn't Capture Motion-Induced Membrane Stiffness

## Second-Order Structural Multibody Theories

- Beam

$$U_b = \frac{1}{2} \int_0^\ell E \left\{ I_{zz} \left( \frac{\partial^2 u_2}{\partial x^2} \right)^2 + I_{yy} \left( \frac{\partial^2 u_3}{\partial x^2} \right)^2 \right\} dx$$

$$U_s = \frac{1}{2} \int_0^\ell EA \left\{ \left[ \left( \frac{\partial u_1}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial u_2}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial u_3}{\partial x} \right)^2 \right]^2 \right\} dx$$

- Thin Rectangular Plates:

$$U_b = \frac{1}{2} \int_0^b \int_0^a \beta \left\{ \left( \frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} \right)^2 \right. \\ \left. - 2(1 - \nu) \left[ \left( \frac{\partial^2 u_3}{\partial x^2} \right) \left( \frac{\partial^2 u_3}{\partial y^2} \right) - \left( \frac{\partial^2 u_3}{\partial x \partial y} \right)^2 \right] \right\} dx dy$$

$$U_s = \frac{1}{2} \int_0^b \int_0^a \gamma \left\{ \left( \frac{\partial u_1}{\partial x} \right)^2 + \left( \frac{\partial u_1}{\partial x} \right) \left( \frac{\partial u_3}{\partial x} \right)^2 + \left( \frac{\partial u_2}{\partial y} \right)^2 \right. \\ \left. + \left( \frac{\partial u_2}{\partial y} \right) \left( \frac{\partial u_3}{\partial y} \right)^2 + \frac{1}{4} \left[ \left( \frac{\partial u_3}{\partial x} \right)^2 + \left( \frac{\partial u_3}{\partial y} \right)^2 \right]^2 \right. \\ \left. + 2\nu \left[ \left( \frac{\partial u_1}{\partial x} \right) \left( \frac{\partial u_2}{\partial y} \right) + \frac{1}{2} \left( \frac{\partial u_2}{\partial y} \right) \left( \frac{\partial u_3}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial u_1}{\partial x} \right) \left( \frac{\partial u_3}{\partial y} \right)^2 \right] \right. \\ \left. + \frac{(1 - \nu)}{2} \left[ \left( \frac{\partial u_1}{\partial y} \right)^2 + 2 \left( \frac{\partial u_1}{\partial y} \right) \left( \frac{\partial u_2}{\partial x} \right) + \left( \frac{\partial u_2}{\partial x} \right)^2 \right] \right. \\ \left. + 2 \left( \frac{\partial u_1}{\partial y} \right) \left( \frac{\partial u_3}{\partial x} \right) \left( \frac{\partial u_3}{\partial y} \right) + 2 \left( \frac{\partial u_2}{\partial x} \right) \left( \frac{\partial u_3}{\partial x} \right) \left( \frac{\partial u_3}{\partial y} \right) \right\} dx dy$$

$$M\ddot{q} + G\dot{q} + (K_I + K_L + K_n)q = F$$

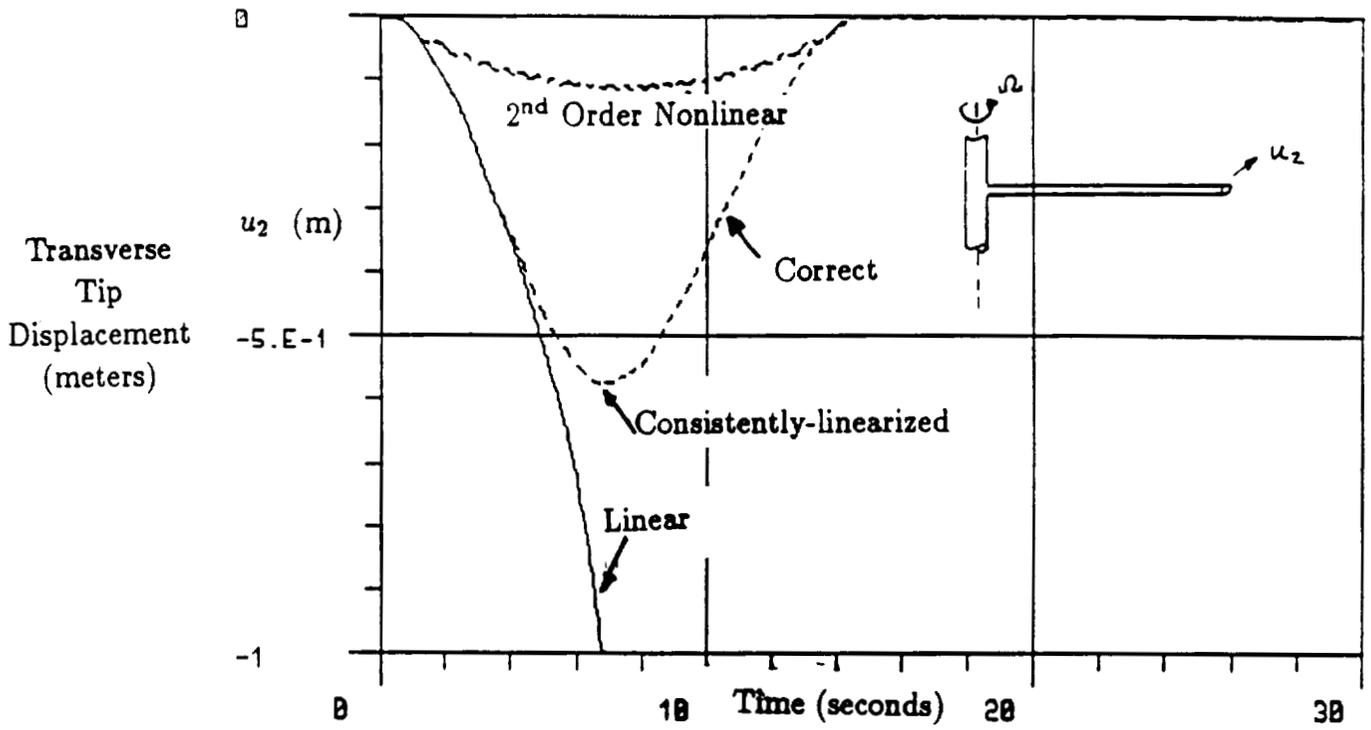
### Advantages:

- Captures Important Motion-Induced Bending AND Membrane Stiffness Variations for Small Strain

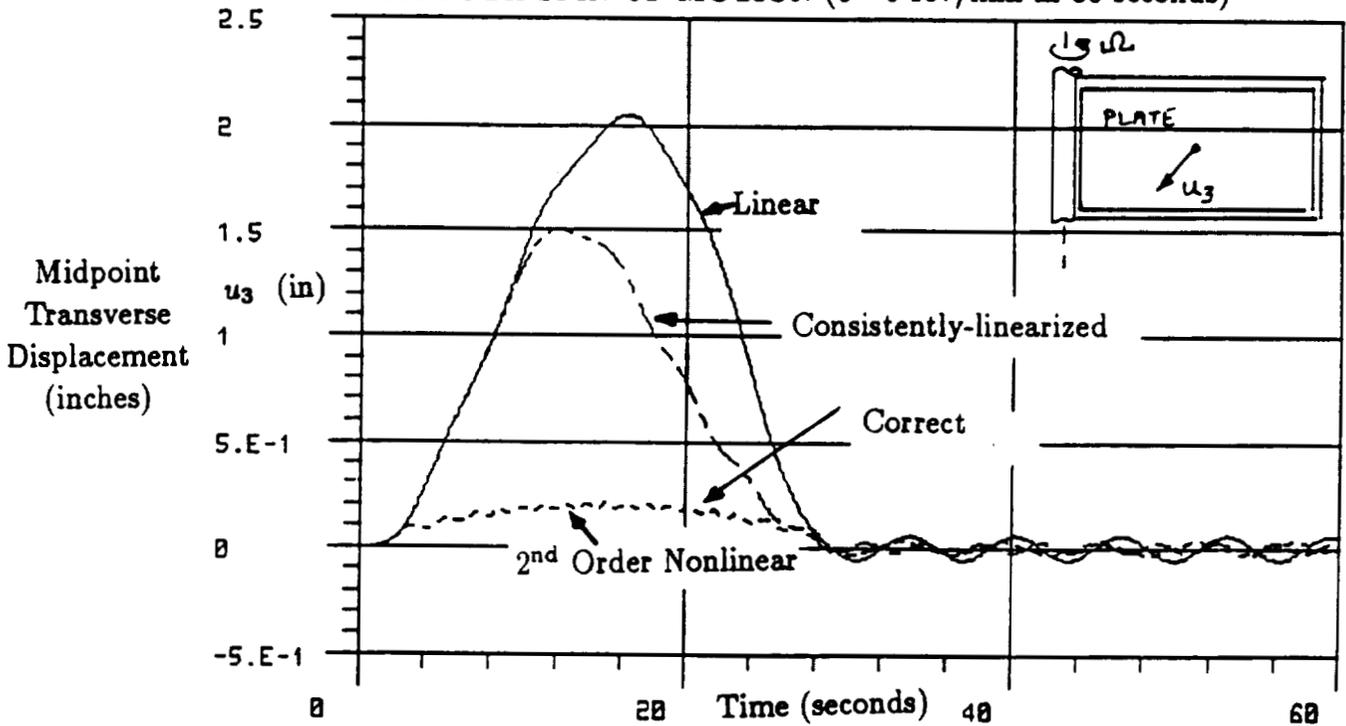
### Disadvantages:

- Poor Convergence With Standard Modes
- Order Reduction Results in Very Inaccurate Models
- Very Costly to Incorporate

SMOOTH SPIN-UP MOTION (0 - 6 rad/sec in 15 seconds)



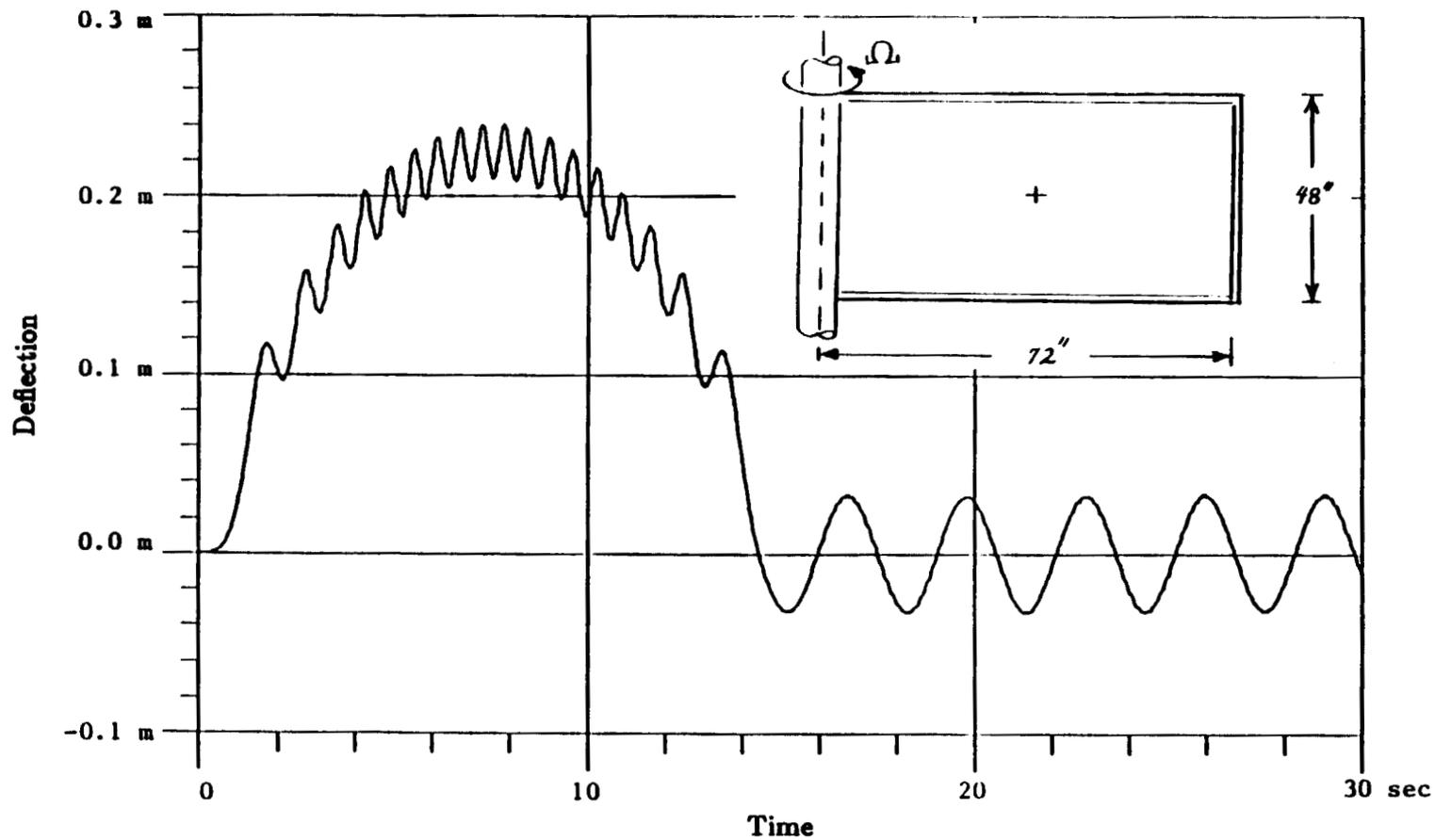
SMOOTH SPIN-UP MOTION (0 - 6 rev/min in 30 seconds)



# SIMPLY SUPPORTED RECTANGULAR PLATE SPIN-UP MOTION

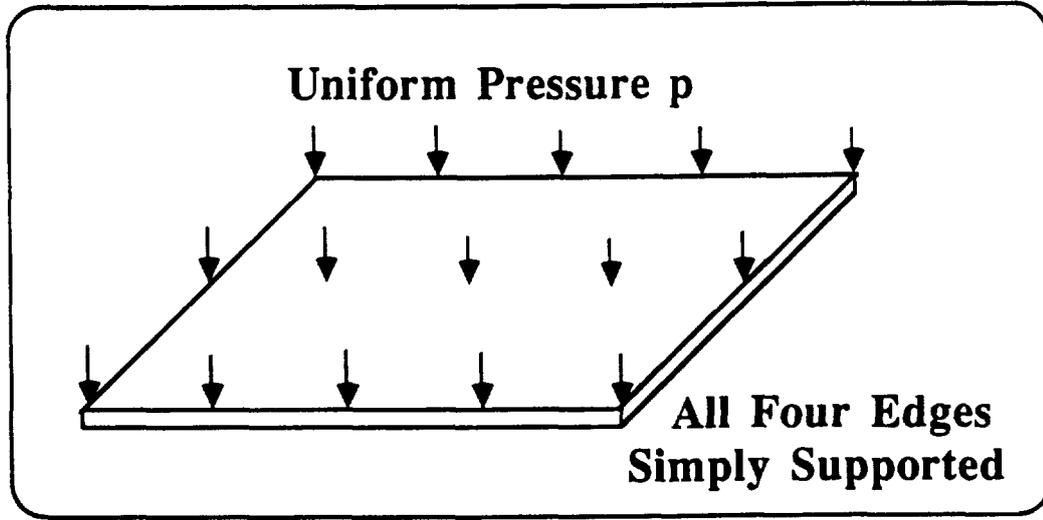
## Assumed Mode Approach with Nonlinear Strain Expression

Transverse Deflection of Center of Plate - Smooth Spin-up From 0 - 6 rad/sec in 15 seconds



Results obtained with 3 Assumed Stretch Modes and 3 Assumed Bending Modes

**Static Analysis of a Square Plate with  
Uniform Pressure Distribution Considering  
only Membrane Stiffness**



**Maximum inertia force per area in the middle of the plate during the spin-up motion is used as uniform pressure distribution.**

**Results of Maximum Lateral Deflection**

$\omega$ \ case	Static	Dynamic	
		Present	Conventional
$\pi/5$	0.18"	0.21"	2.1"
$\pi$	0.32"	0.37"	Divergence

**Fig. 19 - Static Deflections under High Pressure Loads**

## CONCLUSIONS

- Existing Flexible Multibody Formalisms Are Limited in Their Ability to Treat Coupled Large Displacement/Small Deformation.
- Alternative Approaches Include Taking Explicit Account of Constraints Geometrically or Within a Non-linear Strain Measure.
- Consistently-Linearized Models Work Well For Most Problems But Cannot Capture Motion-Induced Membrane Stiffness Variations.
- Second Order Structural Theories Account for Motion-Induced Stiffness Variations But Converge Slowly With Standard Mode Functions.
- Nonlinear Constraint Functions Serve Well as Modal Functions in Order to Improve Convergence in Second Order Structural Theories.

EFFICIENCY AND CAPABILITIES OF MULTI-BODY SIMULATIONS

R.J. VanderVoort  
DYNACS Engineering Co., Inc.  
Clearwater, Fl

ABSTRACT

Simulation efficiency and capability go hand in hand. The more capability you have the lower the efficiency will be. ~~Section 1 of this paper discusses~~ efficiency and ~~section 2 deals with~~ capabilities. The lesson we have learned about generic simulation is: Don't rule out any capabilities at the beginning but keep each one on a switch so it can be bypassed when warranted by a specific application.

1. EFFICIENCY

Efficiency means different things to different people. For the person running simulations interactively on a terminal quick turn around time is efficiency. For the person making 10,000 Monte-Carlo runs low cost is efficiency. For the person running real time simulations minimum CPU time is efficiency.

Three aspects of a simulation should be considered when dealing with efficiency; hardware, software and modeling.

Hardware A fast processor will reduce CPU time for a given simulation but this doesn't necessarily equate to improved efficiency. For example, the Monte-Carlo simulation may take 10 minutes on a super computer and 2 weeks on a PC but if time is free on the PC then that may be an efficient solution. We will not discuss hardware related issues except for two points. 1.) Fast hardware is of primary importance to the real time simulation because it means higher fidelity models can be incorporated 2.) Vector processors and parallel processors should use custom algorithms that take full advantage of the special machine architecture.

Software A fast algorithm will also reduce CPU time but again this doesn't necessarily equate to improved efficiency. For example, it is generally accepted that an ad-hoc simulation is much faster than a generic simulation. The cost of developing and testing the ad-hoc simulation may exceed the run time saving thereby reducing overall efficiency.

Recent work in the area of symbolic programming has shown that significant savings can be achieved by symbolically forming the equation of motion and numerically solving them. Other algorithms have been proposed that promise similar savings. There is one point that software developers should keep in mind. With generic simulations the user must have complete flexibility in retaining or deleting different parts of his model. This is because generic simulations are often used for model development and validation. In that environment an analyst will add or delete certain features to determine the effect on performance and whether or not the feature should be retained in the model.

More on this subject in section 2.

Modeling This is the domain of the simulation user and the area in which many improvements in efficiency can be made. For example, deleting a high order mode in a flexible body model has a compound effect. It reduces the model complexity and at the same time allows a bigger integration step size both of which reduce run time. Often times the reduced fidelity is justified by the savings in run time.

The point to be made is that the analyst is the end authority on the "correct" model for a given application. The more flexibility he has in changing his model the easier it is for him to select the best model for the job.

## 2. CAPABILITIES

Capability in our context is synonymous with flexibility and not with complexity. A simulation may be very detailed and complex but if it can't be changed then it's only useful in a narrow range of applications and has limited capability.

In our experience with TREETOPS and DCAP we have found that it is much easier to generate a model and obtain a response than it is to predict the correct response. In other words, when we don't get the expected response the simulation is usually correct and our expectation is wrong. This is not entirely unexpected because it is very difficult, even for an expert, to solve the equations of anything but the simplest dynamical systems. The solution to this dilemma is flexibility. Start with simple models that have known analytic solutions. Then add complexity one step at a time while gaining confidence in your model and insight into the behavior of your system.

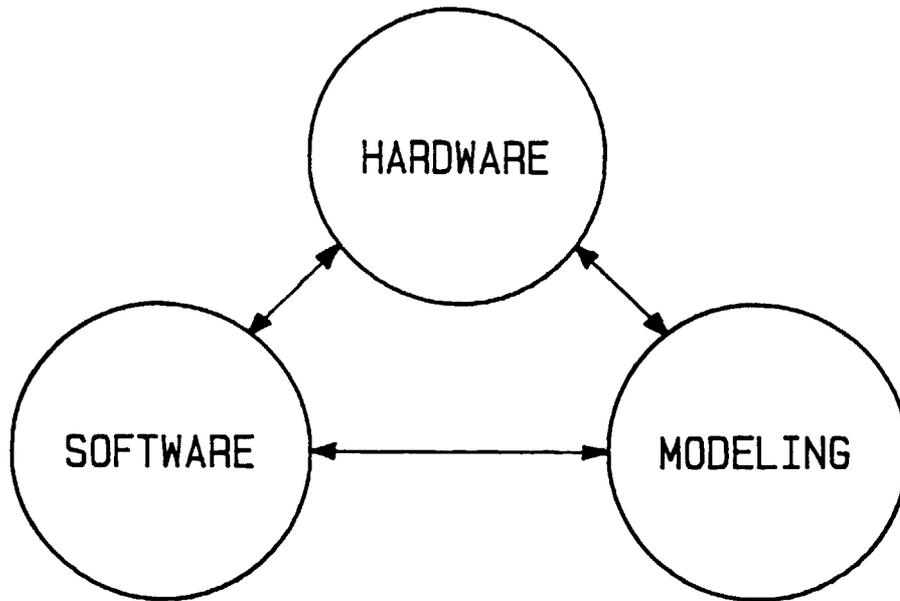
For multibody systems with flexible bodies the same arguments apply but the complexity of the model increases more rapidly than for rigid bodies. The person doing software development makes assumptions that simplify the resulting equations of motion. If this is done carelessly then terms are dropped that may prove essential in specific applications. On the other hand, if simplifications are not made then the computation burden becomes too great.

The lesson we learned is that you must retain as many terms as possible in the kinematics but they must have associated switches so you can easily add or delete them from a specific application. This is done for two reasons. 1.) to give you insight into the effect of various model elements on system response and 2.) to allow the selection of the most efficient model for a given application.

# SIMULATION EFFICIENCY



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- BYPASS TERMS
- MULTI-RATE ALGORITHMS
- SYMBOLIC PROGRAMMING
- ADHOC SIMULATION
- 

- BYPASS TERMS
- INTEGRATION TYPE & STEP SIZE
- REDUCED ORDER
- 
-

# SPEED-UP OPTIONS



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	COORDINATE TRANSFORMATIONS	MASS MATRIX FORMULATION (M)	NON-LINEAR TERMS ( $\underline{\omega} \times \underline{I} \cdot \underline{\omega}$ )	CONSTRAINT FORMULATION (A)
PERFORM ALL COMPUTATIONS				
COMPUTE ONLY ON FIRST PASS OF R-K INTEGRATION				
COMPUTE ONLY ON FIRST PASS OF NTH R-K STEP				
BYPASS COMPUTATIONS				



## TREETOPS SOFTWARE IMPROVEMENTS

	<u>EQUATION FORMULATION</u>	<u>EQUATION SOLUTION</u>	<u>PROCESSING HARDWARE</u>
354 CURRENT STATUS	NUMERIC	NUMERIC	SERIAL
FIRST STEP	∇ SYMBOLIC	NUMERIC	SERIAL
SECOND STEP	SYMBOLIC	NUMERIC	∇ PARALLEL

$$M\ddot{q} = f + A^T \lambda$$

$$A\dot{q} = B$$

## SIMULATION CAPABILITY-MENUS



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<u>BODIES</u>	<u>SENSORS</u>	<u>ACTUATORS</u>	<u>CONSTRAINTS</u>	<u>DEVICES</u>	<u>CONTROLLERS</u>
1. RIGID	1. RATE GYRO	1. REACTION JET	1. CLOSED LOOP	1. SPRINGS	1. CONTINUOUS
2. FLEXIBLE	2. RESOLVER	2. HYDRAULIC	2. VELOCITY-TIME	2. DAMPERS	2. DISCRETE
	3. ANGULAR	CYLINDER	3. VELOCITY	3. COULOMB	3. BLOCK DIAGRAM
	ACCELEROMETER	3. REACTION	-DIRECTION	DAMPER	(FREQUENCY
	4. VELOCITY	WHEEL	4. RATE-TIME	4. QUADRATIC	DOMAIN)
	5. POSITION	4. TORQUE	5. RATE-DIRECTION	SPRING/DAMPER	4. MATRIX
	6. ACCELEROMETER	MOTOR	6. CUT JPINT	5. SOLID DAMPER	(STATE SPACE)
	7. TACHOMETER	5. MOMENT		6. HARDSTOP	5. USER
	8. INTEGRATING	6. BRAKE		7. CONTACT	
	RATE GYRO	7. LOCK		SPRINGS	
	9. SUN SENSOR	8. SINGLE GIMBAL			
	10. STAR SENSOR	CMG			
	11. IMU	9. DOUBLE GIMBAL			
	12. POSITION VECTOR	CMG			
	13. VELOCITY VECTOR	10. MAGNETIC			



## SWITCHES FOR MODAL DATA

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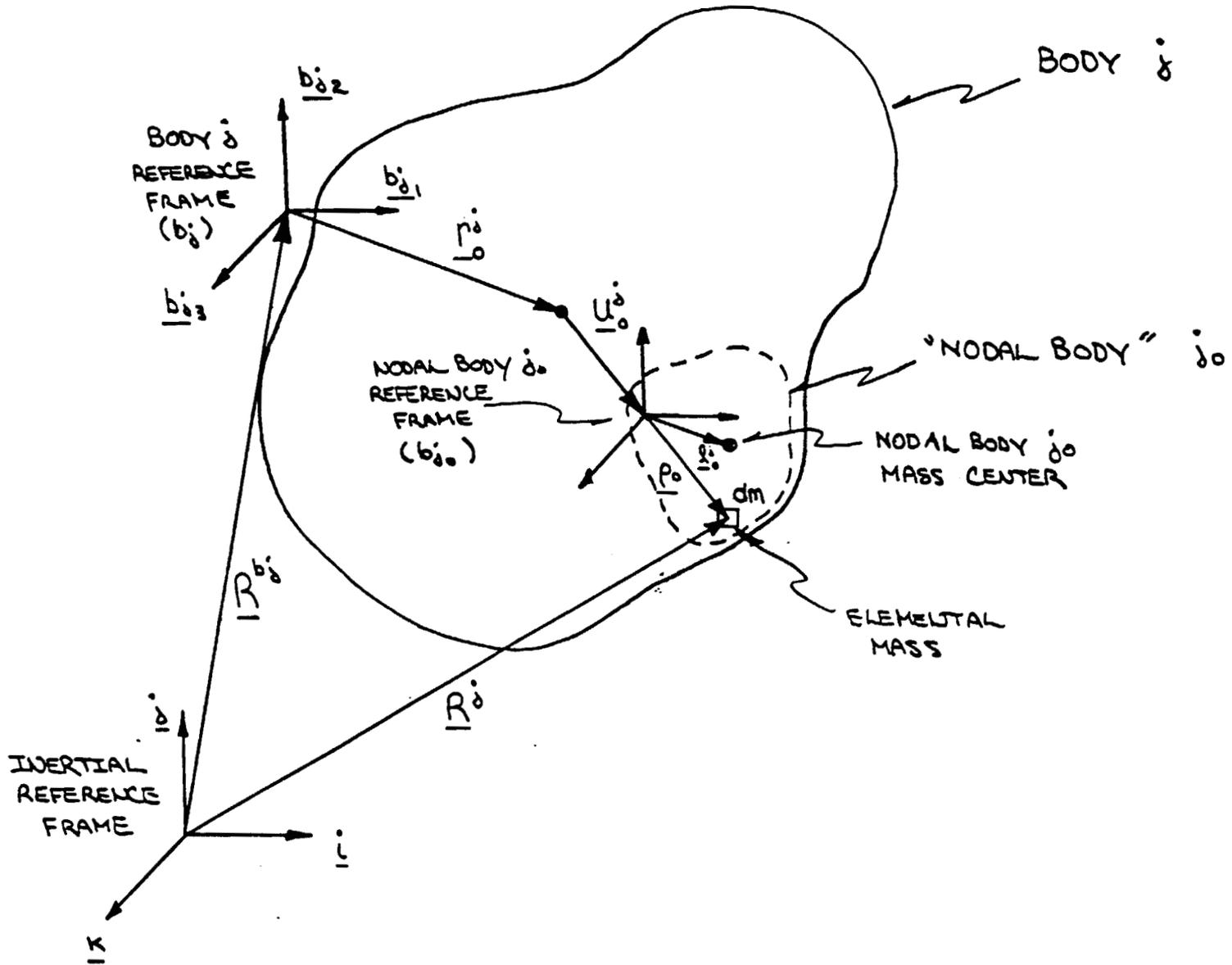
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- HIGH LEVEL
  - LUMPED MASS SWITCH
  
- MID LEVEL
  - FIRST ORDER SWITCH
  - SECOND ORDER SWITCH
  - THIRD ORDER SWITCH
  
- LOW LEVEL
  - ONE SWITCH FOR EACH TERM

FLEXIBLE BODY DISPLACEMENT FIELD



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$$f_k + f_k^* = 0 ; k=1, \dots, N$$

$$f_k = \sum_{o=1}^{N_B} \left\{ \int_{o^i} v_k^{o^i} \cdot df_o^i \right\}$$

$$-f_k^* = \sum_{o=1}^{N_B} \left\{ \sum_{o=1}^{N_B} \int_{o^i} v_k^{o^i} \cdot R_o^i dm \right\}$$

FROM NEWTON'S LAW

$$\int_{o^i} df_o^i - \ddot{R}_o^i dm = \underline{0}$$

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$$v_k^{o^i} = v_k^{o^o} + \underline{\omega}_k^{o^i} \times (\underline{r}_o^{o^i} + \underline{u}_o^{o^i} + \underline{p}_o) + v_k^{o^i}$$

$$R_o^i = \ddot{R}_o^{o^o} + \underline{\dot{\omega}}_o^{o^i} \times (\underline{r}_o^{o^i} + \underline{u}_o^{o^i} + \underline{p}_o) + \underline{\ddot{u}}_o^{o^o} + \underline{\ddot{u}}_o^{o^i} \times \underline{p}_o + 2\underline{\omega}_o^{o^i} \times \underline{\dot{u}}_o^{o^i}$$

$$+ (\underline{\omega}_o^{o^i} \times \underline{\dot{u}}_o^{o^i}) \times \underline{p}_o + \underline{\omega}_o^{o^i} \times (\underline{\omega}_o^{o^i} \times (\underline{r}_o^{o^i} + \underline{u}_o^{o^i} + \underline{p}_o))$$

$$+ \underline{\omega}_o^{o^i} \times (\underline{\dot{u}}_o^{o^i} \times \underline{p}_o) + \underline{\dot{u}}_o^{o^i} \times (\underline{\omega}_o^{o^i} \times \underline{p}_o) + \underline{\dot{u}}_o^{o^i} \times (\underline{\dot{u}}_o^{o^i} \times \underline{p}_o)$$

NOTE: OPEN DOT  
DENOTES LOCAL TIME  
DERIVATIVE, SOLID DOT  
DENOTES INERTIAL TIME  
DERIVATIVE.

COEFFICIENTS OF GENERALIZED SPEEDS

TRANSLATIONAL D.O.F.

LET  $K$  CORRESPOND TO THE  $i$ -TH TRANSLATIONAL D.O.F. OF THE  $q$ -TH HINGE.

$$\underline{V}_K^{qj} = \begin{cases} \underline{q}_i^q ; & j \in E(q) \\ \underline{0} ; & \text{OTHERWISE} \end{cases}$$

$$\underline{W}_K^{qj} = \underline{0}$$

$$\underline{V}_K^{qj} = \underline{0}$$

ROTATIONAL D.O.F.

LET  $K$  CORRESPOND TO THE  $i$ -TH ROTATIONAL D.O.F. OF THE  $q$ -TH HINGE.

$$\underline{V}_K^{qj} = \begin{cases} \underline{l}_i^q(\theta^q) \times (q \underline{x}^i - (\underline{r}_{hq} + \underline{u}_{hq})) ; & j \in E(q) \\ \underline{0} ; & \text{OTHERWISE} \end{cases}$$

$$\underline{W}_K^{qj} = \begin{cases} \underline{l}_i^q(\theta^q) ; & j \in E(q) \\ \underline{0} ; & \text{OTHERWISE} \end{cases}$$

$$\underline{V}_K^{qj} = \underline{0}$$

MODAL D.O.F.

LET  $K$  CORRESPOND TO THE  $i$ -TH MODAL D.O.F. OF THE  $q$ -TH BODY.

$$\underline{V}_K^{qj} = \begin{cases} -\underline{\phi}_i^{qg}(\underline{r}_{hq}) \times \underline{x}^{hj} + \underline{\phi}_i^{qg}(\underline{r}_{p(q)}) \times \underline{S}^{qj} (1 - \delta_{iq}) \\ \underline{0} ; & \text{OTHERWISE} \end{cases}$$

$$+ \underline{\phi}_i^{qg}(\underline{r}_{Kq}) \times (1 - \delta_{iq}) - \underline{\phi}_i^{qg}(\underline{r}_{hq}) ; \quad j \in E(q)$$

$$\underline{W}_K^{qj} = \begin{cases} -\underline{\phi}_i^{qg}(\underline{r}_{hq}) + \underline{\phi}_i^{qg}(\underline{r}_{Kq}) \times (1 - \delta_{iq}) ; & j \in E(q) \\ \underline{0} ; & \text{OTHERWISE} \end{cases}$$

$$\underline{V}_K^{qj} = \begin{cases} \underline{\phi}_i^{qg}(\underline{r}_0^q) + \underline{\phi}_i^{qg}(\underline{r}_0^q) \times \underline{p}_0 ; & j = q \\ \underline{0} ; & \text{OTHERWISE} \end{cases}$$

$\underline{q}_i^g \equiv i$ -TH TRANSLATION AXIS OF THE  $g$ -TH HINGE, FIXED IN  $L(g)$ , BODY INBOARD OF THE  $g$ -TH BODY

$\underline{l}_i^g(\theta^g) \equiv i$ -TH ROTATION AXIS (EULER AXIS) OF THE  $g$ -TH HINGE

$\underline{x}^j \equiv$  VECTOR LOCATING BODY  $j$  REFERENCE WRT BODY  $g$  REFERENCE

$\underline{r}_{hg} \equiv$  VECTOR LOCATING UNDEFORMED HINGE ATTACH POINT ON BODY  $g$  WRT BODY  $g$  REFERENCE.

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$\underline{u}_{hg} \equiv$  DEFORMATION AT  $\underline{r}_{hg}$  ON BODY  $g$  ( $\underline{u}_{hg} = \sum_{i=1}^{NM_g} \phi_i^g(\underline{r}_{hg}) \eta_i^g$ )

$\underline{z}^{bj} \equiv$  VECTOR LOCATING BODY  $j$  REFERENCE WRT BODY  $g$  HINGE ATTACH POINT ON BODY  $g$ .

$\underline{s}^{bj} \equiv$  VECTOR FROM POINT  $P(g)$  ON BODY  $g$  (LEADING TO BODY  $j$ ) TO BODY  $j$  REFERENCE. (IF  $j=g$ ,  $\underline{s}^{bj} = \underline{0}$ )

$E(g) \equiv$  SET OF ALL BODIES OUTBOARD OF THE  $g$ -TH BODY INCLUDING BODY  $g$

"GENERALIZED INERTIAL FORCE"  
(EXPRESSION FOR  $-f_k^*$ )

$$-f_k^* = \sum_{j=1}^{NB} \left\{ m_j (\ddot{R}_{k_j}^i + \ddot{L}_j^i) \cdot \underline{V}_k^i + (\underline{H}_j^i + m_j \underline{L}_j^i \times \ddot{R}_{k_j}^i) \cdot \underline{\omega}_k^i + \sum_{\sigma=1}^{NNB_j^i} \int_{V_{k_j}^i} \underline{V}_k^i \cdot \ddot{R}_j^i dm \right\}$$

MODAL TERMS

$k$  CORRESPONDING TO THE  $i$ -TH MODAL D.O.F. OF THE  $j$ -TH BODY

(I.)  $m_j \underline{\alpha}_j^i \cdot \ddot{R}_{k_j}^i$

(II.)  $\left( \underline{h}_j^i + \sum_{K=1}^{NN_j^i} \underline{Y}_{Kj}^i \eta_K^i + \sum_{K=1}^{NN_j^i} \sum_{M=1}^{NN_j^i} \underline{Z}_{MKj}^i \eta_M^i \eta_K^i \right) \cdot \underline{\omega}_j^i$

(III.)  $2 \underline{\omega}_j^i \cdot \left( \sum_{K=1}^{NN_j^i} \underline{Y}_{Kj}^i \eta_K^i + \sum_{K=1}^{NN_j^i} \sum_{M=1}^{NN_j^i} \underline{Y}_{MKj}^i \eta_M^i \eta_K^i \right)$

(IV.) MODAL MASS (ASSUMED BODY BASIS)

- SCALAR REPRESENTATION FOR "LUMPED APPROACH"

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$$\left\{ \sum_{o=1}^{N_{NB}^i} \left( \{ \phi_{o,i}^i \} \{ \phi_{o,i}^i \}^T \right) \right\} \left( \begin{array}{c|c} \left[ m_o^i \right]_{\text{DIAGONAL}} & \left[ -m_o^i \tilde{l}_o^i \right] \\ \hline \left[ m_o^i \tilde{l}_o^i \right] & \left[ J^{b_o} + \tilde{u}_o^i J^{i_o} - J^{b_o} \tilde{u}_o^i - \tilde{u}_o^i J^{b_o} \tilde{u}_o^i \right] \end{array} \right) \left( \begin{array}{c} \{ \phi_{o,k}^i \} \\ \{ \phi_{o,k}^i \} \end{array} \right) \left. \right\} \eta_k^i$$

(V.)  $\underline{\omega}^i \cdot \left( \underline{W}_i^i + \sum_{k=1}^{N_{NB}^i} \underline{W}_{Ki}^i \eta_k^i + \sum_{k=1}^{N_{NB}^i} \sum_{m=1}^{N_{NB}^i} \underline{W}_{mKi}^i \eta_m^i \eta_k^i \right) \cdot \underline{\omega}^i$

$$\sum_{o=1}^{N_{NB}^i} (\underline{\omega}^i + \underline{u}_{o,i}^i) \cdot \left( \underline{T}_i^i + \sum_{k=1}^{N_{NB}^i} \underline{T}_{Ki}^i \eta_k^i + \sum_{k=1}^{N_{NB}^i} \sum_{m=1}^{N_{NB}^i} \underline{T}_{mKi}^i \eta_m^i \eta_k^i \right) \cdot (\underline{\omega}^i + \underline{u}_{o,i}^i)$$

$$(VI.) \quad \underline{\omega}^i \cdot \left( \sum_{K=1}^{M_i} \underline{W}_{Ki}^i \eta_K^i + \sum_{K=1}^{M_i} \sum_{M=1}^{M_i} \underline{W}_{MKi}^i \eta_M^i \eta_K^i \right)$$

$$(VII.) \quad \sum_{i=1}^{M_i} (\underline{\omega}^i \times \underline{u}^i) \cdot \left( \underline{D}_{ii}^{i,i} + \sum_{K=1}^{M_i} \underline{D}_{Ki}^{i,i} \eta_K^i + \sum_{K=1}^{M_i} \sum_{M=1}^{M_i} \underline{D}_{MKi}^{i,i} \eta_M^i \eta_K^i \right)$$

RATE OF CHANGE OF BODY  $i$  ANGULAR MOMENTUM  
(CONSOLIDATED EXPRESSION FOR  $\dot{H}_i$ )

$$\dot{H}_i = \underline{\underline{I}}_i \cdot \underline{\underline{\dot{\omega}}}_i + \sum_{c=1}^{N_i} \left( \underline{\underline{r}}_{ci} + \sum_{k=1}^{N_i} Y_{ki} \underline{\underline{\eta}}_k + \sum_{k=1}^{N_i} \sum_{m=1}^{N_i} Z_{mki} \underline{\underline{\eta}}_m \underline{\underline{\eta}}_k \right) \underline{\underline{\dot{\eta}}}_c + \underline{\underline{\omega}}_i \times \underline{\underline{I}}_i \cdot \underline{\underline{\omega}}_i + \left( 2 \sum_{c=1}^{N_i} \left\{ \underline{\underline{M}}_{ci} + \sum_{k=1}^{N_i} \underline{\underline{P}}_{ki} \underline{\underline{\eta}}_k \right\} \underline{\underline{\eta}}_c + \underline{\underline{K}}_i \right) \cdot \underline{\underline{\omega}}_i + \underline{\underline{R}}_i$$

WHERE :

$$\underline{\underline{I}}_i = \underline{\underline{I}}_R + \sum_{c=1}^{N_i} \left\{ \underline{\underline{M}}_{ci} + \underline{\underline{N}}_{ci} \right\} \underline{\underline{\eta}}_c + \sum_{c=1}^{N_i} \sum_{k=1}^{N_i} \underline{\underline{P}}_{ki} \underline{\underline{\eta}}_k \underline{\underline{\eta}}_c$$

## DEFINITION OF VECTORS AND DYADICS

DEFINE:  $\underline{b}^i = \begin{pmatrix} \underline{b}_{i1} \\ \underline{b}_{i2} \\ \underline{b}_{i3} \end{pmatrix}$  ;  $\underline{b}^{iT} = (\underline{b}_{i1} \ \underline{b}_{i2} \ \underline{b}_{i3})$  , BODY  $i$  REFERENCE BASIS

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VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{q}_i^i$	0	MODAL	$\frac{1}{m_i} \left\{ \sum_{n=1}^{NNB_i} m_i \phi_{ni}^i - m_i \underline{l}_i \times \phi_{ni}^i - \sum_{k=1}^{MB_i} m_i (\phi_{kn}^i \times \underline{l}_i) \times \phi_{ni}^i \right\} \eta_k^i$
$\underline{u}_i^i$	0, 1, 2	ROTATIONAL / MODAL	$\underline{b}^T \left\{ \underline{J}^{i0} + \tilde{\underline{u}}_0^i \underline{J}^{i0} - \underline{J}^{i0} \tilde{\underline{u}}_0^i - \tilde{\underline{u}}_0^i \underline{J}^{i0} \tilde{\underline{u}}_0^i \right\} \underline{b}$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><math>\underline{J}^{i0}</math> IS INERTIA MATRIX (3x3) OF NODAL BODY <math>i_0</math> WRT NODAL BODY <math>i_0</math> REFERENCE FRAME, <math>\underline{b}_{i0}</math></p> </div>
$\underline{r}_i^i$	0	ROTATIONAL / MODAL	$\underline{b}^T \left\{ \sum_{n=1}^{NNB_i} \left[ m_i \tilde{\underline{r}}_0^i \{ \phi_{ni}^i \} - \tilde{\underline{r}}_0^i m_i \underline{l}_0^i \{ \phi_{ni}^i \} + m_i \underline{l}_0^i \{ \phi_{ni}^i \} + \underline{J}^{i0} \{ \phi_{ni}^i \} \right] \right\}$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><math>\{x\}</math> DENOTES COLUMN MATRIX</p> </div>

VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{Y}_{ki}^i$		ROTATIONAL/ MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NMB_i} \left( m_o^i \tilde{\phi}_{ok}^i \{ \phi_{oi}^i \} + \tilde{r}_o^i (m_o^i \tilde{l}_o^i \{ \phi_{ok}^i \}) \{ \phi_{oi}^i \} \right. \right.$ $\left. - \tilde{\phi}_{ok}^i m_o^i \tilde{l}_o^i \{ \phi_{oi}^i \} - m_o^i ( \tilde{l}_o^i \{ \phi_{ok}^i \} ) \{ \phi_{oi}^i \} \right.$ $\left. + ( \tilde{\phi}_{ok}^i J_{oo}^i - J_{oo}^i \tilde{\phi}_{ok}^i ) \{ \phi_{oi}^i \} \right\}$
$\underline{Z}_{mki}^i$		ROTATIONAL/ MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NMB_i} \left( \tilde{\phi}_{om}^i (m_o^i \tilde{l}_o^i \{ \phi_{ok}^i \}) \{ \phi_{oi}^i \} - \tilde{\phi}_{om}^i J_{oo}^i \tilde{\phi}_{ok}^i \{ \phi_{oi}^i \} \right) \right\}$
$\underline{Y}_{ki}^i$		MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NMB_i} \left( m_o^i \tilde{\phi}_{ok}^i \{ \phi_{oi}^i \} - \tilde{\phi}_{ok}^i m_o^i \tilde{l}_o^i \{ \phi_{oi}^i \} \right) \right\}$
$\underline{Y}_{mki}^i$		MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NMB_i} \left( \tilde{\phi}_{ok}^i (m_o^i \tilde{l}_o^i \{ \phi_{ok}^i \}) \{ \phi_{oi}^i \} \right) \right\}$
$\underline{W}_{ci}^i$		MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NMB_i} \left( m_o^i \tilde{\phi}_{oi}^i \tilde{r}_o^i - (m_o^i \tilde{l}_o^i \{ \phi_{oi}^i \}) \tilde{r}_o^i \right) \right\} \mathbf{b}$
$\underline{W}_{ki}^i$		MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NMB_i} \left( (m_o^i \tilde{l}_o^i \{ \phi_{ok}^i \}) \{ \phi_{oi}^i \} \tilde{r}_o^i + m_o^i \tilde{\phi}_{oi}^i \tilde{\phi}_{ok}^i \right. \right.$ $\left. - (m_o^i \tilde{l}_o^i \{ \phi_{oi}^i \}) \tilde{\phi}_{ok}^i \right\} \mathbf{b}$

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VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{\underline{W}}_{mki}^i$		MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{m_i} \left[ \left( m_o \tilde{l}_o \{ \phi_{o,k}^i \} \right) \{ \phi_{o,i}^i \} \tilde{\phi}_{o,m}^i \right] \right\} \mathbf{b}$
$\underline{\underline{T}}_i^{b_i o}$		MODAL	$\mathbf{b}^T \left\{ \tilde{\phi}_{o,i}^i m_o \tilde{l}_o + J^{b_i o} \tilde{\phi}_{o,i}^i \right\} \mathbf{b}$
$\underline{\underline{T}}_{ki}^{b_i o}$		MODAL	$\mathbf{b}^T \left\{ -\tilde{\phi}_{o,i}^i \left( m_o \tilde{l}_o \{ \phi_{o,k}^i \} \right) + \tilde{\phi}_{o,k}^i J^{b_i o} \tilde{\phi}_{o,i}^i - J^{b_i o} \tilde{\phi}_{o,k}^i \tilde{\phi}_{o,i}^i \right\} \mathbf{b}$
$\underline{\underline{T}}_{mki}^{b_i o}$		MODAL	$\mathbf{b}^T \left\{ -\tilde{\phi}_{o,m}^i J^{b_i o} \tilde{\phi}_{o,k}^i \tilde{\phi}_{o,i}^i \right\} \mathbf{b}$
$\underline{\underline{W}}_{ki}^i$		MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{m_i} \left[ \tilde{\phi}_{o,k}^i m_o \tilde{l}_o \{ \phi_{o,i}^i \} \right] \right\}$
$\underline{\underline{W}}_{mki}^i$		MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{m_i} \left[ -\tilde{\phi}_{o,k}^i \left( m_o \tilde{l}_o \{ \phi_{o,m}^i \} \right) \{ \phi_{o,i}^i \} \right] \right\}$
$\underline{\underline{D}}_i^{b_i o}$		MODAL	$\mathbf{b}^T \left\{ J^{b_i o} \{ \phi_{o,i}^i \} \right\}$
$\underline{\underline{D}}_{ki}^{b_i o}$		MODAL	$\mathbf{b}^T \left\{ \tilde{\phi}_{o,k}^i J^{b_i o} \{ \phi_{o,i}^i \} - J^{b_i o} \tilde{\phi}_{o,k}^i \{ \phi_{o,i}^i \} \right\}$
$\underline{\underline{D}}_{mki}^{b_i o}$		MODAL	$\mathbf{b}^T \left\{ -\tilde{\phi}_{o,m}^i J^{b_i o} \tilde{\phi}_{o,k}^i \{ \phi_{o,i}^i \} \right\}$

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VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\mathbb{I}_R^i$	0	ROTATIONAL	$\mathbb{B}^T \left\{ \sum_{o=1}^{NM_i} \left[ J_{o,i}^{i_0} + m_o^i \tilde{r}_o^i \tilde{r}_o^i - \tilde{r}_o^i m_o^i \tilde{l}_o^i - m_o^i \tilde{l}_o^i \tilde{r}_o^i \right] \right\} \mathbb{B}$
$\mathbb{M}_{r,i}^i$	1	ROTATIONAL	$\mathbb{B}^T \left\{ \sum_{o=1}^{NM_i} \left[ -m_o^i \tilde{r}_o^i \tilde{\varphi}_{o,i}^i + \tilde{r}_o^i (m_o^i \tilde{l}_o^i \{\varphi_{o,i}^i\}) - m_o^i \tilde{l}_o^i \tilde{\varphi}_{o,i}^i - J_{o,i}^{i_0} \tilde{\varphi}_{o,i}^i \right] \right\} \mathbb{B}$
$\mathbb{N}_{r,i}^i$	1	ROTATIONAL	$\mathbb{B}^T \left\{ \sum_{o=1}^{NM_i} \left[ -m_o^i \tilde{\varphi}_{o,i}^i \tilde{r}_o^i - \tilde{\varphi}_{o,i}^i m_o^i \tilde{l}_o^i + (m_o^i \tilde{l}_o^i \{\varphi_{o,i}^i\}) \tilde{r}_o^i + \tilde{\varphi}_{o,i}^i J_{o,i}^{i_0} \right] \right\} \mathbb{B}$
$\mathbb{P}_{\kappa,i}^i$	2	ROTATIONAL	$\mathbb{B}^T \left\{ \sum_{o=1}^{NM_i} \left[ -m_o^i \tilde{\varphi}_{o,\kappa}^i \tilde{\varphi}_{o,i}^i + \tilde{\varphi}_{o,\kappa}^i (m_o^i \tilde{l}_o^i \{\varphi_{o,i}^i\}) + (m_o^i \tilde{l}_o^i \{\varphi_{o,\kappa}^i\}) \tilde{\varphi}_{o,i}^i - \tilde{\varphi}_{o,\kappa}^i J_{o,i}^{i_0} \tilde{\varphi}_{o,i}^i \right] \right\} \mathbb{B}$
$\mathbb{K}_i^i$	MULTIPLE	ROTATIONAL	$\mathbb{K}_i^i = \sum_{i=1}^{NM_i} \mathbb{K}_{i,i}^i \eta_i^i + \sum_{i=1}^{NM_i} \sum_{\kappa=1}^{NM_i} \mathbb{K}_{\kappa,i}^i \eta_{\kappa}^i \eta_i^i + \sum_{i=1}^{NM_i} \sum_{\kappa=1}^{NM_i} \sum_{\alpha=1}^{NM_i} \mathbb{K}_{\alpha\kappa i}^i \eta_{\alpha}^i \eta_{\kappa}^i \eta_i^i$

VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{\underline{R}}_{\cdot\cdot}^{\cdot\cdot}$		ROTATIONAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NNS} \left[ \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} + \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \right] \right\} \mathbf{b}$
$\underline{\underline{R}}_{ki}^{\cdot\cdot}$		ROTATIONAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NNS} \left[ \tilde{\phi}_{\cdot\cdot k}^{\cdot\cdot} \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} + \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot k}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} + \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot k}^{\cdot\cdot} \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \right. \right. \\ \left. \left. - \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot k}^{\cdot\cdot} + 2 \tilde{r}_o^{\cdot\cdot} \left[ \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} (m_o^{\cdot\cdot} \tilde{l}_o^{\cdot\cdot} \{ \phi_{\cdot\cdot k}^{\cdot\cdot} \}) \right] \right] \right\} \mathbf{b}$
$\underline{\underline{R}}_{mki}^{\cdot\cdot}$		ROTATIONAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NNS} \left[ 2 \tilde{\phi}_{\cdot\cdot k}^{\cdot\cdot} \left[ \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} (m_o^{\cdot\cdot} \tilde{l}_o^{\cdot\cdot} \{ \phi_{\cdot\cdot m}^{\cdot\cdot} \}) \right] + \tilde{\phi}_{\cdot\cdot m}^{\cdot\cdot} \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot k}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} \right. \right. \\ \left. \left. - \tilde{\phi}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot k}^{\cdot\cdot} \mathbf{J}_{\cdot\cdot}^{\cdot\cdot} \tilde{\phi}_{\cdot\cdot m}^{\cdot\cdot} \right] \right\} \mathbf{b}$
$\underline{\underline{R}}_{\cdot\cdot}^{\cdot\cdot}$	MULTIPLE	ROTATIONAL	$\sum_{o=1}^{NNS} \left\{ \underline{\underline{\omega}}_o^{\cdot\cdot} \times \underline{\underline{\Pi}}_o^{\cdot\cdot} \cdot \underline{\underline{u}}_o^{\cdot\cdot} + \underline{\underline{u}}_o^{\cdot\cdot} \times \underline{\underline{\Pi}}_o^{\cdot\cdot} \cdot \underline{\underline{u}}_o^{\cdot\cdot} \right. \\ \left. + (\underline{\underline{r}}_o^{\cdot\cdot} + \underline{\underline{u}}_o^{\cdot\cdot}) \times (\underline{\underline{u}}_o^{\cdot\cdot} \times (\underline{\underline{u}}_o^{\cdot\cdot} \times (m_o^{\cdot\cdot} \underline{\underline{l}}_o^{\cdot\cdot} + \underline{\underline{u}}_o^{\cdot\cdot} \times m_o^{\cdot\cdot} \underline{\underline{l}}_o^{\cdot\cdot}))) \right\}$

**EXPLICIT MODELING AND COMPUTATIONAL LOAD DISTRIBUTION  
FOR CONCURRENT PROCESSING SIMULATION OF THE SPACE STATION**

By

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**ABSTRACT**

Analytical simulation of the dynamics/control interaction of large space structures is essential during the design process as full-scale tests of these vehicles in the laboratory are impractical. Furthermore, the operational manifests of large space systems on-orbit may call for significant changes in their mass and stiffness distributions as well as for substantial growth during the vehicles' lifetimes, and these can be studied only by analytical simulation.

Current methodologies for simulating large space structures involve implicit mathematical models and solutions on serial digital computers. These methodologies require unacceptably long computer processing time and exorbitant costs as the models become larger and more complex. Potential orders-of-magnitude reductions in simulation time and cost of multibody dynamic systems can be attained using: (1) enhanced analytical models for simulation, and (2) special-purpose, concurrent computational hardware and system software.

*also discussed*  
This paper deals with two important aspects of concurrent processing under development at TRW. These are: (1) the derivation of explicit mathematical models of multibody dynamic systems, and (2) a balanced computational load distribution (BCLD) among loosely coupled computational units (processors) of a concurrent processing system. The developed methodologies will be demonstrated in the paper by way of an application to the Phase 1 of the Space Station - a task being performed by TRW under NASA/JSC contract NAS9-17778.

The mathematical model of the Space Station consists of three interconnected flexible bodies capable of undergoing large, rigid-body motion with respect to each other. Body 1 is the main central body and contains the pressurized modules inboard of the two Alpha gimbals. Bodies 2 and 3 are the starboard and port bodies connected to Body 1 at the Alpha gimbals and include all components on the transverse booms outboard of the Alpha gimbals (including the solar arrays). The control systems in the model maintain Body 1 in a prescribed 3-axis attitude control mode, while producing large-angle rotations of the flexible solar arrays to position them normal to the sun-line.

The BCLD methodology for concurrent processing developed by TRW employs a static allocation strategy in which a separate software package is used off-line and at leisure prior to the execution of the simulation program. The load distribution, in this methodology, is carried out in a manner transparent to the user who, nevertheless, exercises control over the procedure with pre-selected constraint conditions.

The distributed model of the Space Station is now complete and ready to undergo benchmark tests on TRW's Custom Architected Parallel System during the June-July 1988 timeframe.



**Engineering & Test Division**  
TRW Space & Technology Group

**NASA/OAST Workshop on Computational Aspects  
in the Control of Flexible Systems**

**Explicit Modeling and Computational Load  
Distribution for Concurrent Processing Simulation  
of the Space Station**

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**Dr. R. Gluck**

**July 12-14, 1988**

**Williamsburg, Virginia**

# Introduction



**Engineering & Test Division**  
TRW Space & Technology Group

This paper presents the application of concurrent processing technology developed at TRW Space & Technology Group over the past several years to the simulation of the Space Station. The effort is funded by NASA Johnson Space Center under Contract NAS9-17778 and monitored by Mr. John W. Sunkel. The period of performance extends from April 1987 to November 1988

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## OBJECTIVE

This project was established to provide NASA with quantitative data to determine the cost effectiveness of utilizing a specialized processing system such as the Custom Architected Parallel Processing System (CAPPS) for development and verification of the operational Space Station flight control system. The CAPPS is a concurrent processor consisting of loosely coupled, high speed array processors [computational units (CUs)] - each containing its own input/output capability and memory banks. The specially designed CUs are capable of concurrent computation and communication, thereby placing a very low overhead on the latter operation. Furthermore, the system's architecture provides for direct communication between each CU and any other CU, facilitating considerable flexibility in adapting the CAPPS architecture to a specific simulation problem.

## Objective



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The objective of this project is to develop, verify and demonstrate the simulation of an explicit model of the controls/structure interaction of the Space Station on CAPPS

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## PROJECT WORK BREAKDOWN STRUCTURE

Applying previously developed application and system software at TRW to the concurrent processing of an explicit model of the control/structure interaction of the Space Station on CAPPs, the project described herein consists of three sequential tasks as stated in the figure. This paper focuses on the completed Tasks (a) and (b). Task (c), at the time of this writing, is in progress.

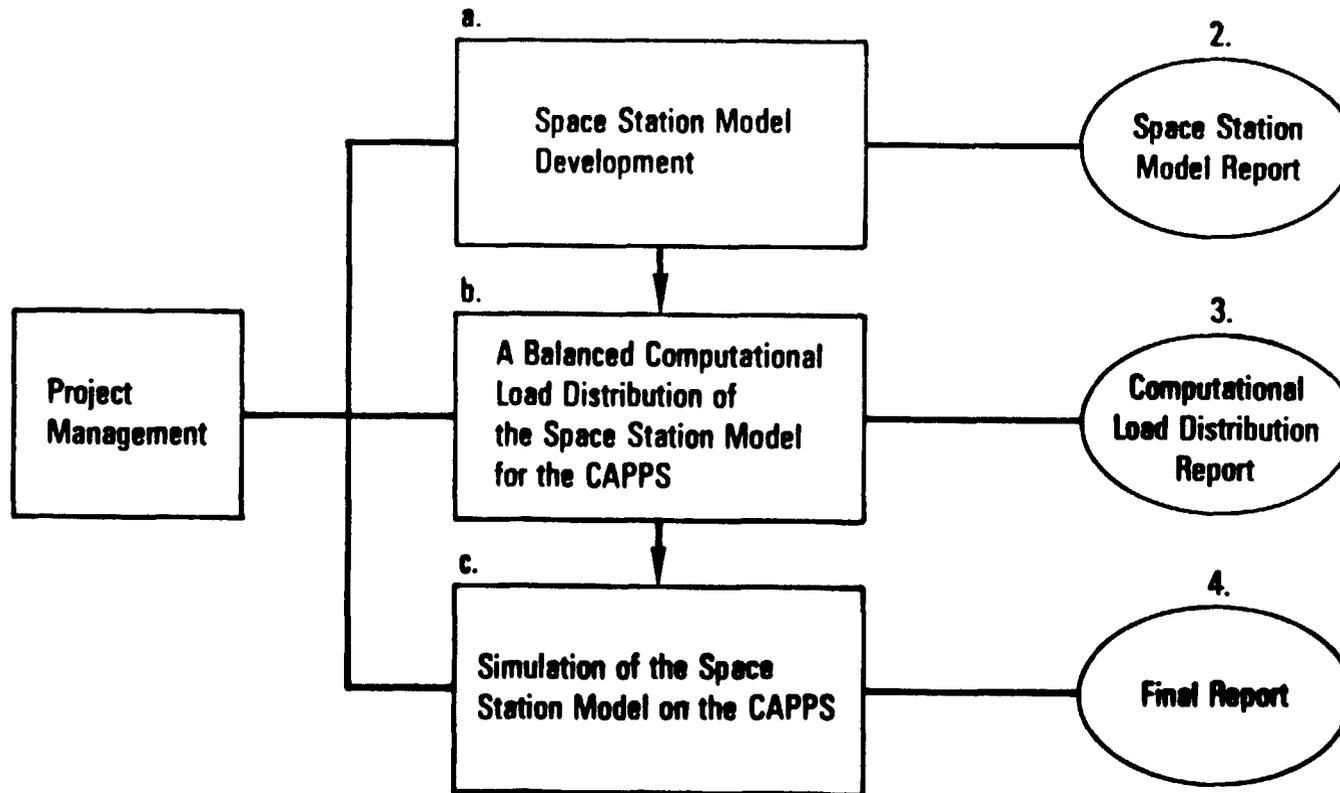
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# Project Work Breakdown Structure



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# The Space Station Model Development



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- Methodology
- Model's Contents

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## ADVANTAGES OF EXPLICIT EQUATIONS OF MOTION

The derivation of the equations of motion by symbol manipulation has several important attributes compared to the conventional (implicit) methodology. Symbol manipulation, i.e., the nonnumerical computation with a digital computer, yields scalar equations of motion specifically tailored to complex dynamical systems, where the analyst has the freedom and insight to incorporate any required fidelity in the model. Furthermore, the output of symbol manipulation is a completely portable FORTRAN code in the format of  $A(x,t)\dot{x} = b(x,t)$ , which can be delivered via file to either serial or concurrent processors without requiring any programming. This reduces development cost by at least one order-of-magnitude or more compared to that of a special-purpose implicit code. Finally, symbolically derived scalar equations of motion require a substantially reduced simulation time compared to those of conventional codes. Benchmark tests conducted at TRW yielded improvements in run times by factors of approximately 4 and 3 for rigid-body and flexible-body systems, respectively.

# Advantages of Explicit Equations of Motion



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- Useful engineering insights into the dynamic characteristics of the system
- No major programming effort required to perform simulation
- Large reduction of time required for simulation as compared to that required for implicit formulation
  - Implicit formulation requires the derivation of the equations of motion to be performed numerically at each integration step
  - Explicit formulation requires the derivation to be performed only once

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## ADVANTAGES OF USING PROGRAM SMP TO DERIVE EXPLICIT EQUATIONS OF MOTION

Program SMP was selected for the TRW symbol manipulation methodology following a thorough analysis which proved it superior in both versatility and speed to other available symbol manipulation codes such as MACSYMA, Reduce and FORMAC. The SMP program is implemented in the C language and is available on a variety of mainframes and workstations. Its capability of handling very large amounts of data is ideally suited for the derivation of explicit equations of motion of multibody spacecraft. The program's other attributes are listed in the accompanying figure.

## **Advantages of Using SMP to Derive Explicit Equations of Motion**



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- **Relieves the drudgery and distasteful tasks of manual algebraic manipulation**
- **Reduces cost and time by orders-of-magnitude as compared to manual derivation**
- **Allows the analysts to fully participate in the derivation process to achieve the most efficient mathematical model**
- **Leads to equations with no wasteful operations, such as additions of zeros, multiplications by unity, and dot product of orthogonal vectors**

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## PROGRAM SYMBOD

Program SYMBOD for the derivation of the explicit equations of motion of multibody flexible dynamical systems was developed within the general framework of SMP. A menu is provided to the analyst for introducing the system's topology and appropriate logic is available for the definition/computation of the essential elements of Kane's Dynamical Equations as shown in the figure; however, the analyst can, at his discretion, override the logic imbedded in each of the program's modules. The procedure is considered to be optimal in the sense that it leaves with the analyst the tasks he is best trained to perform, while transferring to the computer the manually prohibitive algebraic manipulation and long derivation operations.

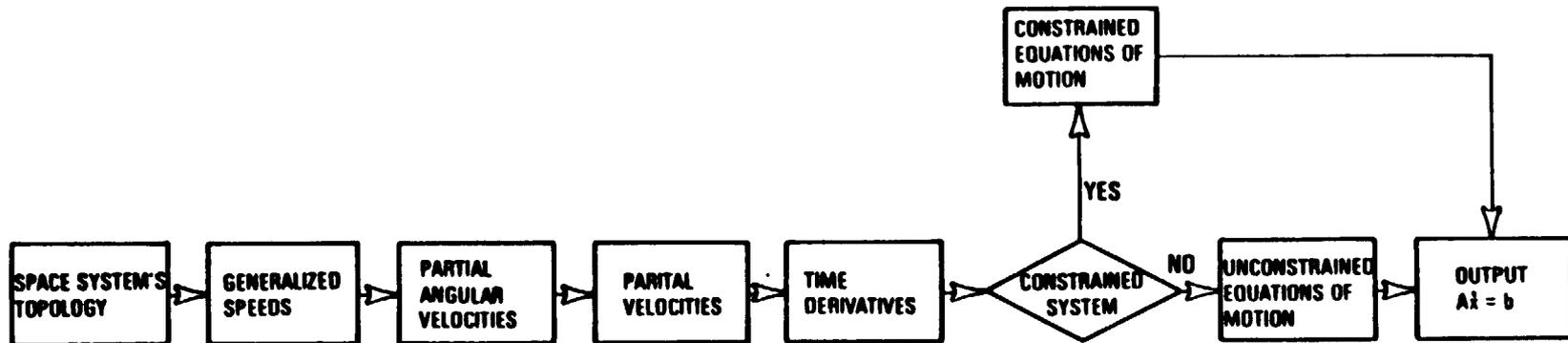
Two methodologies were incorporated in Program SYMBOD for dealing with the presence of  $m$  geometrical and velocity constraints in a multibody dynamic system. In the first methodology (by Wampler et al, Reference 1), the dynamic equations governing a constrained system are generated symbolically directly from expressions comprising the equations governing the system without constraints. This methodology for constraint elimination (which requires a symbolic inversion of a matrix of order  $m$ ) is applied when the number of constraint equations in the system is small ( $m < 6$ ) and no working constraints are involved; otherwise, the Lagrange multiplier methodology is used, where the stabilized penalty procedure of Reference 2 offers an attractive way for stabilizing the constraint equations now retained in the mathematical model.

# Top Level Flow Diagram for the Explicit Formulation of Kane's Dynamical Equations with the SMP Program (Program SYMBOD)



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## ATTRIBUTES OF PROGRAM SYMBOD

Program SYMBOD contains several innovations which combine to produce an efficient mathematical model. These are listed in the figure and explained briefly below.

The generation of equations of motion by symbol manipulation requires a systematic method of reducing the number of algebraic operations in the formation of Kane's equations. Frequently, the intermediate computations of expressions, such as the velocity terms, produce a multitude of expressions so large that their storage requirements exceed the computer's capacity. A procedure for systematically introducing new intermediate symbols to replace recurring combinations of algebraic subexpressions was developed. This procedure eliminates repetitious calculations and results in efficient computational algorithms requiring fewer arithmetic operations.

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The formulation of Kane's dynamical equations associated with the flexible-body degrees-of-freedom (dof) of a body are iterative in the number of assumed admissible functions required to represent the flexibility. The totality of the flexible-body dof for each body was, therefore, represented in Program SYMBOD by a single dof of that body. This allows postponement of the final selection of the required number of assumed admissible functions until after the development of the explicit mathematical model (including the control system) is completed, i.e., the assumed admissible functions in this formulation need not be selected prematurely.

Program SYMBOD provides for direct elimination (by command) of superfluous higher order terms in the explicit equations of motion when these terms are inconsistent with basic assumptions made in the formulation or with the variance of input parameters.

# Attributes of Program SYMBOD



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- Systematic introduction of intermediate variables for algebraic subexpressions
  - Eliminates repetitious calculations
  - Reduces computer memory requirements
  - Produces efficient computational equations involving fewer arithmetic operations
- Totality of flexible-body degrees-of-freedom (dof) of each flexible body is represented by a single dof of that body. Selection of desired flexible functions in simulation is performed **after** control system is specified
- Elimination of superfluous higher-order terms in equations
- Error-free translation of explicit equations of motion into FORTRAN
  - Eliminates manual mistyping of equations
  - Eliminates debugging of code

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**The Space Station Model Contents**  
(A joint effort of NASA & TRW)

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## Space Station Model Simulation Objective



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Simulate a transient maneuver involving a large-angle rigid-body motion of the flexible solar arrays connected to their respective transverse booms, while the central body is maintained in a three-axis attitude control mode

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## SPACE STATION MODEL

The mathematical model of the Space Station consists of three interconnected flexible bodies capable of undergoing large, rigid-body motion with respect to each other. Body 1 is the main central body and contains the pressurized modules inboard of the two Alpha gimbals. Bodies 2 and 3 are the starboard and port bodies connected to Body 1 at the Alpha gimbals, and include all components on the transverse booms outboard of the Alpha gimbals (including the solar arrays).

The three-body Space Station model contains eight (8) large-motion, rigid-body degrees of freedom, three translational and three rotational for the central body and one rotational for each of the extraneous bodies relative to the central body. Full coupling between the rigid- and flexible-body degrees of freedom are facilitated in the model. The flexibility of Body 1 is described by 1 to 45 "free-free" natural modes, excluding the six rigid-body modes, used here as assumed admissible functions. The flexibilities of Bodies 2 and 3 are each described by 1 to 45 "fixed-free" natural modes serving also as assumed admissible functions. The assumed spatial admissible functions are general 3-dimensional functions which satisfy the boundary conditions of the body in question but do not provide solutions to its differential equations of motion.

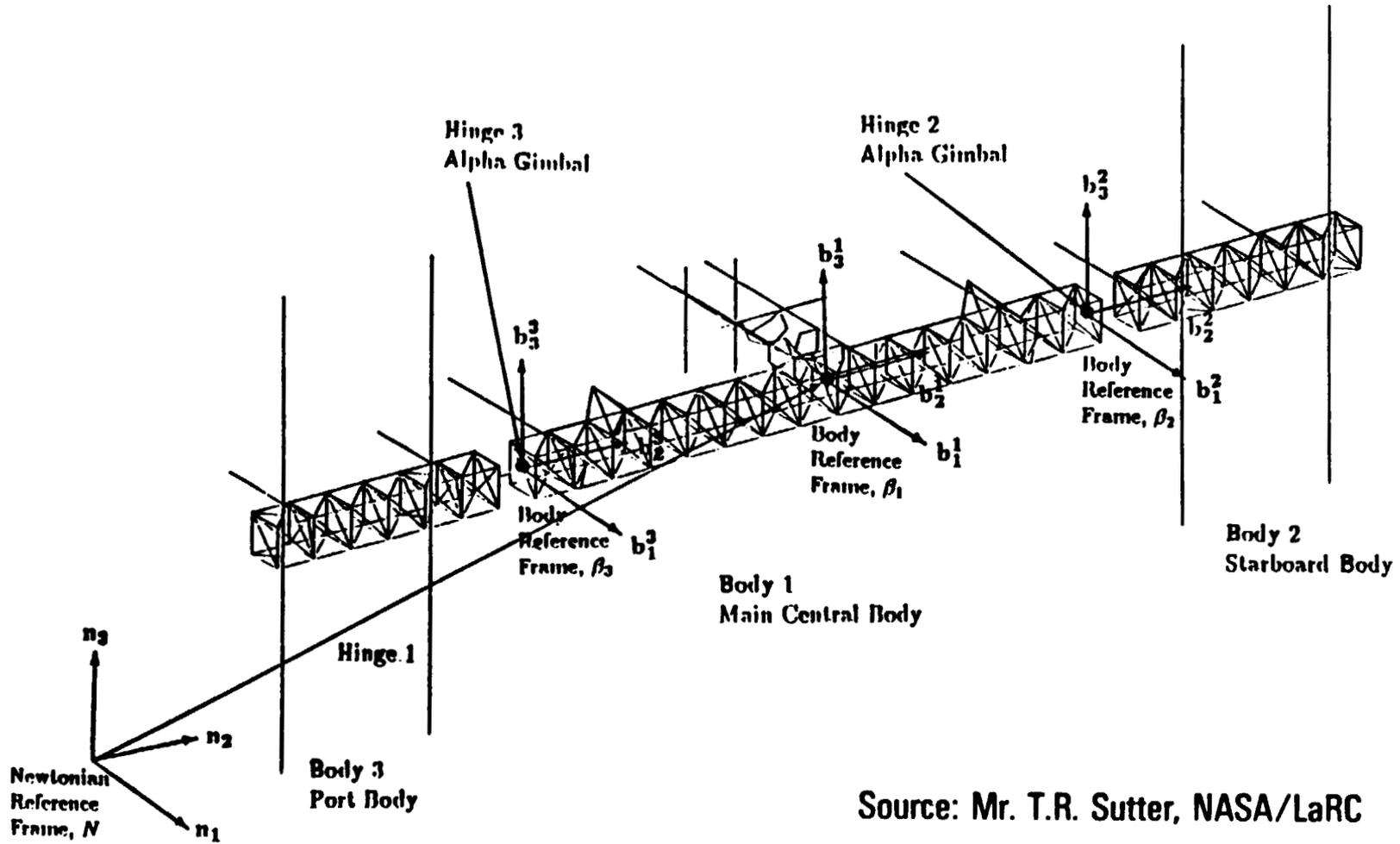
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# Space Station Model



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Source: Mr. T.R. Sutter, NASA/LaRC

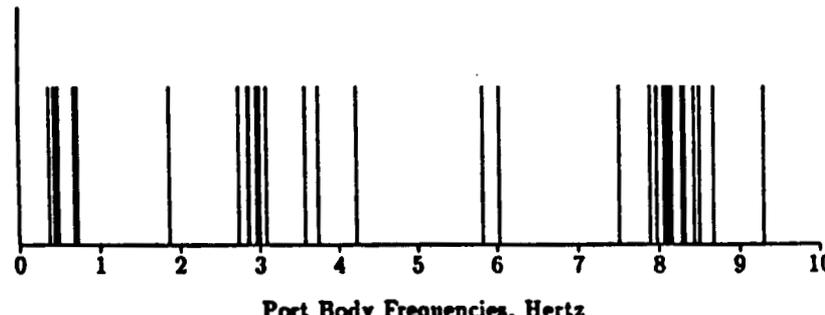
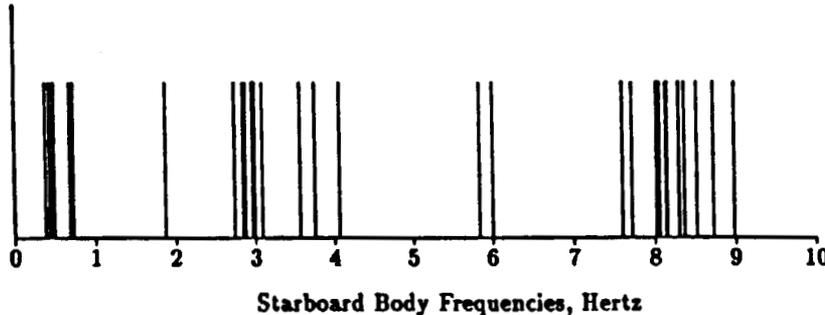
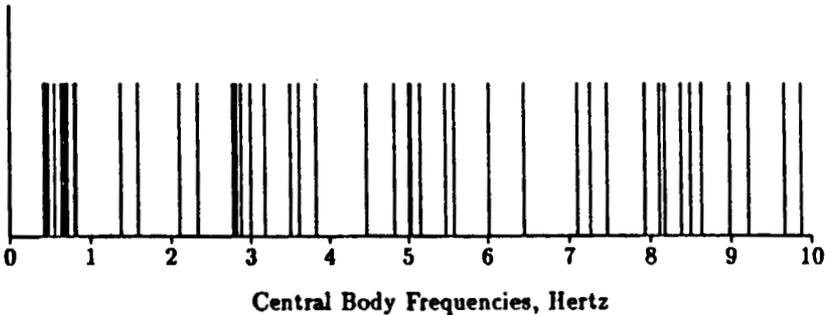
## FREQUENCY SPECTRA OF THE SPACE STATION MODEL

The assumed admissible functions in the Space Station model were obtained from finite element models developed for each of the bodies. These included an unconstrained (free-free) model for the central body and two constrained (fixed-free) models for the starboard and port bodies, cantilevered at the Alpha gimbals. A MSC/NASTRAN code was used to obtain the natural modes of vibration within a 10 Hz frequency bandwidth. The spectrum of natural frequencies for each of the three finite element models is shown in the figure. Note that these are characterized by a number of low frequency modes (below 1 Hz) spaced closely together.

# Frequency Spectra of the Space Station Model



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## ATTITUDE CONTROL SYSTEM FOR SPACE STATION MODEL

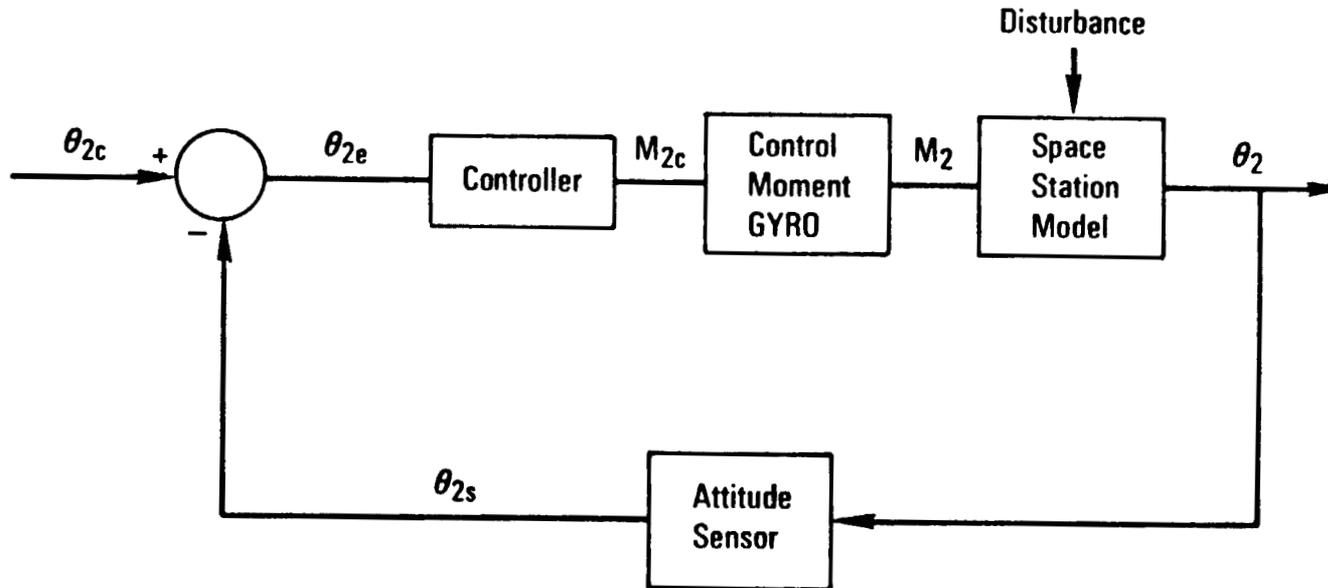
The attitude control system of the Space Station was designed to regulate its orientation and keep its longitudinal axis aligned with the local vertical vector while maintaining its plane perpendicular to the velocity vector. The control system consists of attitude sensing instrumentation, control moment gyros, and electronics to cause corrective control moments to be applied to the Space Station's central body whenever it moves away from the commanded attitude. The attitude and rate sensors and the control moment gyros (CMG's) are co-located at the origin of the coordinate system of the central body placed at its undeformed center of mass.

394 A block diagram of the control law for the  $X_2$  axis is shown in the figure. Similar control laws were designed for the remaining two axes (the three control laws are uncoupled from each other). Attitude sensing instrumentation provides electronic representations of the attitude angle  $\theta_2$  and its time rate of change. The sensed attitude angle is subtracted from the commanded attitude angle ( $\theta_{2c}$ ) to form the attitude error signal ( $\theta_{2e}$ ). The electronic controller mechanizes a control law, specified in the form of a transfer function, to produce a commanded control moment ( $M_{2c}$ ) based on the error signal. The CMG generates control moments ( $M_2$ ) according to the commanded moments to drive the attitude error towards zero. External disturbances are not considered in this simulation and the commanded attitude is set nominally to zero.

# Attitude Control System for Space Station Model - X<sub>2</sub> Axis



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- |                                    |                                     |
|------------------------------------|-------------------------------------|
| $\theta_2$ — Central-Body Attitude | $\theta_{2e}$ — Attitude Error      |
| $\theta_{2s}$ — Senced Attitude    | $M_2$ — Control Moment              |
| $\theta_{2c}$ — Commanded Attitude | $M_{2c}$ — Commanded Control Moment |

Source: Mr. J.W. Young, NASA/LaRC

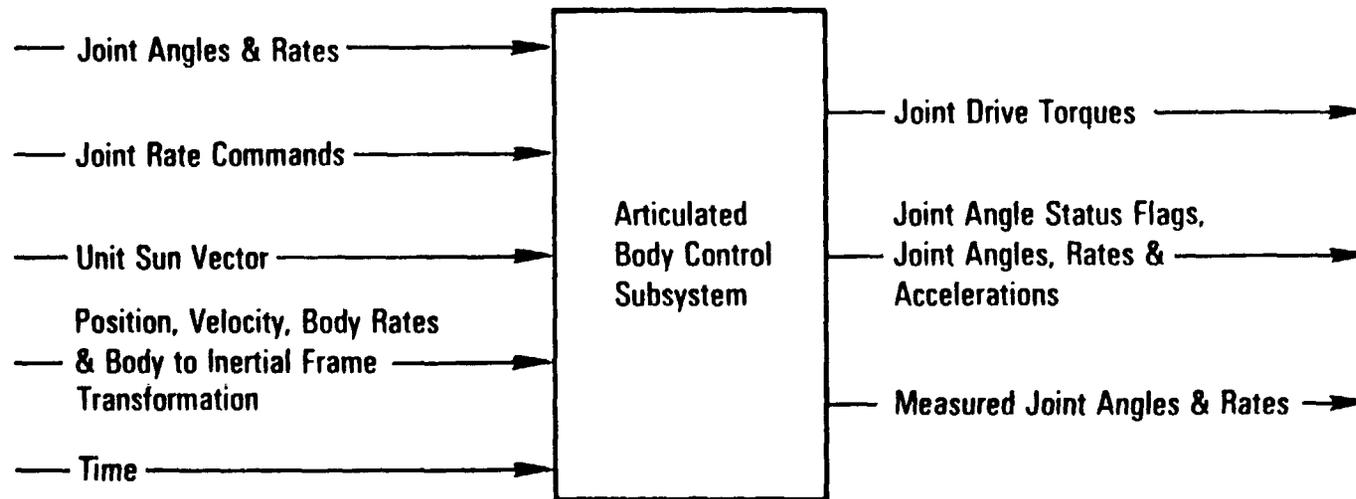
## ARTICULATED-BODY CONTROL FOR SPACE STATION MODEL

In addition to the attitude control system, the Space Station model includes a second control system to maintain the solar arrays pointing in a direction perpendicular to the sun line. The control law is based on an angular position and rate feedback scheme with options provided to rewind the solar arrays during eclipse. The commanded angular position of the Alpha gimbal is utilized in a second order transfer function to calculate the controller's motor torque. Input and output parameters for the articulated body control system are shown in the figure.

# Articulated-Body Control for Space Station Model



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Source: Mr. J. Mapor, LEMSCO; Houston, Texas



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# **A Balanced Computational Load Distribution Methodology**

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C-5

## MATHEMATICAL CHARACTERISTICS OF THE SPACE STATION MODEL

The balanced computational load distribution methodology described herein is aimed at a broad class of multibody dynamic systems, which includes every variety of spacecraft, robot, rotary aircraft and mechanism. This class is characterized by a set of first-order, ordinary differential equations, known as Kane's Dynamical Equations, as depicted in the figure.

The methodology for computational load allocation adopted here takes advantage of the fact that the mathematical model involved, although generally very complex, remains essentially unchanged for many hundreds (if not thousands) of simulation runs made in the course of the development and verification of the dynamic system in question. For these simulation runs, which feature different combinations of initial conditions, input functions and parameter values, it is possible to distribute the computational load statically, off-line, and thereby gain a significant advantage in execution speed during simulation compared to that achievable with a dynamic load allocation methodology.

# MATHEMATICAL CHARACTERISTICS OF THE SPACE STATION MODEL



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- KANE'S DYNAMICAL EQUATIONS CONSTITUTE A SET OF FIRST-ORDER ORDINARY DIFFERENTIAL EQUATIONS IN THE MATRIX FORM

$$\begin{aligned} \mathbf{A}(\mathbf{q}, t) \dot{\mathbf{u}} &= \mathbf{b}(\mathbf{q}, \mathbf{u}, t) \\ \dot{\mathbf{q}} &= \mathbf{f}(\mathbf{q}, \mathbf{u}, t) \end{aligned}$$

WHERE  $\mathbf{u}$  IS THE VECTOR OF GENERALIZED SPEEDS,  
 $\mathbf{q}$  IS THE VECTOR OF GENERALIZED COORDINATES,  
 $t$  IS THE TIME, AND A DOT INDICATES TIME DIFFERENTIATION

- THE FORM OF THE EQUATIONS IS QUITE GENERAL, AND IS THE FORM ADDRESSED BY THE LOAD BALANCING METHODOLOGY

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## LOAD BALANCING DIAGRAMS

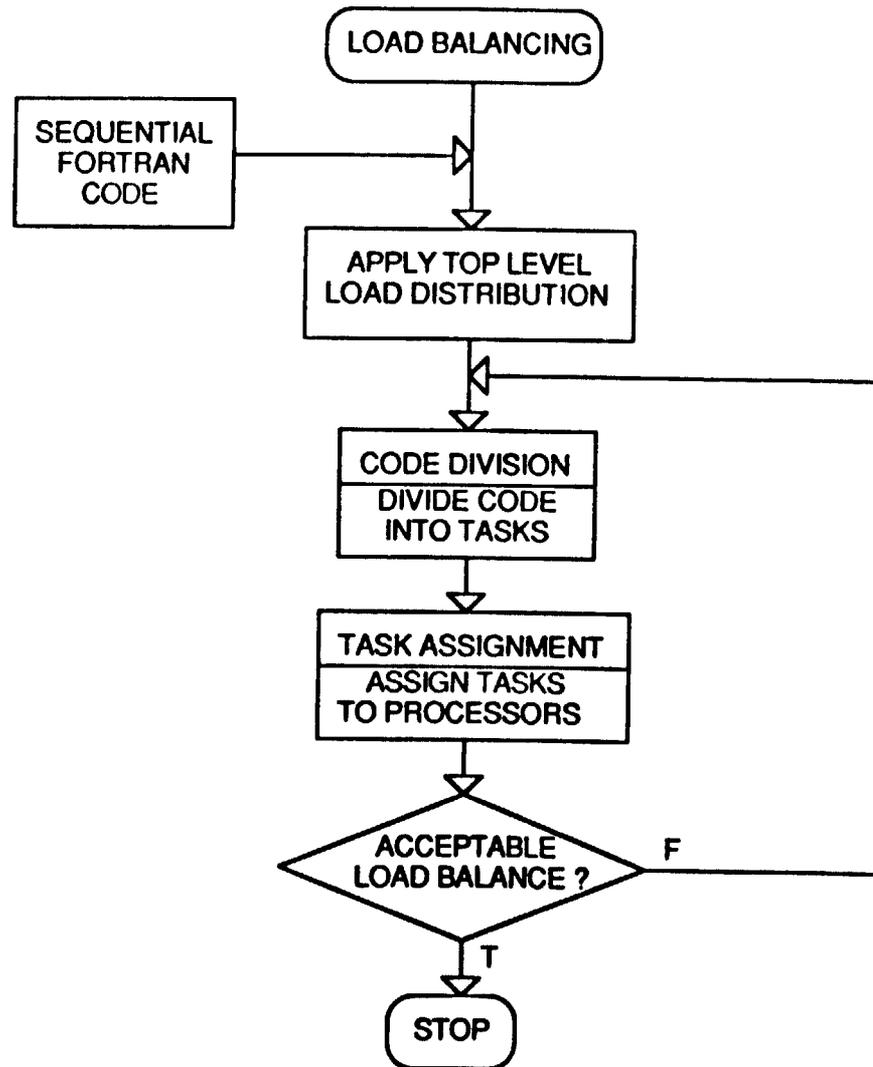
The basic input to the load balancing software is the sequential FORTRAN code developed using Kane's Dynamical Equations and the symbol manipulation program SYMBOD/SMP.

The code is inspected by the user to determine large scale operations that may be done in parallel. This provides the software with a top level load distribution that it may then refine and balance. Mathematical models of the sort considered here will have certain computational features that are ideally suited to parallel execution, and these may be used to provide a preliminary code division into tasks for each processor.

# LOAD BALANCING DIAGRAMS



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## LOAD BALANCING DIAGRAMS (CONT'D)

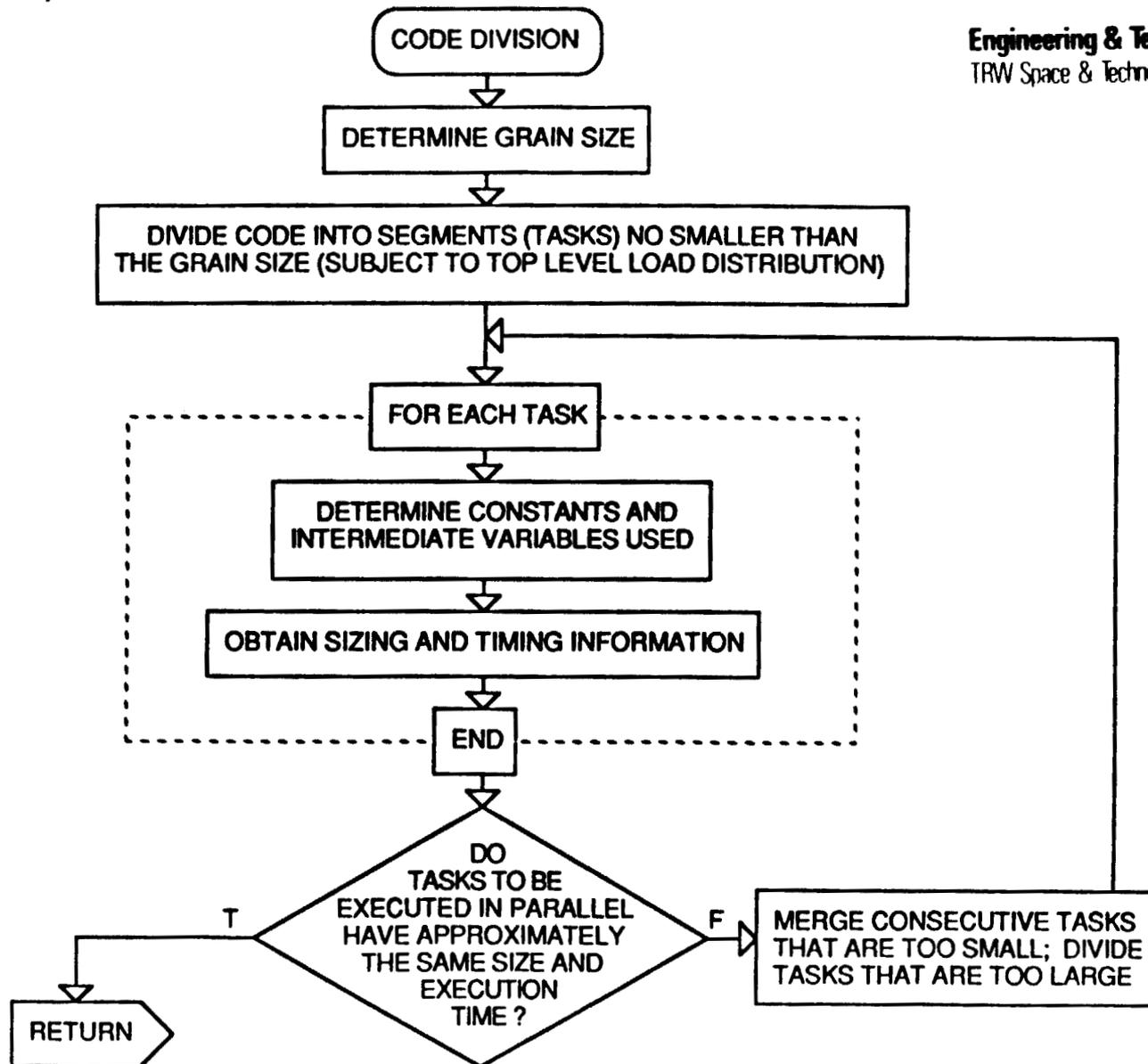
The code is then divided into smaller tasks using the program TASK. This partitioning incorporates the divisions already given in the top level distribution, and results in tasks which are no smaller in size than some predetermined "grain" size. The user chooses whether or not the code division is to be fine-grained or coarse-grained. Different choices will be a result of, e.g., a different number of available processors, the need to examine values for intermediate variables in a convenient way, etc.

Once tasks are obtained, TASK checks to see that the size and execution time for tasks that are to be executed in parallel are approximately the same (according to criteria determined, in part, by the user), in order to have a balanced computational load. Those tasks found to be too large are divided while those that are too small are merged with tasks to be executed before or afterwards. This division and merging continues until the criteria mentioned above are satisfied.

# LOAD BALANCING DIAGRAMS (CONTINUED)



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## LOAD BALANCING DIAGRAMS (CONT'D)

At the conclusion of the code division, the tasks are evaluated in terms of how much communication they require with other tasks, and how many variables and parameters they share with other tasks. The program ASSIGN takes the results of the first of these evaluations and constructs what is called the connectivity matrix, with each entry indicating how much communication from task S, say, to task T is required, where S and T range over all tasks. ASSIGN uses the second evaluation to produce the parameter overlap matrix, where each entry indicates the number of parameters shared by the two tasks.

Tasks to be executed in parallel are now assigned to processors by ASSIGN. The connectivity and parameter overlap of each task with tasks already assigned to the processors is examined and a task is assigned to a particular processor according to a set of previously specified constraints.

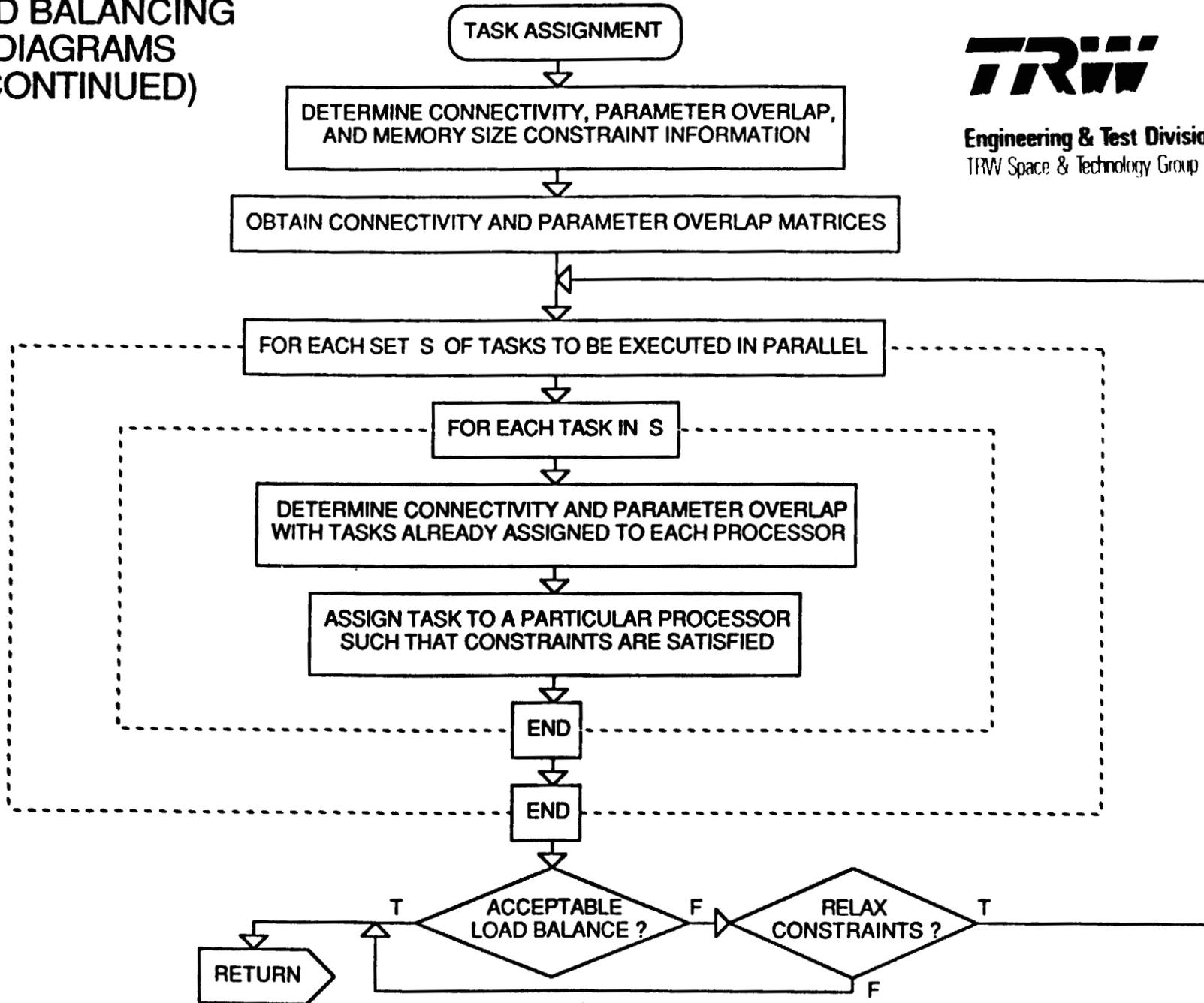
At the end of the assignments, ASSIGN checks the computational load for balance among the processors. If the result is acceptable, then the software is finished. If not, then the constraints may be relaxed (in a way predefined by the user) and task assignment may be attempted again. If the constraints have been relaxed as far as possible, and the load is still not acceptably balanced, then TASK will attempt a different code division (and subsequent task assignment using ASSIGN) where the grain size may be different than before.

LOAD BALANCING  
DIAGRAMS  
(CONTINUED)



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## DATA FLOW GRAPH

The data dependencies in the Space Station model are shown in the figure. The graph depicts the functional form of the model's equations.

The state vector,  $\underline{x}_n$ , at a given time  $t_n$  is composed of the generalized speeds ( $\underline{u}_n$ ), the generalized coordinates ( $\underline{q}_n$ ) and the control variables ( $\underline{c}_n$ ).

The generalized coordinates are used in calculating time derivatives for all the state variables, as are the generalized speeds (though these are not used in the computation of the matrix  $\underline{A}_n$ ). The control variables affect only the control torques and thus influence only the vector  $\underline{b}_n$  and the derivative of  $\underline{u}_n$ .

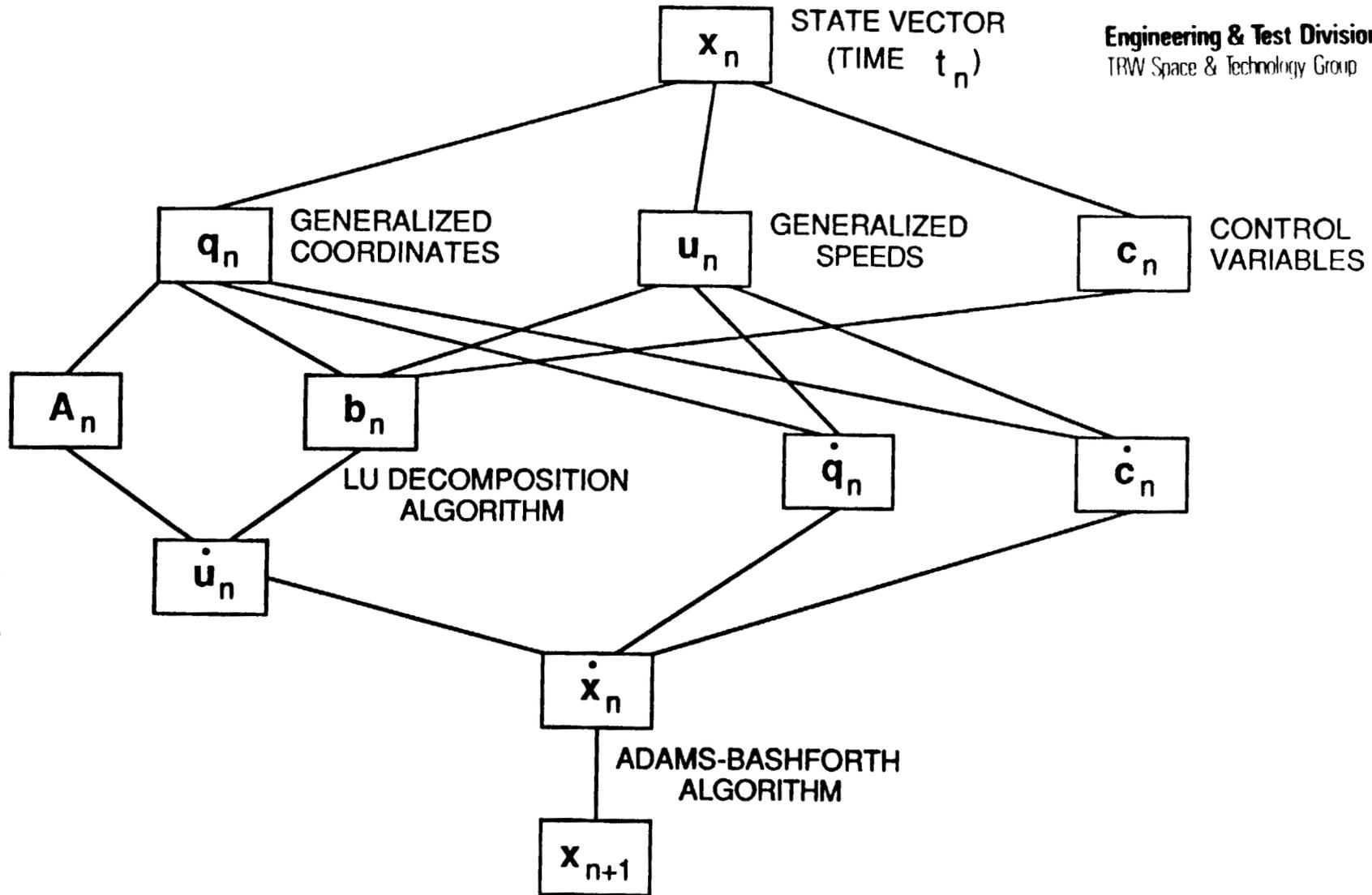
Time derivatives of  $\underline{q}_n$  and  $\underline{c}_n$  are found directly from the generalized coordinates and speeds. Gaussian elimination is used to solve the matrix equation  $\underline{A}_n \dot{\underline{u}}_n = \underline{b}_n$  giving the derivative of  $\underline{u}_n$ .

The time derivatives of each of the components of the state vector are collected to give the derivative of  $\underline{x}_n$ . This is then integrated using the Adams-Bashforth algorithm to give the state vector at time  $t_{n+1}:\underline{x}_{n+1}$ .

# DATA FLOW GRAPH



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## TOP LEVEL LOAD BALANCING (SPACE STATION MODEL)

Some opportunities for parallel execution of the Space Station model code are immediately apparent from even a casual inspection of the model, as shown in the figure.

The coordinate transformation matrices between frames in the three bodies, and between the body frames and an inertial frame are used frequently and must be calculated first. Each matrix, however, is calculated by a processor only if that processor will subsequently use it.

The outputs of the control subroutine are used only in the computation of the vector  $\underline{b}$ . Thus, this subroutine may be executed in parallel with sections of code computing general intermediate variables used by both  $\underline{A}$  and  $\underline{b}$  (such as partial angular velocities, partial velocities, etc.).

The calculation of elements of  $\underline{A}$  and  $\underline{b}$  may also be done in parallel, as may be the computation of the time derivative of  $\underline{q}$ .

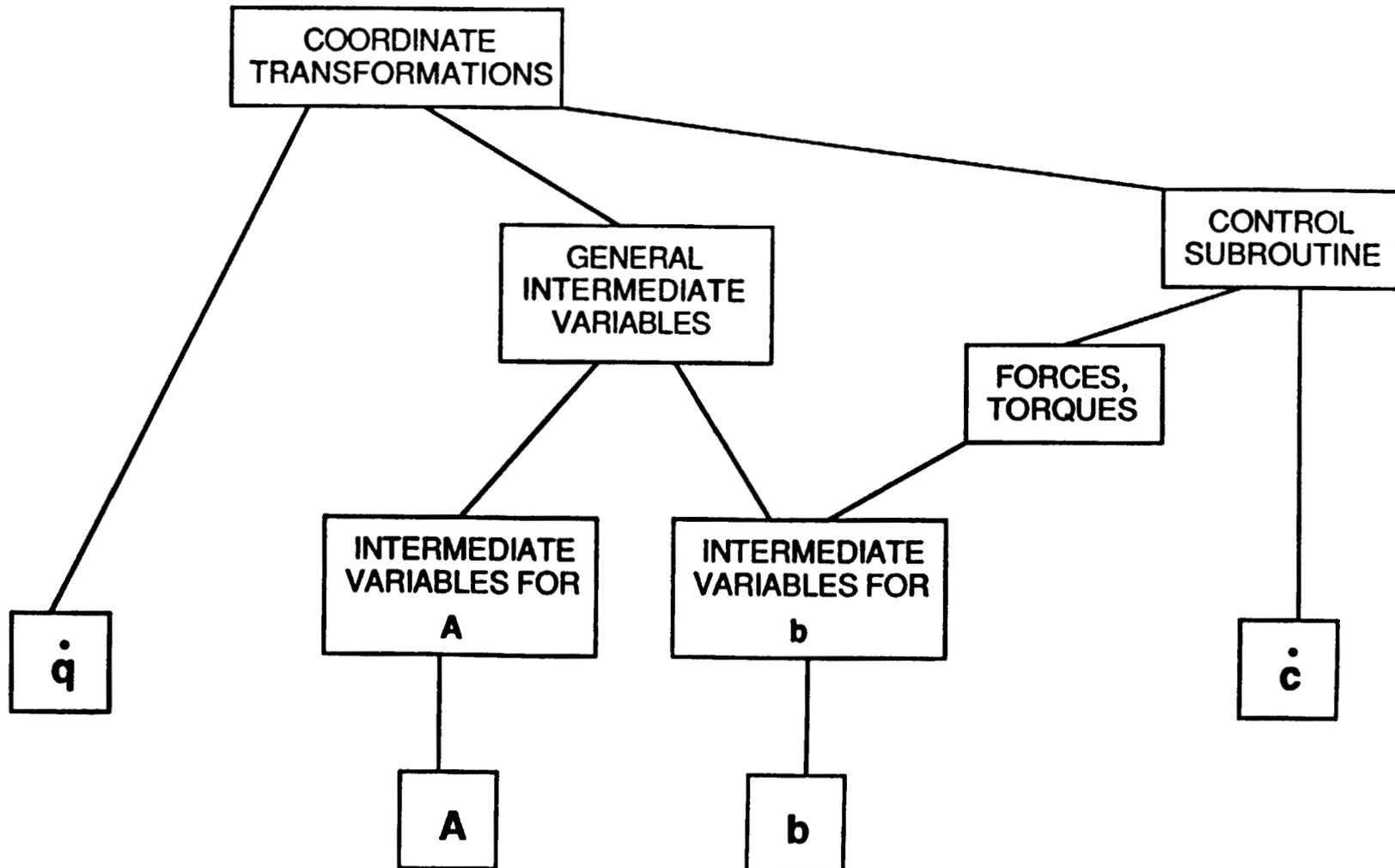
A general division of the code may also be made according to whether computations involving Body 2 or Body 3 are needed. When possible, therefore, a given processor will compute quantities related only to Body 2 or only to Body 3, thus reducing interprocessor communication.

# TOP LEVEL LOAD BALANCING (SPACE STATION MODEL)



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## AN OVERVIEW OF THE CAPPS SIMULATION METHODOLOGY

An overview of the CAPPS simulation methodology is shown in the figure. The methodology is divided into four phases. The analysis phase constitutes the development of the mathematical model and requires an intensive interaction between the analyst and the previously described Program SYMBOD/SMP. The derivation phase follows, in which Program SMP carries out the instructions imbedded in SYMBOD to yield a matrix of ordinary differential equations in FORTRAN format. In addition to providing an accurate reflection of the analyst's intentions in the derivation of the equations of motion, this procedure also leads to equations which are virtually free of wasteful operations (such as additions of zeros, multiplications by unity and taking dot products of orthogonal vectors), as well as superfluous high order terms. The FORTRAN equations are delivered via file to the CAPPS computational load distribution software to begin the processing phase which is described in more detail below. It should be noted that the procedure completely eliminates the costly and time consuming programming effort which is normally required at this stage. The CAPPS system software transforms the derived equations from their original FORTRAN format to a binary format executable in concurrent operations by the CAPPS's CUs.

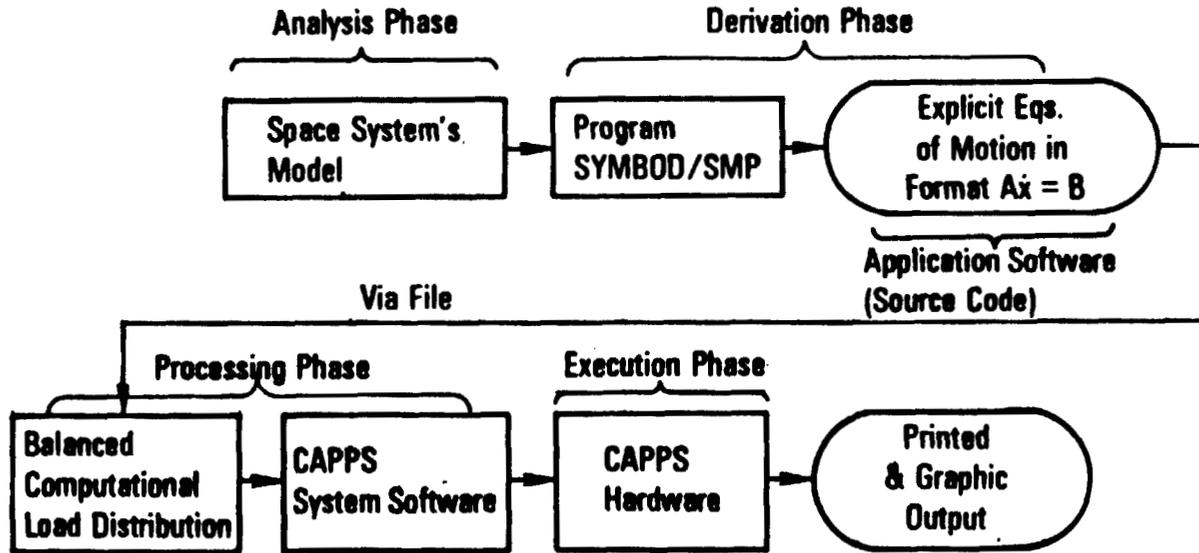
# An Overview of the CAPPS Simulation Methodology



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An Efficient Mathematical Model is Fed into a very Fast Computing System

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## REFERENCES

- (1) C. Wampler, K. Buffinton, and J. Shu-hui, "Formulation of Equations of Motion for Systems Subject to Constraints"; ASME Journal of Applied Mechanics, Vol. 52, June 1985.
- (2) K. C. Park and J. C. Chiou, "Stabilization of Computational Procedures for Constrained Dynamical Systems"; University of Colorado, College of Engineering, Report No. Cu-CSSC-87-05, June 1987.

N90-10097

**SIMULATION OF FLEXIBLE STRUCTURES WITH IMPACT: EXPERIMENTAL  
VALIDATION**

By

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University of Michigan  
Ann Arbor, Michigan

**ABSTRACT**

The dynamics of flexible systems, such as mechanisms and robotic manipulators, is becoming increasingly important due to high-speed operation, high accuracy requirements, and lightweight designs. Such flexible devices can undergo impact during operation, and this may lead to: (1) component failure due to high stresses, and (2) excitation of higher structural modes due to impulsive forces. The latter issue, the simulation of the dynamic behavior of flexible structures with impact, is experimentally and numerically investigated.

A radially rotating flexible beam attached to a rigid shaft is considered. Both experimental and numerical studies are undertaken. Experimental studies show excellent agreement with simulation studies using both the momentum balance (coefficient of friction), and spring-dashpot (impact pair) models. The simulation studies are even capable of predicting the high-speed multiple impacts which occur due to structural flexibility and which were experimentally observed using high-speed video photography. The results of the studies show that a simple momentum balance (coefficient of restitution) method for simulating the impact is sufficiently accurate in predicting the dynamic behavior of the system for most engineering applications. The momentum balance method cannot simulate the impact force which develops during the contact duration, but is computationally very efficient. The spring-dashpot model is more difficult to develop and requires significantly larger computation time, but can simulate impact forces and stresses due to impact.

The momentum balance (coefficient of restitution) method, although strictly not applicable to flexible bodies, has been shown to provide an accurate and computationally efficient method for simulating the dynamic behavior of flexible structures with impact when contact stresses are not needed.

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N90-10098

**SIMULATION AND CONTROL PROBLEMS IN ELASTIC ROBOTS**

By

S. S. K. Tadikonda and H. Baruh  
Rutgers University  
New Brunswick, New Jersey

**ABSTRACT**

Computational issues associated with modeling and control of robots with revolute joints and elastic arms are considered. A manipulator with one arm and pinned at one end is considered to investigate various aspects of the modeling procedure and the model, and the effect of coupling between the rigid-body and the elastic motions.

The rigid-body motion of a manipulator arm is described by means of a reference frame attached to the "shadow beam," and the linear elastic operator denoting flexibility is defined with respect to this reference frame. The small elastic motion assumption coupled with the method of assumed modes is used to model the elasticity in the arm. ~~The complete model coupling the rigid-body and the elastic motion is highly nonlinear, and contains terms up to quartic in powers of the amplitudes of the assumed modes.~~ It is shown that only terms up to quadratic in these model amplitudes need to be retained.

An important aspect of the coupling between the rigid-body and the elastic motion is the centrifugal stiffening effect. This effect stiffens the elastic structure, as to be expected on physical grounds, gives rise to a time-varying inertia term for the rigid-body motion, and, in general, results in an effective inertia term smaller than the rigid-body inertia term. In fact, this reduction in inertia ~~determines the limitation of the small motion assumption.~~ If the elastic behavior is excited sufficiently so as to cause a vanishing effective rigid-body motion inertia term, one should either modify the manipulator model, or consider the forcing profiles that excite the elastic motion least. The Fourier series expansion of a few such profiles is examined to provide insight in this regard.

Simulation results are presented for an elastic beam pinned at one end and free at the other, and rotating in a horizontal plane, and control issues such as the order of the model, number of sensors, and modal extraction are examined within this context. It is shown that the effect of centrifugal stiffening is pronounced on the rigid-body motion during transition, and ignoring it in the control model leads to gross inaccuracies in response. The effect of including varying amounts of flexibility on the response is studied.

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# SIMULATION AND CONTROL PROBLEMS IN ELASTIC ROBOTS

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H. Baruh\*\*

Mechanical & Aerospace Engineering Dept.,  
Rutgers University, New Brunswick, NJ.

\* Graduate Assistant

\*\* Assistant Professor

## **OBJECTIVE**

To investigate modeling, control, and computational issues associated with elastic manipulators

## **SCOPE**

Revolute joints

Actuators at joints only

Shadow beam approach

Small elastic motion, and limit of such an assumption

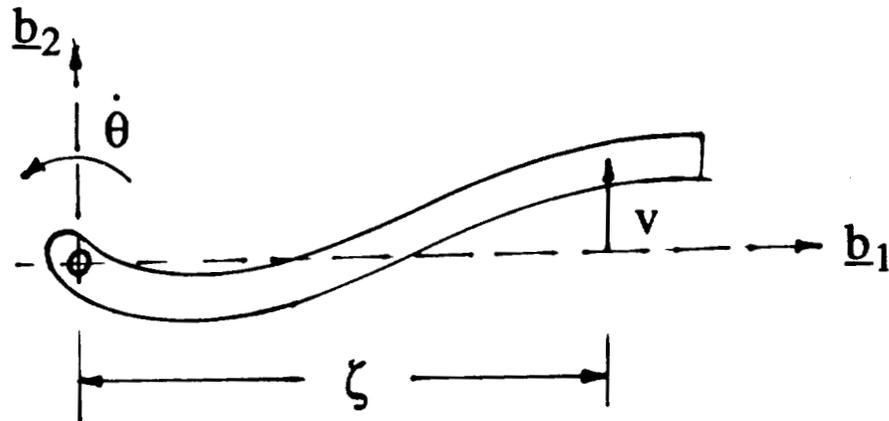
Nonlinear model

Control issues

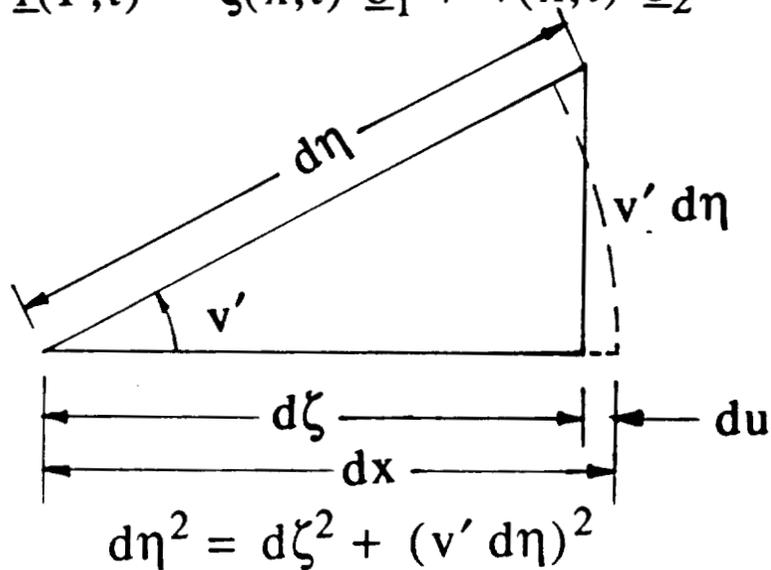
Illustrative example

## Pinned - free link

Reference frame located at the pin joint; describes rigid-body motion. Elastic motion is defined with respect to this frame



$$\underline{r}(P,t) = \zeta(x,t) \underline{b}_1 + v(x,t) \underline{b}_2$$



Notes:  $x$  is the position of the point in the undeformed configuration  
The beam rotates in a horizontal plane

$u(x,t)$  is obtained by integrating

$$d\zeta = [ 1 - (\frac{\partial v}{\partial x})^2 ]^{1/2} d\eta$$

where

$$\eta(x,t) = x + s(x,t)$$

$$\zeta(x,t) = x - u(x,t)$$

On integration,

$$\zeta(x,t) \approx \eta - 1/2 \int_0^\eta [ (\frac{\partial v}{\partial \sigma})^2 ] d\sigma$$

or

$$u(x,t) \approx - s(x,t) + 1/2 \int [ (\frac{\partial v}{\partial \sigma})^2 ] d\sigma$$

$s(x,t)$  : axial vibration term

Integral : results in centrifugal stiffening term

Neglect axial vibration

- 5 -

## Elastic Displacement

$$\underline{u}(x,t) = - u(x,t) \underline{b}_1 + v(x,t) \underline{b}_2$$

## Position

$$\underline{r} = \zeta(x,t) \underline{b}_1 + v(x,t) \underline{b}_2$$

## Velocity

$$\dot{\underline{r}} = - \frac{\partial u}{\partial t} \underline{b}_1 + \frac{\partial v}{\partial t} \underline{b}_2 + \dot{\theta} \underline{X} \underline{r}$$

## Kinetic Energy

$$K = 1/2 \int \dot{\underline{r}} \cdot \dot{\underline{r}} \, dm$$

## Potential Energy

$$V = 1/2 \int_0^L EI(x) (v'')^2 \, dx$$

## Lagrangian

$$L = K - V$$

Notes :  $(\dot{\quad})$  corresponds to partial derivative with respect to time,  $(\quad)'$  corresponds to spatial derivative,  $\rho$  is the mass per unit length, and  $EI(x)$  is the flexural rigidity

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$$\begin{aligned} L = & \frac{1}{2} \int_0^L \rho (\dot{v}^2 + \dot{\theta}^2 v^2 + 2 x \dot{v} \dot{\theta} + x^2 \dot{\theta}^2) dx \\ & - \frac{1}{2} \int_0^L EI (v'')^2 dx \\ & - \frac{1}{2} \dot{\theta}^2 \int_0^L \rho x \int_0^x (v')^2 d\sigma dx \\ & + \frac{1}{2} \int_0^L \left[ -\frac{1}{2} \int_0^x \frac{d}{dt} ((v')^2) d\sigma \right]^2 \rho dx \\ & + \frac{1}{2} \dot{\theta} \int_0^L \rho v \int_0^x \frac{d}{dt} ((v')^2) d\sigma dx \\ & - \frac{1}{2} \dot{\theta} \int_0^L \rho \dot{v} \int_0^x (v')^2 d\sigma dx \\ & + \frac{1}{2} \dot{\theta}^2 \int_0^L \rho/4 \left[ \int_0^x (v')^2 d\sigma \right]^2 dx \end{aligned}$$

## Assumed Modes

$$v(x,t) = \sum_{i=1}^{N_1} \phi_i(x) a_i(t)$$

$\phi_i(x)$  : Admissible functions

**Define**

$$m_{ij} = \int_0^L \rho \phi_i(x) \phi_j(x) dx$$

$$k_{ij} = \int_0^L EI(x) \phi_i''(x) \phi_j''(x) dx$$

$$s_{ij}(x) = \int_0^x \phi_i'(x) \phi_j'(x) dx$$

$$p_{ij} = \int_0^L \rho x s_{ij}(x) dx$$

$$s_{ijkl} = \int_0^L s_{ij}(x) s_{kl}(x) dx$$

$$q_{ijk} = \int_0^L \rho \phi_k(x) s_{ij}(x) dx$$

$$r_i = \int_0^L \rho x \phi_i(x) dx$$

**Then,**

$$\int_0^x (v')^2 dx = s_{ij}(x) a_i a_j$$

$$\int_0^L \rho x \left[ \int_0^x (v')^2 d\sigma \right] dx = p_{ij} a_i a_j$$

$$\int_0^L \left[ \int_0^x \frac{d}{dt} ((v')^2) d\sigma \right]^2 \rho dx = 4 s_{ijkl} a_i \dot{a}_j a_k \dot{a}_l$$

$$\int_0^L \rho v \left[ \int_0^x \frac{d}{dt} ((v')^2) d\sigma \right] dx = 2 q_{ijk} a_i \dot{a}_j a_k$$

## Substituting in the Lagrangian,

$$\begin{aligned} L = & 1/2 I_o \dot{\theta}^2 + 1/2 m_{ij} \dot{a}_i \dot{a}_j + \dot{\theta} r_i \dot{a}_i \\ & - 1/2 [k_{ij} + (p_{ij} - m_{ij}) \dot{\theta}^2] a_i a_j \\ & + 1/2 s_{ijkl} a_i \dot{a}_j a_k \dot{a}_l + \dot{\theta}^2 / 8 s_{ijkl} a_i a_j a_k a_l \\ & + \dot{\theta} q_{ijk} a_i \dot{a}_j a_k - \dot{\theta} / 2 q_{ijk} a_i a_j \dot{a}_k \end{aligned}$$

### Example :

#### Beam parameters

$$\begin{aligned} \text{Cross-section : } & 6 \text{ in } \times 3/8 \text{ in} \\ \text{Length} = & 3.6576 \text{ m (12 ft)} \\ \rho = & 4.015 \text{ kg/m} \\ EI = & 756.65 \text{ N} \cdot \text{m} \end{aligned}$$

Admissible functions : Normalized eigenfunctions of a pinned-free beam

$$m_{ij} = \delta_{ij} ; \quad k_{ij} = \omega_i^2 \delta_{ij}$$

where  $\delta_{ij}$  is the Kronecker delta

Notes : The summation convention,  $\sum \sum m_{ij} a_i a_j = m_{ij} a_i a_j$ , etc., will be employed for conciseness - i.e., repeated indices in an expression indicate summation over appropriate range.

Natural frequencies and Centrifugal stiffening coefficients

i	$\omega_i$	$P_{ij}$		
		j = 1	j = 2	j = 3
1	15.82	6.397	1.861	-0.366
2	51.282	1.861	17.905	6.195
3	106.983	-0.366	6.195	35.999

Coriolis terms,  $q_{ijk}$

i	j	k = 1	k = 2	k = 3
1	1	-0.152	0.143	0.008
1	2	0.415	-0.144	0.169
1	3	0.077	0.347	-0.143
2	1	0.415	-0.144	0.169
2	2	-0.175	0.152	-0.117
2	3	0.883	-0.196	0.145
3	1	0.077	0.347	-0.143
3	2	0.883	-0.196	0.145
3	3	-0.178	0.171	-0.152

Other coupling terms  $s_{ijkl}$

Note:  $s_{ijkl} = s_{jikl} = s_{ijlk} = s_{jilk}$

i	j	k	l = 1	l = 2	l = 3
1	1	1	0.669	0.099	0.001
1	1	2	0.099	1.800	0.444
1	1	3	0.001	0.444	3.570

1	2	1	0.099	0.275	-0.050
1	2	2	0.275	0.397	0.594
1	2	3	-0.050	0.594	0.901
1	3	1	0.001	-0.050	0.183
1	3	2	-0.050	-0.005	-0.074
1	3	3	0.183	-0.074	-0.113
2	1	1	0.099	0.275	-0.005
2	1	2	0.275	0.397	0.594
2	1	3	-0.050	0.594	0.901
2	2	1	1.800	0.397	-0.005
2	2	2	0.397	5.010	0.147
2	2	3	-0.005	0.147	9.94
2	3	1	0.444	0.594	-0.074
2	3	2	0.594	1.470	1.500
2	3	3	-0.074	1.500	3.160
3	1	1	0.001	-0.050	.183
3	1	2	-0.050	-0.005	-0.074
3	1	3	0.183	-0.074	-0.113
3	2	1	0.444	0.594	-0.074
3	2	2	0.594	1.470	1.500
3	2	3	-0.074	1.500	3.160
3	3	1	3.57	0.901	-0.113
3	3	2	0.901	9.94	3.160
3	3	3	-0.113	3.160	20.000

Notes : The magnitudes of the terms  $q_{ijk}$  and  $s_{jkl}$  are small. In addition, they are multiplied by the cubic and quartic powers of modal amplitudes. Hence they will be dropped from further development.

Retaining terms only up to quadratic in modal amplitudes,

$$L = 1/2 [I_0 - (p_{ij} - m_{ij}) a_i a_j] \dot{\theta}^2 + 1/2 m_{ij} \dot{a}_i \dot{a}_j - 1/2 k_{ij} a_i a_j$$

**The equation for rigid-body motion is**

$$\frac{d}{dt} [(I_0 - (p_{ij} - m_{ij}) a_i a_j) \dot{\theta}] = T$$

**And the elastic motion is described by**

$$m_{ij} \ddot{a}_j + [k_{ij} + (p_{ij} - m_{ij}) \dot{\theta}^2] a_j = T \phi_i'(0),$$
$$i = 1, 2, \dots, N_1$$

**Measurements at  $x = 0$**

$$\theta_1 = \theta + v'(0,t)$$

$$\dot{\theta}_1 = \dot{\theta} + \dot{v}'(0,t)$$

## Choices for the control model

1. Ignore elastic effects completely

$$\begin{aligned} \text{Control model : } I_o \ddot{\theta} &= T \\ \theta &= \theta_1 & \dot{\theta} &= \dot{\theta}_1 \end{aligned}$$

2. Rigid-body model, with the shadow frame angle properly extracted

$$\begin{aligned} \text{Control model : } I_o \ddot{\theta} &= T \\ \theta &= \theta_1 - v'(0,t) \\ \dot{\theta} &= \dot{\theta}_1 - \dot{v}'(0,t) \end{aligned}$$

3. A few elastic modes are included, and the modal coordinates are approximated

Control model:

$$\begin{aligned} \frac{d}{dt} [(I_o - (p_{ij} - m_{ij}) a_i a_j) \dot{\theta}] &= T \\ i, j &\leq N_2, \quad N_2 < N_1 \end{aligned}$$

Notes :  $N_1$  is the number of modeled modes.  $N_2$  is the number of modes used for controller design.  $N_1 = 3$  for the following simulation results.

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4. Appropriate number of sensors used to obtain accurate modal coordinates.

$$N_2 = N_1$$

## **Control synthesis**

Computed torque method

Pointwise-optimal control method

# Open-loop Maneuver

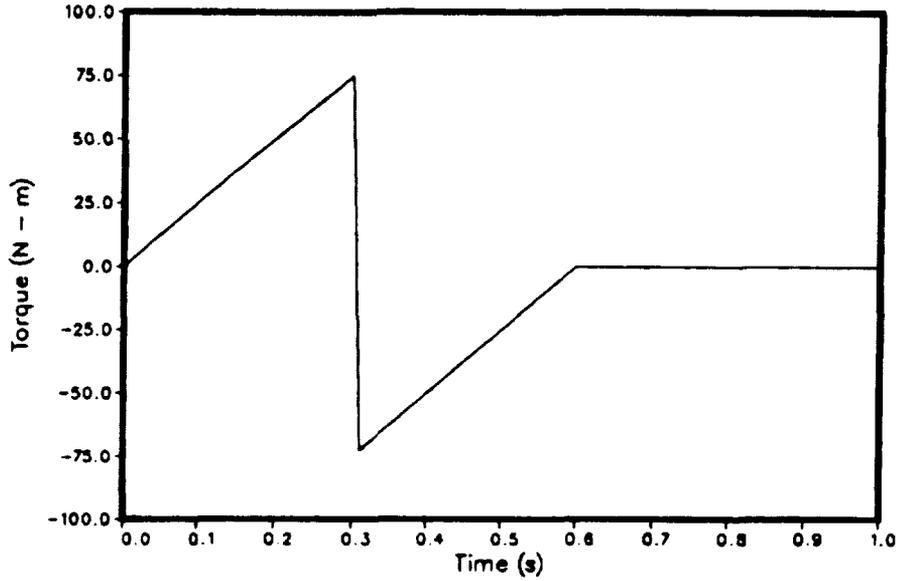


Fig. 1 : Torque Profile for Open-Loop Maneuver

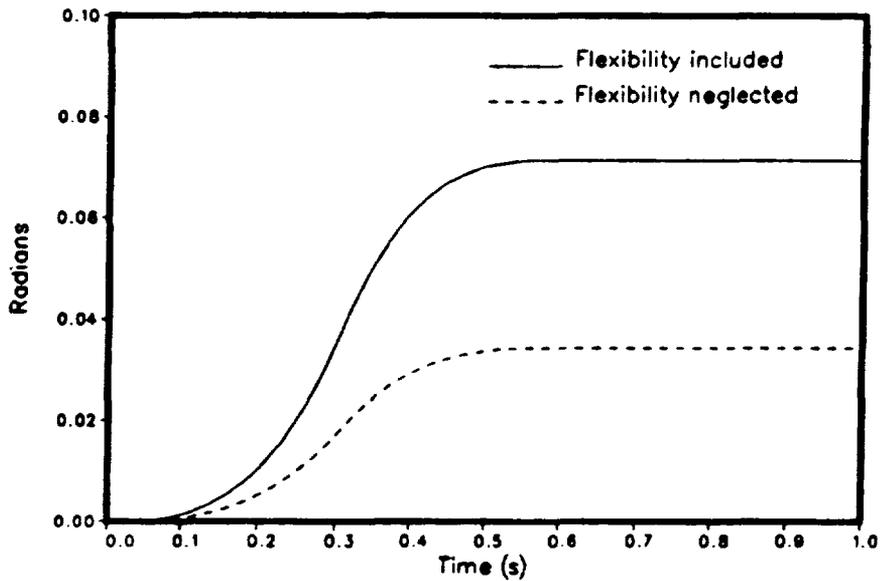


Fig. 2 : Position Response of the Beam for the Torque Above

## Feedback Control

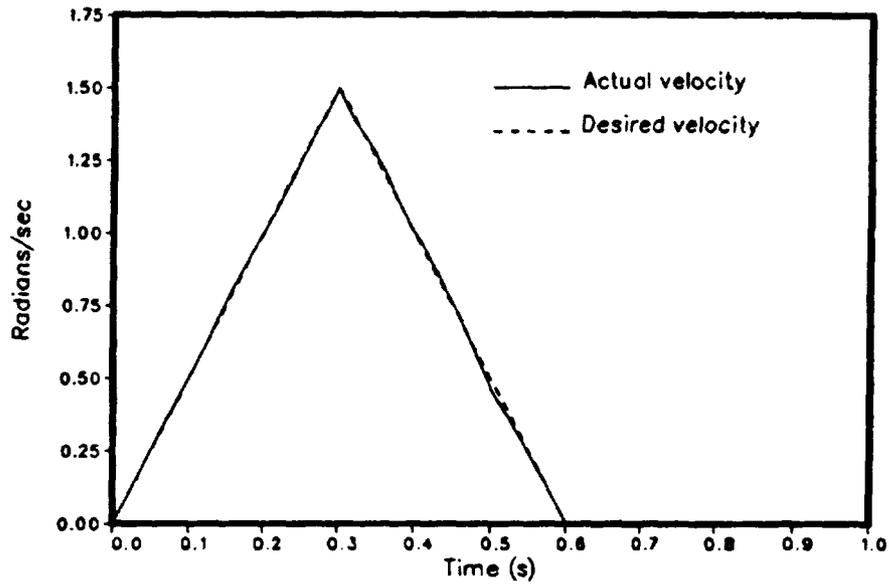


Fig. 3 : Rigid Model, Velocity response

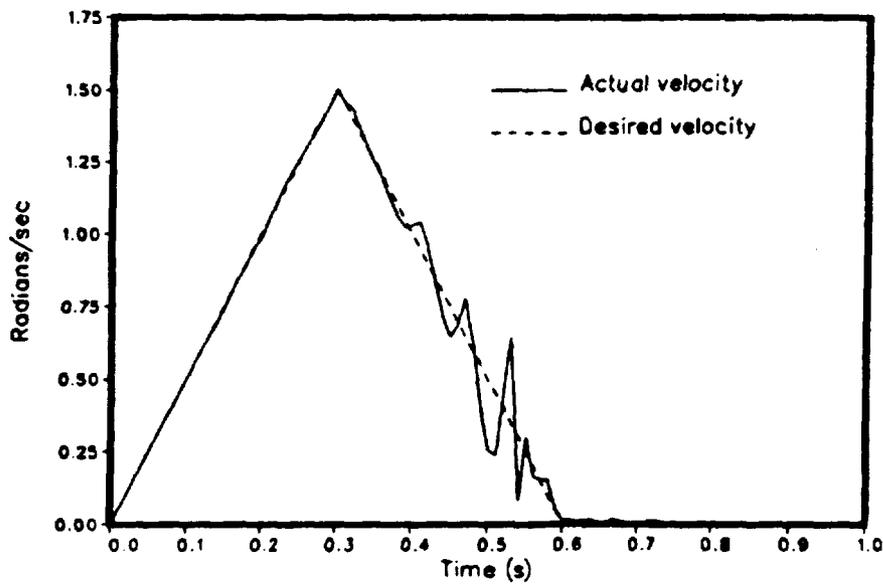


Fig. 4 : One Flexible Mode Included in the Model

### Feedback Control, contd.

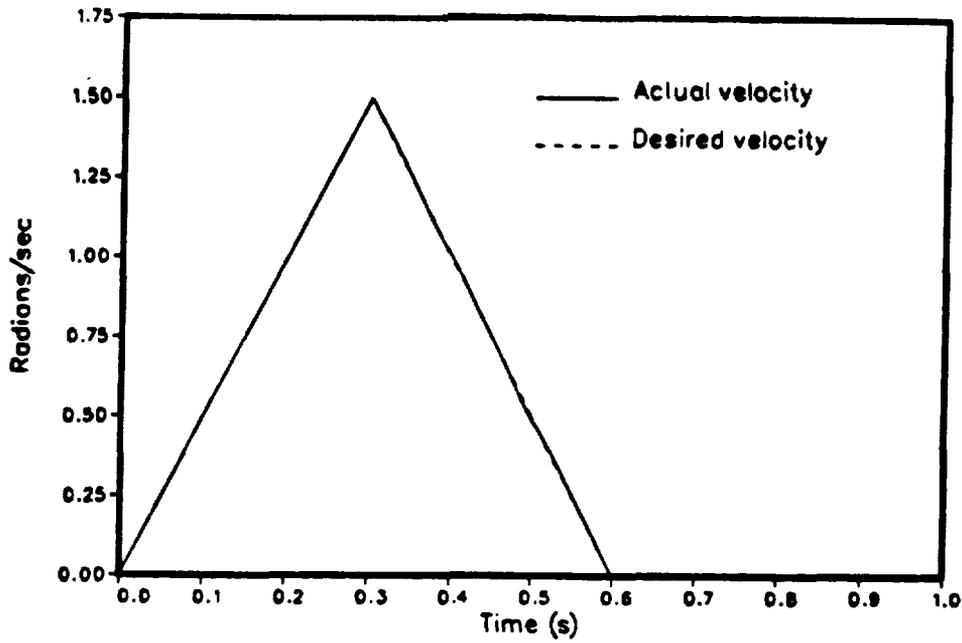


Fig. 5 : Three Flexible Modes Included in the Model

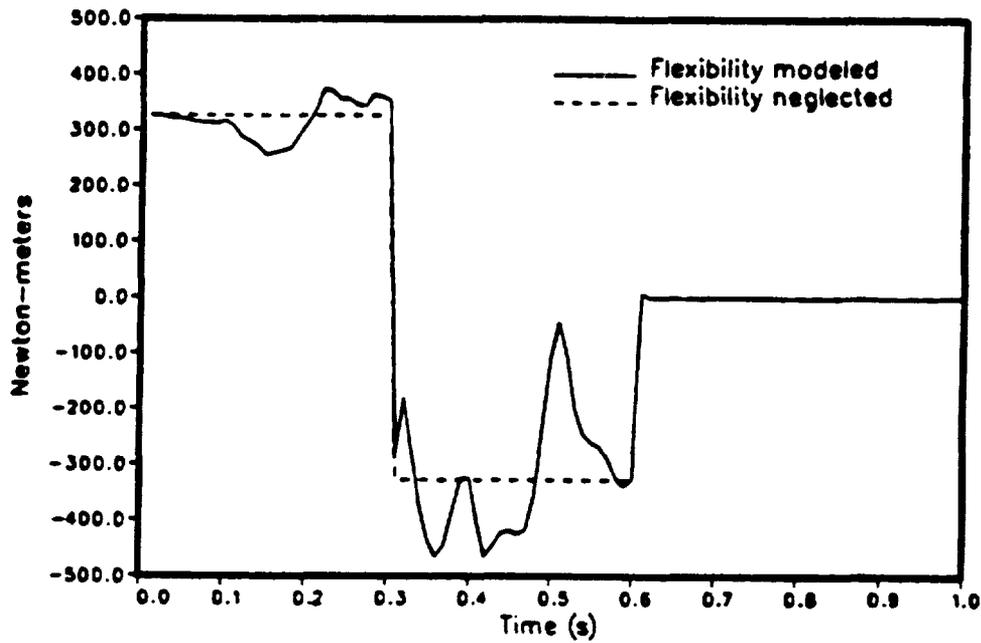


Fig. 6 : Comparison of Open- and Closed-Loop Torques

## **Effects of centrifugal stiffening**

1. Provides a strong coupling between the rigid-body and elastic motion
2. Increases the stiffness of the structure
3. Reduces the effective rigid-body inertia term. Can cause it to vanish if the elastic motion is large. May have to modify the model, or vary the torque profiles.

# Torque profiles and their Fourier coefficients

	Coefficients of	
	$\sin(n\omega_0 t)$	$\cos(n\omega_0 t)$
	$\frac{(-1)^{m-1}}{n\omega_0}$	0
	$\frac{(1 - (-1)^m)}{3n\omega_0}$	$\frac{2((-1)^m - 1)}{3\pi\omega_0 n^2}$
	$\frac{(1 - (-1)^m)}{n\omega_0}$	0
	$n = 2m + 1$	
	$\frac{8}{3\pi\omega_0 n^2}$	0

$\int_0^T \int_0^T \text{Torque} \cdot dt$  is the same for all cases

$$\omega_0 = \frac{2\pi}{T}$$

## **Computational Issues for Control of multi-link flexible robot arm**

1. The dynamic model can be arrived at by modeling each link independently and imposing constraints at the joints
2. The link geometry may not be simple
3.  $s_{ijkl}$ ,  $q_{ijk}$ , may not be negligible, and the control model may include all the terms
4. The choice of admissible functions for each of the links may be different
5. Sampling rates - should not excite elastic motion
6. Control input computation may pose formidable burden.

The above issues can be adequately addressed by selecting pointwise-optimal control law for control input computations, where, the inputs can be computed at least one time step ahead.

## Conclusions

1. A complete model for control of a flexible link is developed
2. Modeling issues are examined within the context of an example
3. Several control issues are investigated
4. It is shown that centrifugal stiffening effect on rigid-body motion is significant
5. There is a strong coupling between rigid-body and elastic motions; ignoring this coupling results in gross inaccuracies in response.

N 9 0 - 1 0 0 9 9 .

**LINEARIZED FLEXIBILITY MODELS IN MULTIBODY DYNAMICS AND CONTROL**

By

William W. Cimino  
Boeing Aerospace  
Seattle, Washington

**ABSTRACT**

*Discussed.*

~~This presentation discusses~~ simulation of structural response of multi-flexible-body systems by linearized flexible motion combined with nonlinear rigid motion. Advantages and applicability of such an approach for accurate simulation with greatly reduced computational costs and turnaround times are described, restricting attention to the control design environment. Requirements for updating the linearized flexibility model to track large angular motions are discussed. Validation of such an approach by comparison with other existing codes is included. Application to a flexible robot manipulator system is described.

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# **Linearized Flexibility Models in Multibody Dynamics and Control**

**12 July 1988**

**William W. Cimino**

**Boeing Aerospace  
Seattle, Washington**

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- **Some controls requirements of multibody codes**
- **Introduction to SADACS**
- **Validation**
- **Applications**
  - **Spacecraft**
  - **Robot manipulators**

## Some Controls Requirements

- 1) **General purpose dynamic module**
- 2) **Models can be merged in any configuration without creating new structural models**
- 3) **Very fast (computationally inexpensive)**
  - **Short simulation turnover time**
    - **Time domain analysis with nonlinear controllers**
      - **Sensitivity studies**
      - **Stability analysis**
  - **Control design iteration**

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# **SADACS**

## **Spacecraft Appendage Dynamics and Control Simulation**

- **Dynamics simulation of multi-flexible-body systems**
- **Designed for controls engineers/controls environment**
- **Approximate code to address controls requirements**
  - **Linearized flexible modal analysis with "configuration update"**

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## **SADACS is designed for controls environment**

- **Used as general purpose dynamics module in a control simulation environment**
- **Allows multibody systems to be merged in any desired configuration without creating new structural models**
- **Very fast (computationally inexpensive) for system design and sensitivity/stability analysis**

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- **How fast is SADACS?**
  - **Problem dependent**
  - **Large complex models 100-500 times faster than DISCOS**
- **Why is it fast?**
  - **System modes**
    - **Diagonalized, linear, constant coefficient flexible equations of motion**
    - **Truncation (with residualization) to increase  $\Delta t$**
    - **Use 'explicit' integration**
- **Propagate linear system until 'update'**

## CPU Time Comparisons for 3-Body Problem

---

CODE	RUN TIMES	REMARKS
DISCOS	> 26 hours	No component modal truncation
DISCOS	5 hours 9 minutes	With component modal truncation
SADACS	10 minutes	With system modal truncation

Other Test Problems :

*High Speed Simulation of Flexible Multibody Dynamics*

Presented at MSFC, April 22-24, 1986

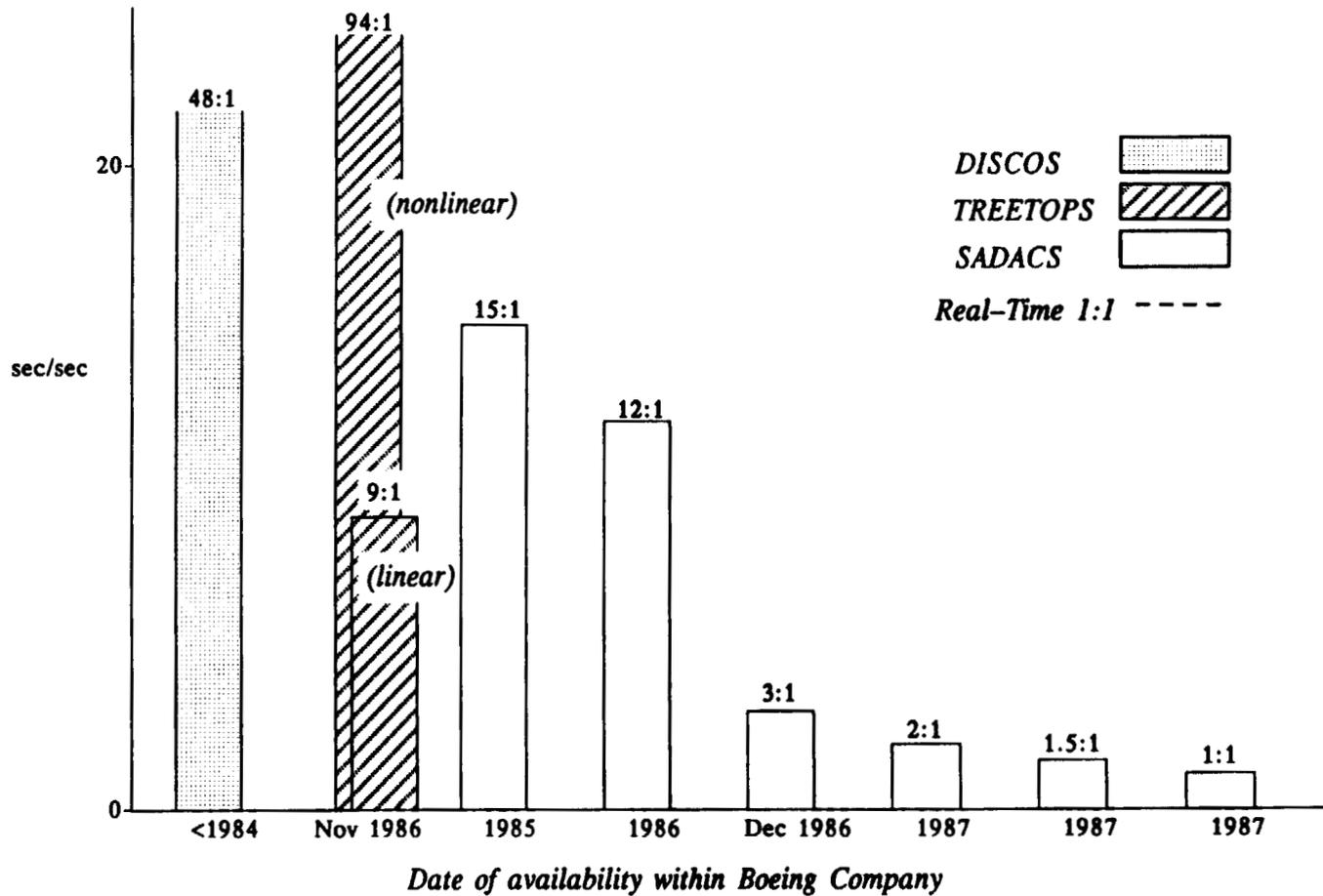
CODE	RUN TIMES	REMARKS
DISCOS	≈ 50 Hours	With component modal truncation
SADACS	≈ 15 minutes	With system modal truncation

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# Multi-Flexible-Body Run-Times

3-Body simulation CPU (seconds) / Real-Time (seconds)

4.49



## **SADACS Structure**

- 1) Nonlinear rigid body code  
(SD/EXACT, TREETOPS, MBDYN, etc.)**
  
- 2) Linear flexible dynamics**
  - **System mode formulation**
  - **Retain truncated modes quasi-statically**
  
- 3) System mode update/restart**

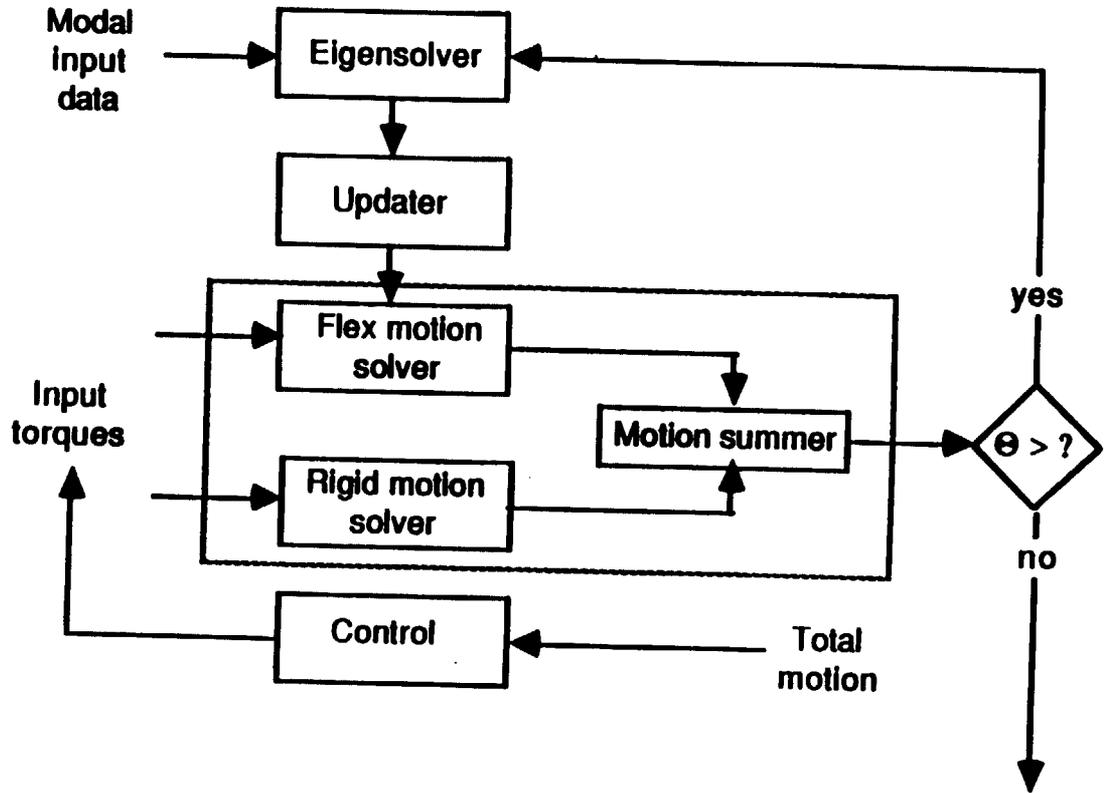
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### Simplified Diagram of the SADACS Code

Inputs to the system (generalized forces and torques) are applied to both the rigid and flex motion solvers. The rigid motion solver computes the nonlinear rigid body response. The flex motion solver uses a "system mode" formulation to compute the linearized flexible response. The outputs are combined in the motion summer and tested for an "update condition". If an update is not required the outputs are passed out to the simulation. If an update is required, a new eigensolution is performed on the new configuration and the mode shapes, frequencies, and system mode state vector are adjusted.

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*Simplified Diagram of SADACS Code*

# **The Difficult Technical Problem:**

## **Large angle motions with linear flexibility**

- **System modes change (shape, frequency) with angular position**
- **Track changes by 'updating' system modes**
  - **Update at predetermined angles or time**
    - **Shape**
    - **Frequencies**
    - **Transfer functions**
  - **Restart dynamic analysis**

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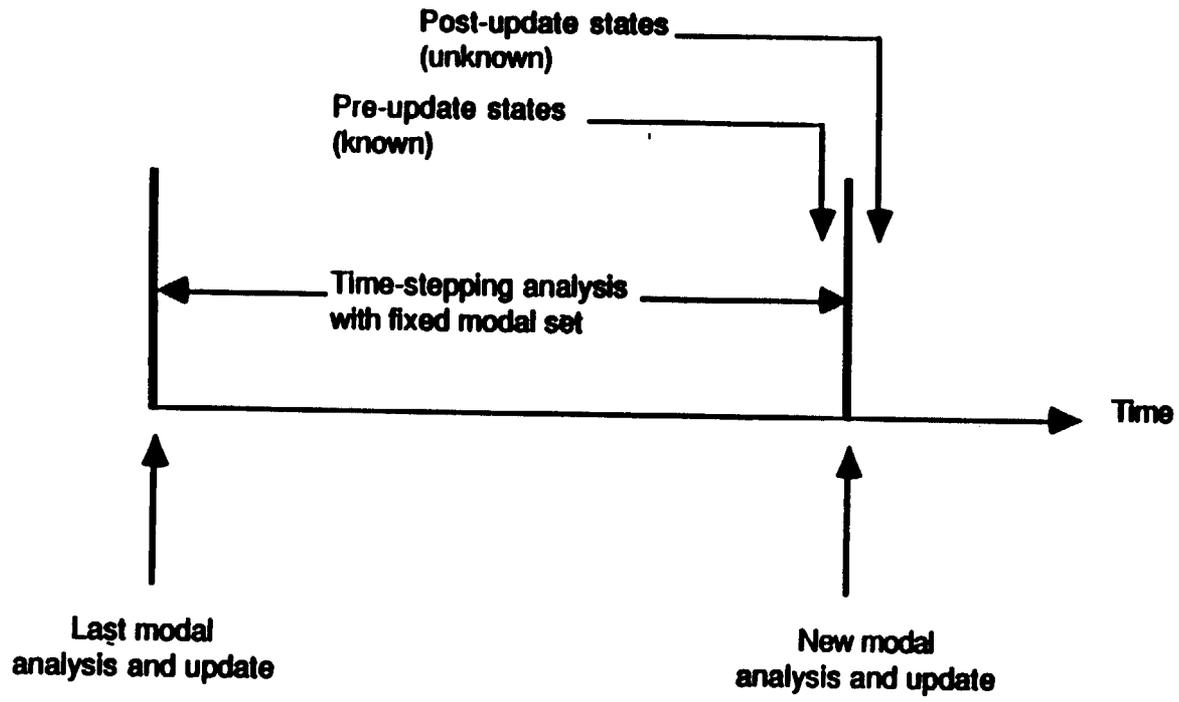
#### Updating Time-Line Overview

A fixed modal set is maintained during a given epoch (time between updates). At the end of the current epoch the pre-update states are known. Following the eigensolution on the new system matrices, the new modes shapes and frequencies are known. The difficult part of the update is then to assign new values to the post-update states.

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*Updating Time-Line Overview*

### **Why is updating a problem?**

**When gimbal rotations and rates (which include structural deformations) developed in one configuration are imposed on a new configuration, they excite the structure in shapes (modes) that would not have occurred in a 'continuous' solution, and in addition fail to preserve energy.**

**The problem is nonlinearity-induced trading of excitation, or coupling, between the modes.**

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**Example problem: chosen to emphasize 'trade' in modal participation**

- **Want update that**
  - 1) **Doesn't ring**
  - 2) **Maintains energy**
  - 3) **Tracks frequencies**
  - 4) **Correct shapes**
  
- **Coupling of flex into rigid neglected**
  - **SADACS not intended for problems where flex nonlinearities drive rigid motion**
  
- **Address update entirely with component modal variables**

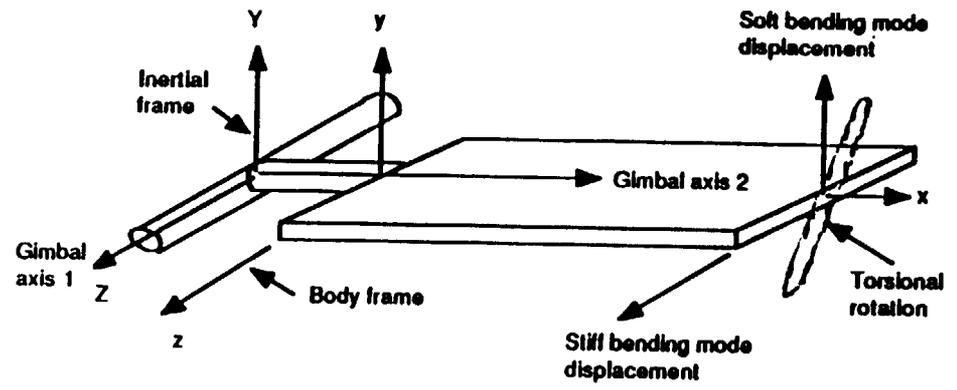
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### Example Problem

This figure shows the system used to examine the update. The system has two flexible modes with coordinates  $q_1$  (soft mode) and  $q_2$  (stiff mode).

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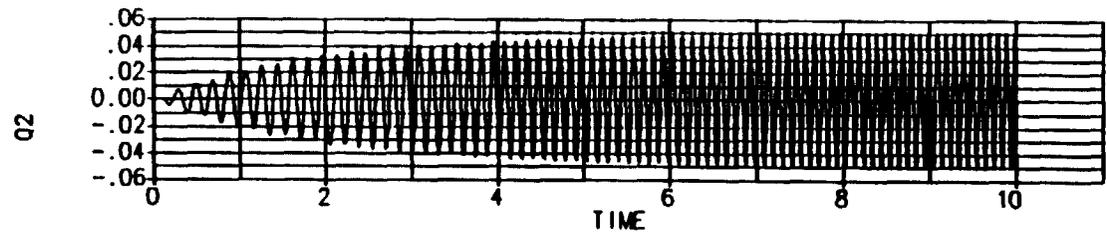
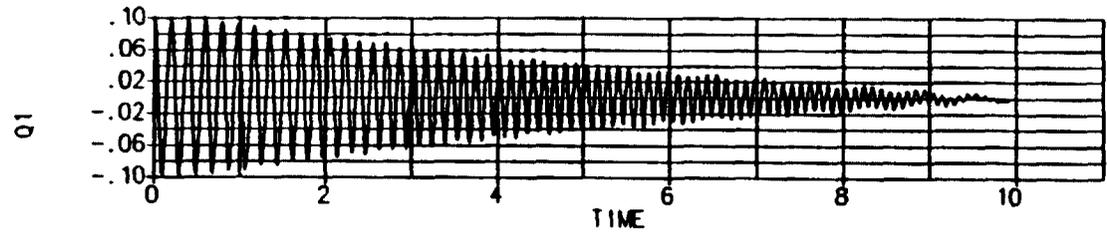


- **Rigid coordinates  $\theta_1, \theta_2$**
- **Flex coordinates  $q_1, q_2$**
- **Soft mode = 1.59 Hz**
- **Stiff mode = 3.18 Hz**
- **$\dot{\theta}_2 = 9$  deg/sec (constrained)**
- **Initial conditions:**
  - $\theta_1 = -.03$  rad
  - $q_1 = .1$

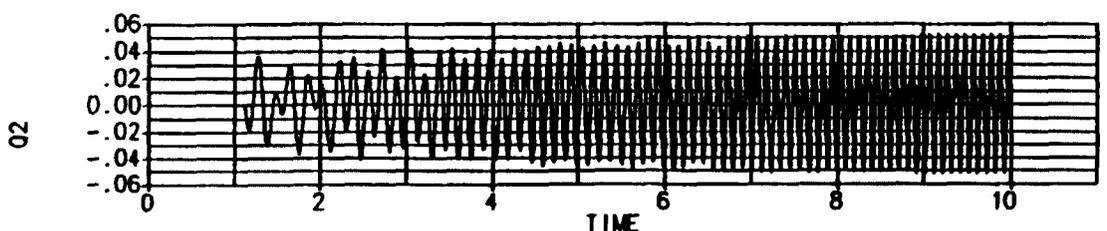
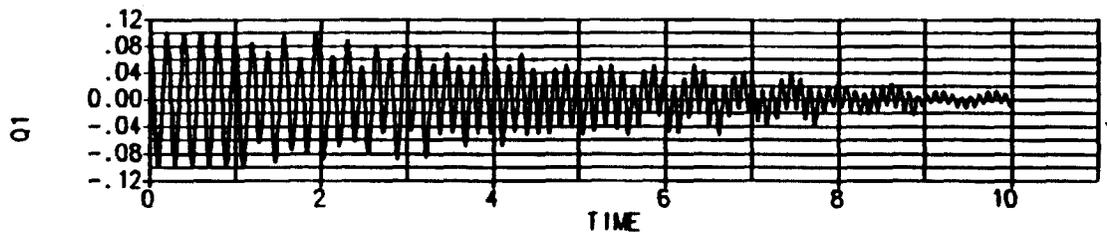
#### SADACS vs. Continuous Solution

The top set of plots show the flexible coordinate response when the equations of motion include flex/rigid coupling and are integrated in a continuous manner. The bottom set of plots show the flexible coordinate response using the SADACS approach.

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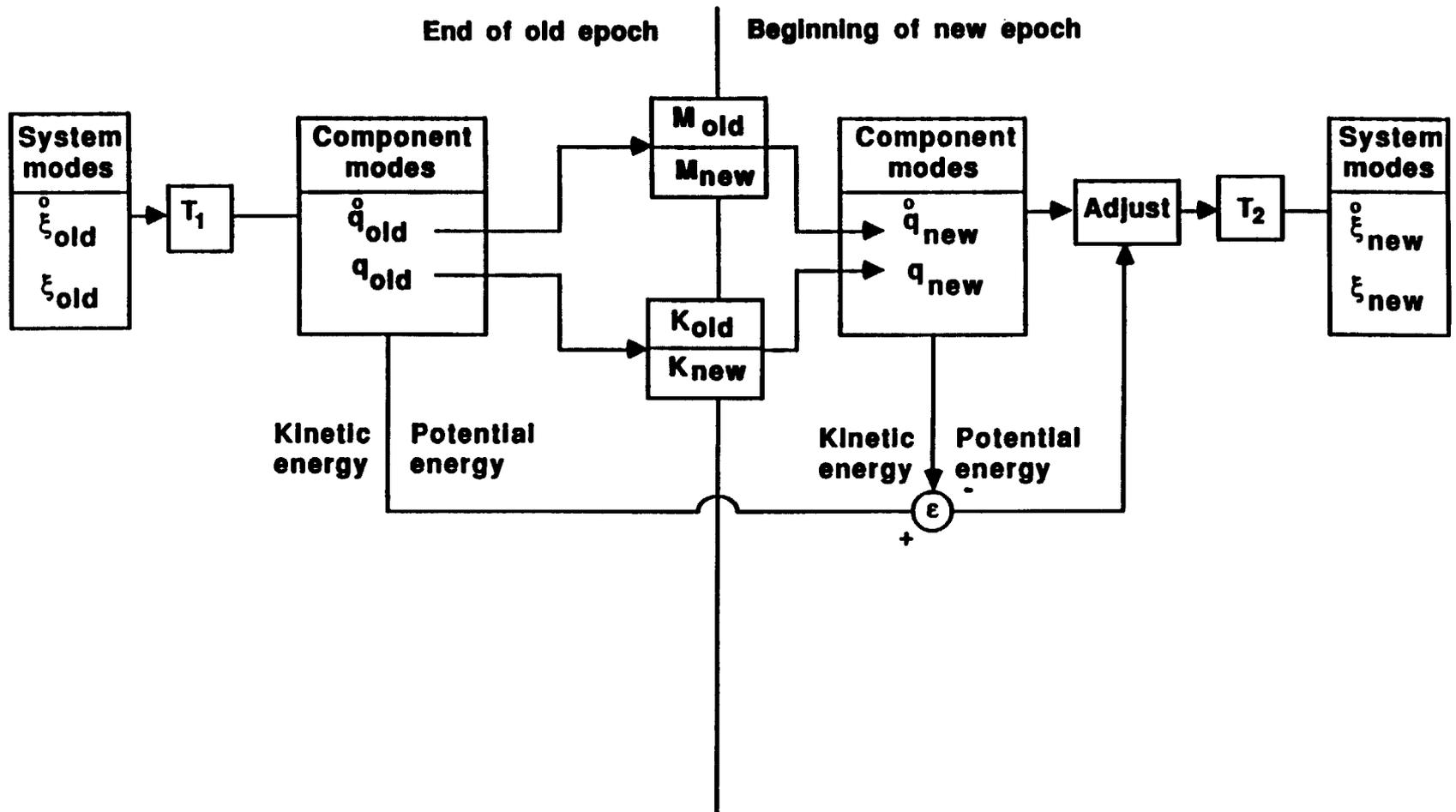


Continuous Solution with Nonlinear Loads

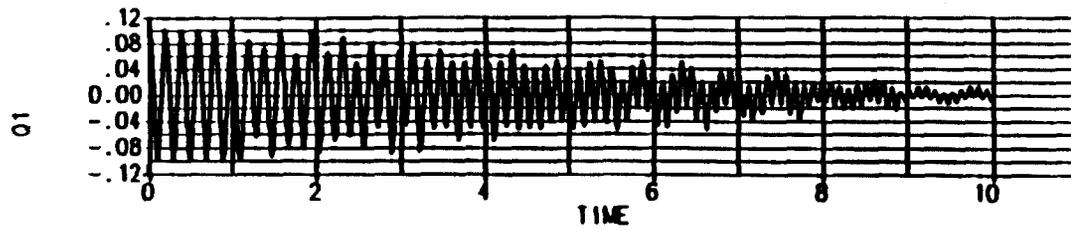


Updated Solution Based on Momentum with Energy Balance

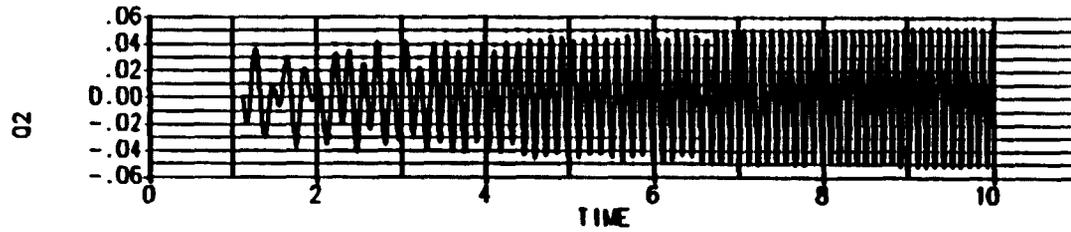
# Momentum/Stress Update with Energy Balance



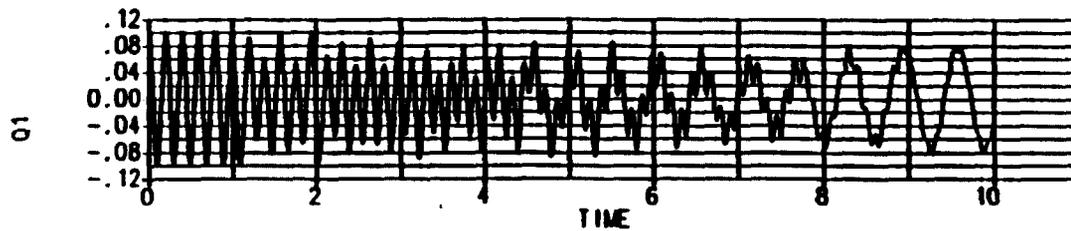
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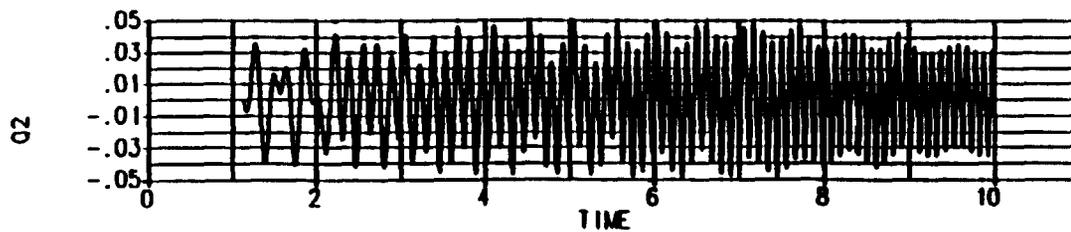
**Updated Solution Based on Momentum  
with Energy Balance**



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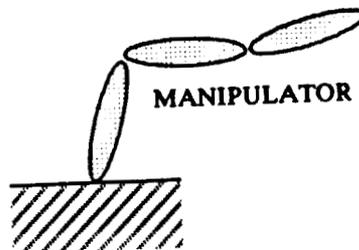
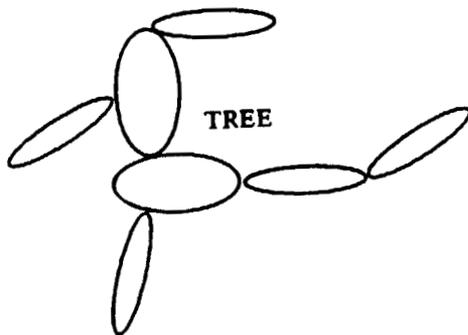
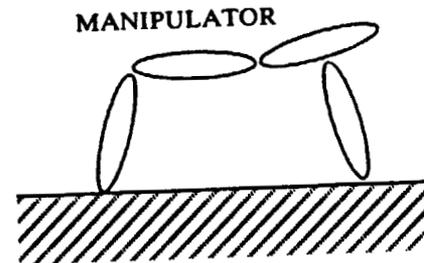
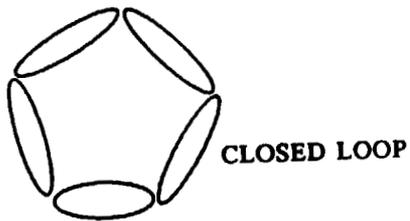
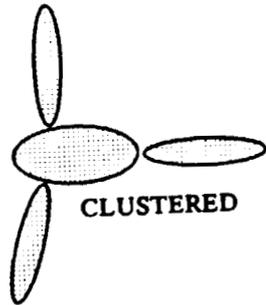


**Updated Solution Based on Phase  
Variables with Energy Balance**



# FB2 Topology Capabilities

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# SADACS program capabilities Summary

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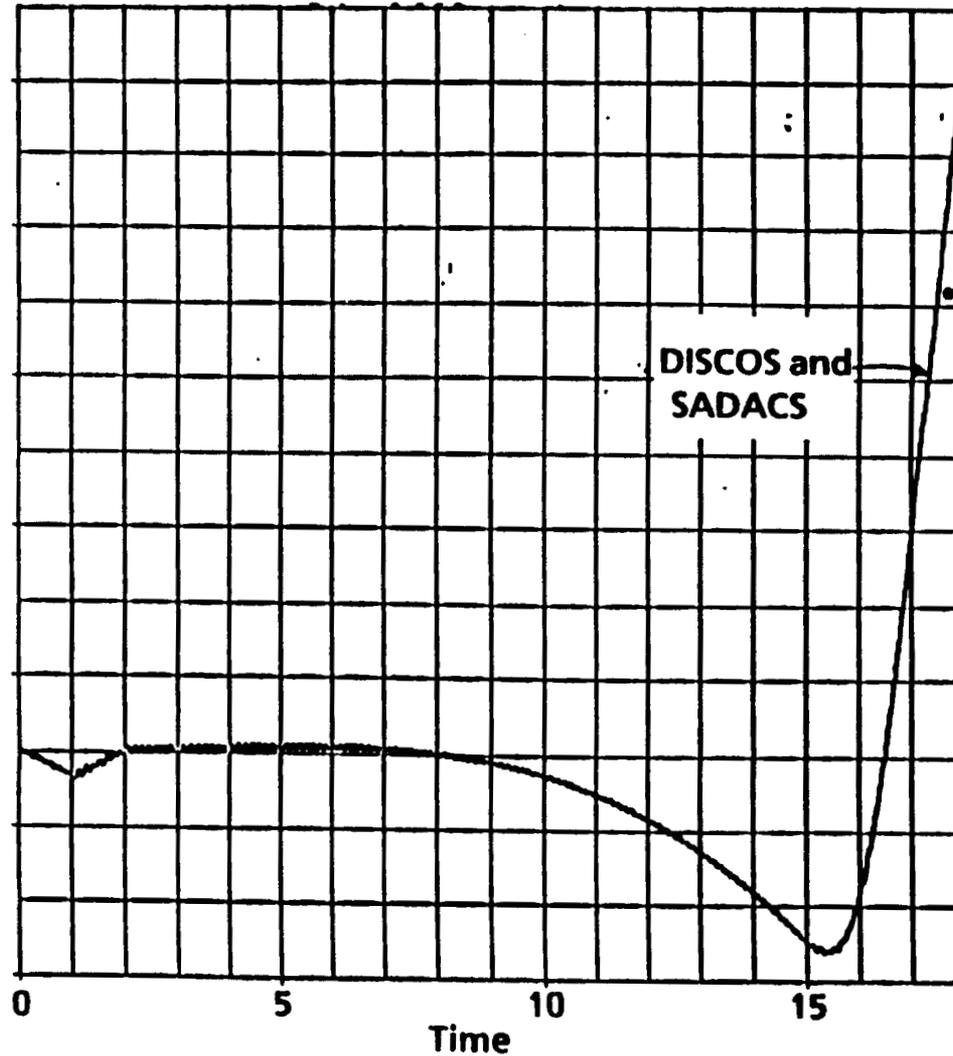
- Rigid body analyser
  - based upon the code used ( MBDYN, TREETOPS, SD/EXACT etc )
- Flex body analyser ( FB2 )
  - Number of bodies no limit
  - Number of flexural modes/body no limit
  - Number of gimbals no limit
  - Configurations :
    - cluster
    - linked
    - tree
    - closed loop
    - manipulator
    - multiple closed loop
    - multiply grounded manipulator
  - Degrees of freedom at gimbals 0 → 6 ( totally locked to totally free )

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# DISCOS-SADACS Comparison: Main Body Sensor X Rotation Due To Appendage Command

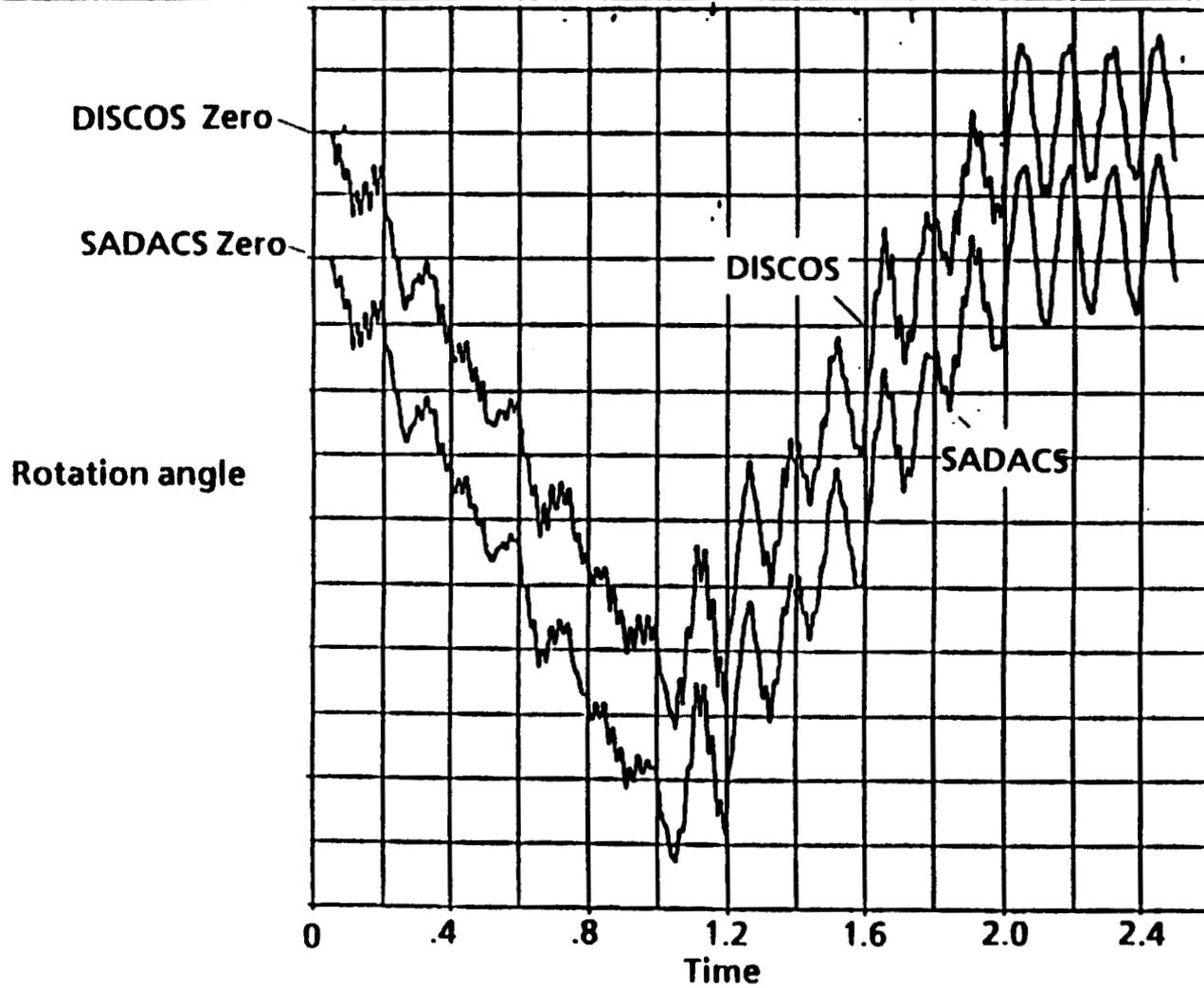
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Rotation angle



022P1-RJ-136/New

# DISCOS-SADACS Comparison: Main Body Sensor X Rotation Due To Appendage Command



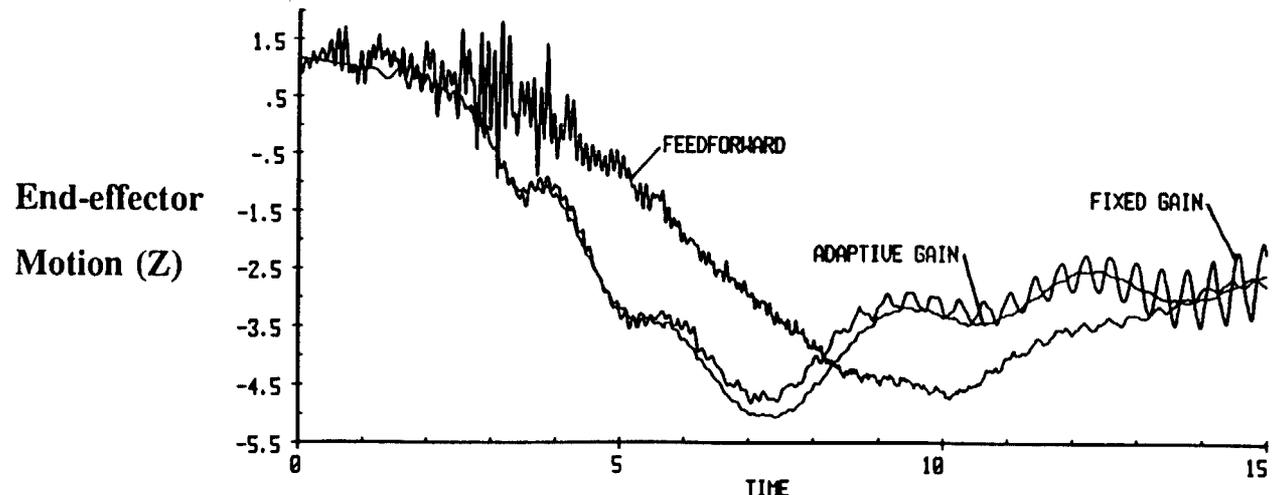
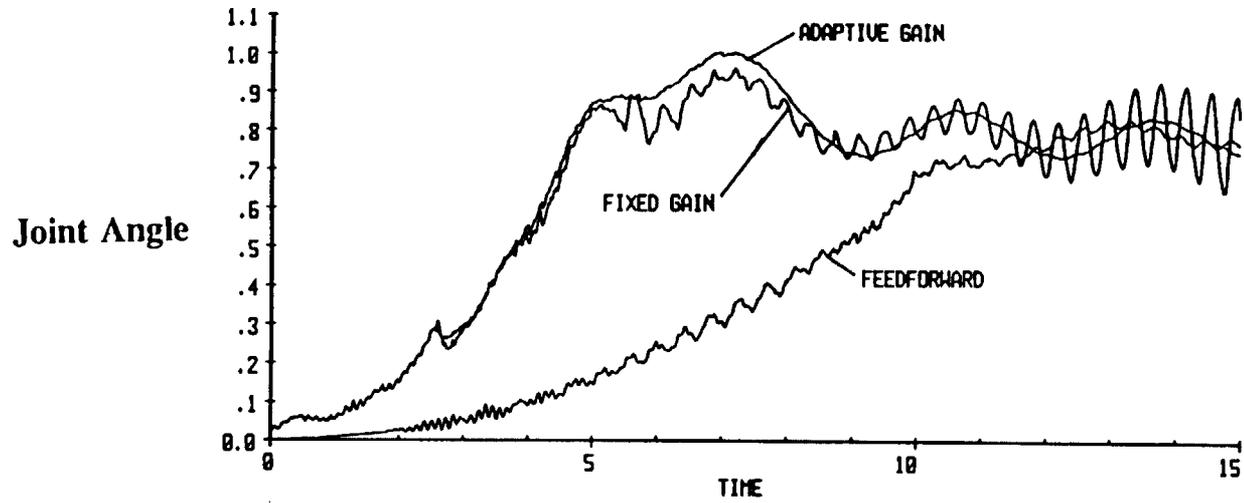
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### Controller Comparison - Closed Loop

The figure below shows the closed-loop response of a flexible model of the SPAR robot manipulator with three different controllers. The top plot is the first joint angle (waist) and the bottom plot is the z motion of the end-effector (up and down). The three different controllers are feedforward, semi-adaptive gain, and fixed gain.

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CONTROLLER COMPARISON - CLOSED LOOP

## **Applicability**

- **Large body of common problems**
  - **Non-spinners**
  - **Problems not dominated by nonlinear flexible response**
- **Each new problem should be validated against 'full code' (TREETOPS/DISCOS)**

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## Conclusions

- **SADACS fast, efficient multi-flexible-body simulation code**
- **Designed for use in controls environment**
- **New 'update' procedure improves accuracy, efficiency, works better**
- **Numerical example compared well with 'truth code' solution (DISCOS)**

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N90-10100

**SIMULATION OF SHUTTLE FLIGHT CONTROL SYSTEM  
STRUCTURAL INTERACTION WITH RMS DEPLOYED PAYLOADS**

By

Joe Turnball  
C. S. Draper Laboratories  
Cambridge, MA

**ABSTRACT**

In support of NASA/JSC, the C. S. Draper Laboratory (CSDL) has implemented a simulation of the system made up of the Orbiter, Remote Manipulator System (RMS), and payload grappled by the RMS. CSDL has used the simulation to study the stability of this overall system when its attitude is under control of the Orbiter's On-orbit Flight Control System (FCS). CSDL has also used the simulation to study the dynamics of the system when the RMS and its associated command software are in active control of the relative Orbiter to payload position and orientation.

The simulation models all of the following elements:

- RMS boom bending (represented by two cubic bending models)
- RMS boom Torsion
- RMS joint gearbox compliance (represented by a non-linear wind-up model)
- Flexibility at the RMS to Orbiter interface
- Flexibility at the RMS to payload interface
- Joint motor dynamics
- Joint servo-loop dynamics
- RMS on-board computer command logic
- Data transfer delays between the RMS sensor and the RMS on-board computer and between the RMS on-board computer and RMS joint servos
- On-orbit flight control nonlinear control logic
- Reaction Control System (both Primary and Vernier) jet forces and moments.

The Draper RMS Simulation (DRS) has close to a decade of development effort behind it. During that time, it has been used to analyze a wide range of RMS questions. Payload weights have run from zero (i.e., an unloaded arm) to weights in excess of the original design limit of the arm (65,000 lbs.). Types of interactions studied have ranged from interactions between failure detection algorithms in the RMS command software and high frequency motor transients to interactions between the On-orbit FCS and the

fundamental bending mode of the composite system with a 20,000 to 20,000 lb payload (0.05 to 0.2 hz).

For all its complexity the DRS is reasonably economical. A run simulating one minute of real time costs on the order of \$10 when run as a low priority over night batch job. Nevertheless, increases in economy can be of benefit for flight control/structural interaction studies which will involve increasing numbers of simulations with longer and longer simulation durations. Consequently, an effort has been under way for the last several years at CSDL on a so called Limited Singing and Dancing (LSAD) simulation that would sacrifice high frequency motor dynamics but retain good representation of bending modes pertinent to the interaction of the On-orbit FCS with the Orbiter/RMS/Payload structural system. LSAD shows approximately a ten-fold increase in economy as compared to similar DRS simulations

# **Simulation of Shuttle Flight Control System Interaction with RMS Deployed Payloads**

A Presentation by Joseph Turnbull to the

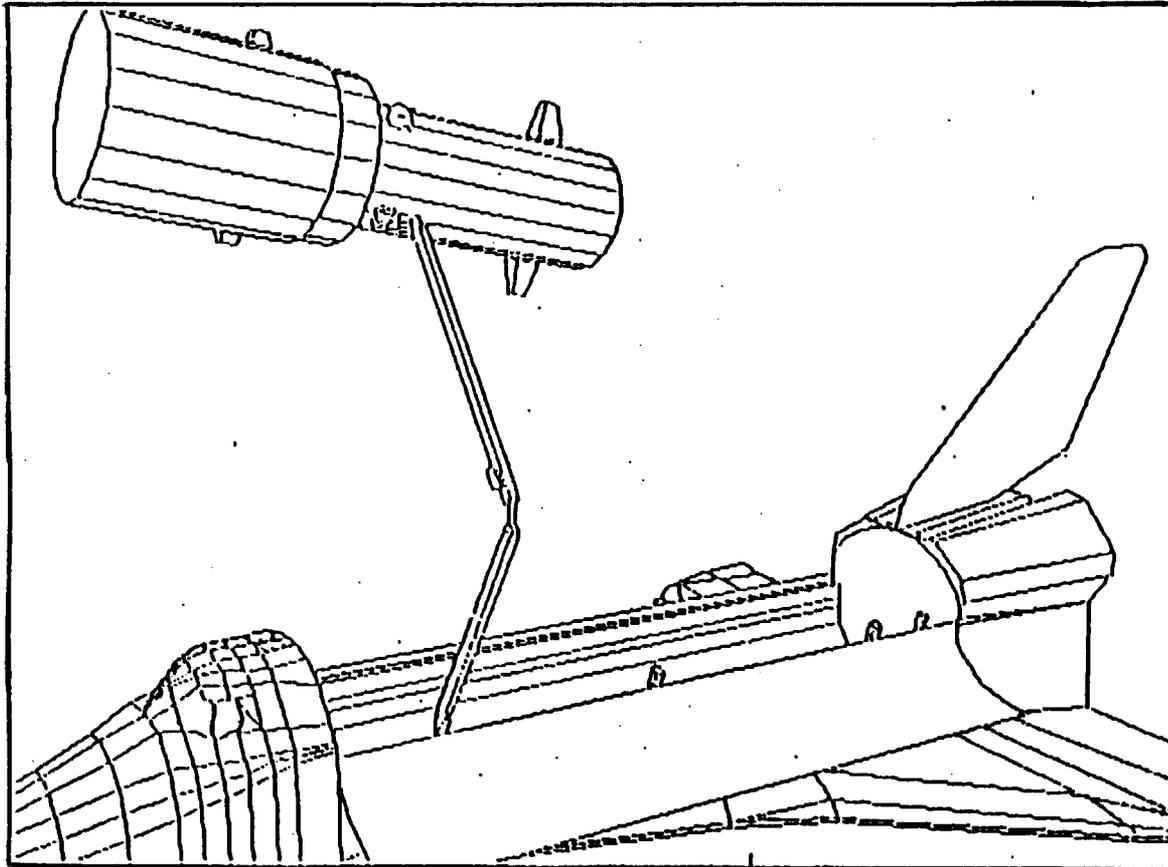
**WORKSHOP ON COMPUTATIONAL ASPECTS IN THE  
CONTROL OF FLEXIBLE SYSTEMS**

*July 12-14, 1988*



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# Orbiter / RMS / Payload



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# The Draper RMS Simulation (DRS)

Modeled in the DRS are:

- Transverse bending in the long booms  
Two cubic modes each in-plane and cross axis
- Torsion in the long booms  
Modeled as a torsional spring
- RMS to Orbiter stiffness  
Lumped with this are the flexibilities of the short links between the shoulder pitch joint and the Orbiter
- RMS to payload interface stiffness  
Lumped with this are the flexibilities of the short links between the wrist pitch joint and the payload
- Orbiter and payload as rigid bodies

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## DRS Model Elements (cont)

- RMS joint non-linear gearbox compliance
- Joint servo and motor dynamics
- Data transfer delays between the RMS sensors and the on-board computer and between the RMS on-board computer and the RMS joint servos
- RMS on-board computer command logic
- On-orbit flight control non-linear control logic
- Reaction Control System (both Primary and Vernier) jet forces and moments

## Fundamental DRS Equation of Motion

$$\ddot{x} = A^{-1} [u - Kx]$$

where:

x is the state vector (dimension 25)

A is the "inertia" matrix

K is the "stiffness" matrix

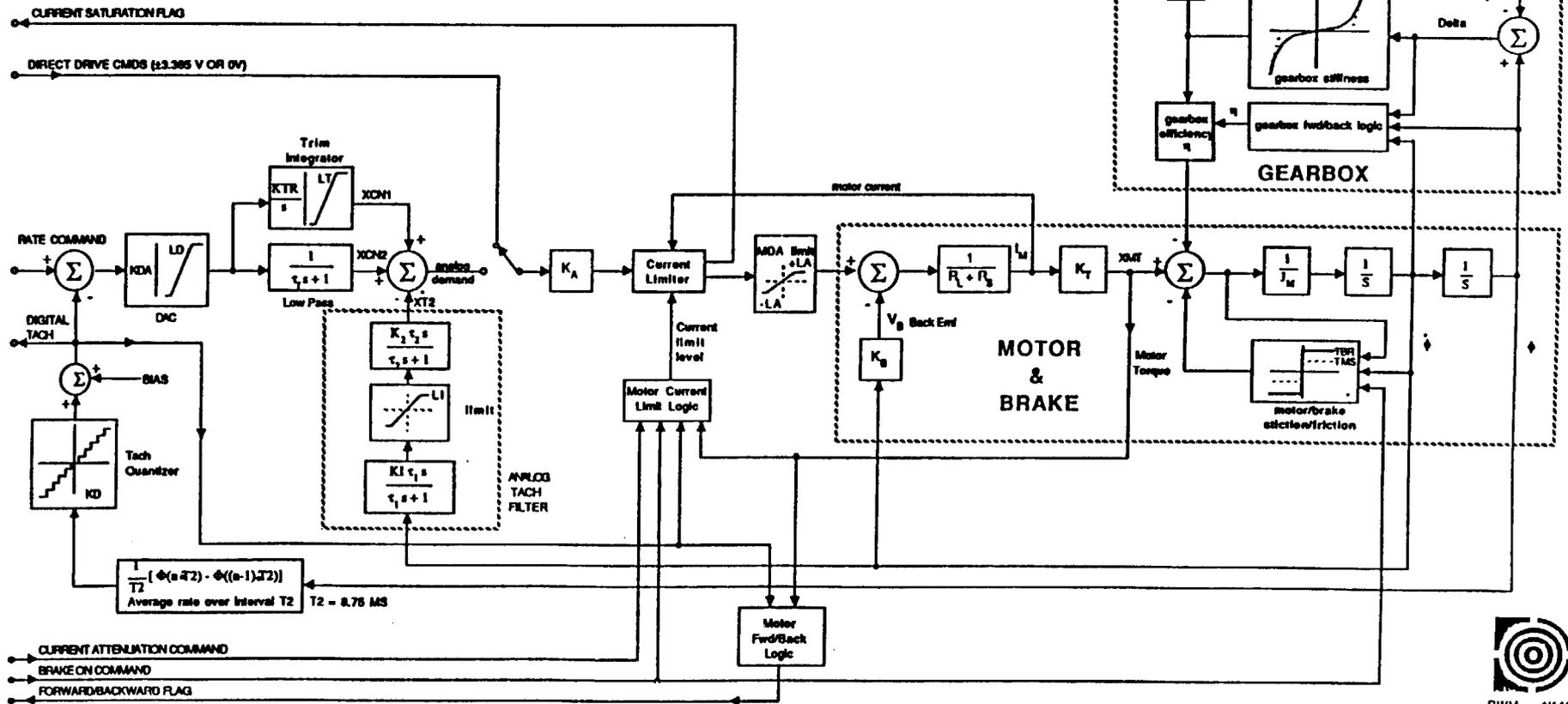
u is the "torque" vector (a function, in part, of the servo loop)

Equation is integrated using a first order predictor/corrector scheme with a 1 ms integration step size.

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# DRAPER RMS SIMULATION SERVO MODEL APRIL 1988



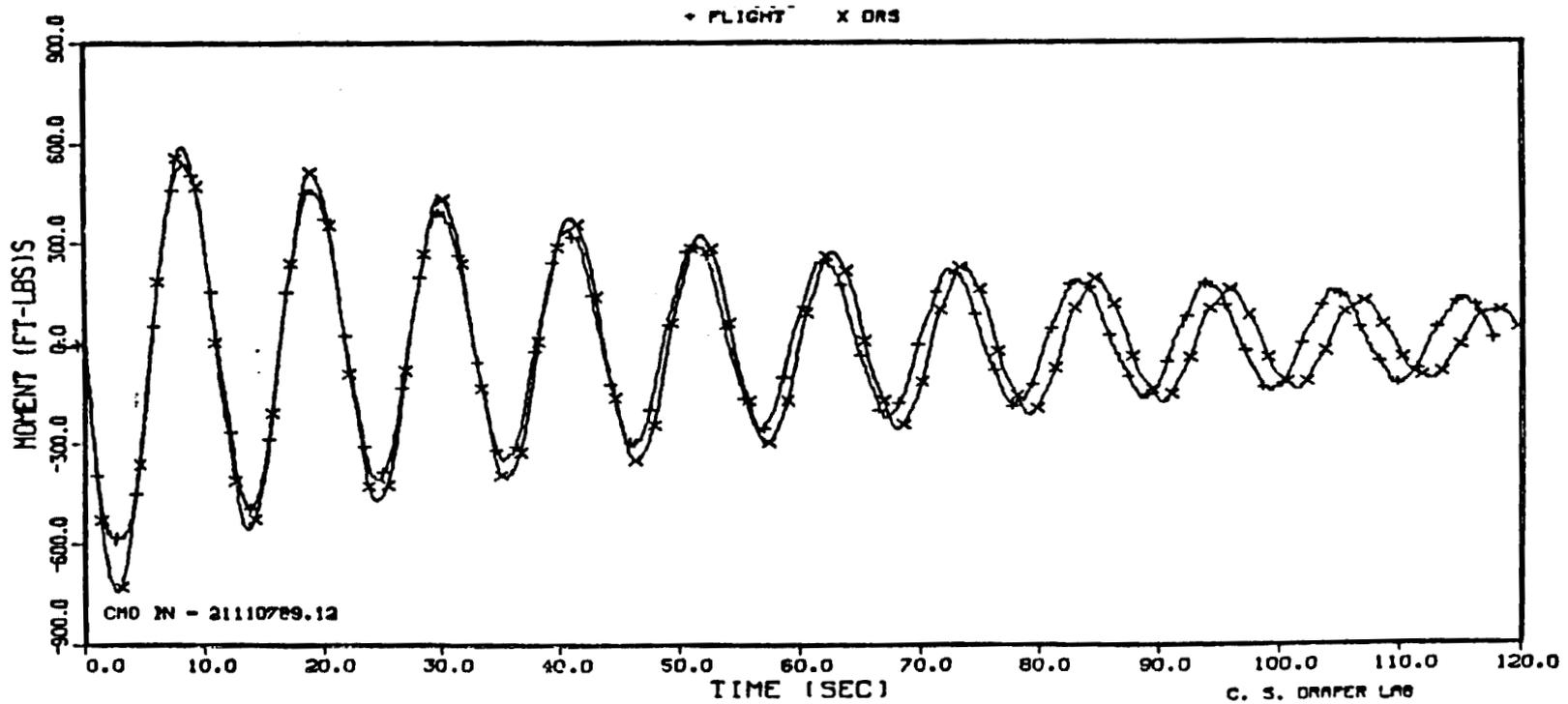
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## DRS Verification

- Extensive simulation to flight comparison has been done.
- Model changes and parameter adjustments have been made to produce an excellent sim to flight agreement.
- Attention has been paid to both the low frequency bending and high frequency transients,

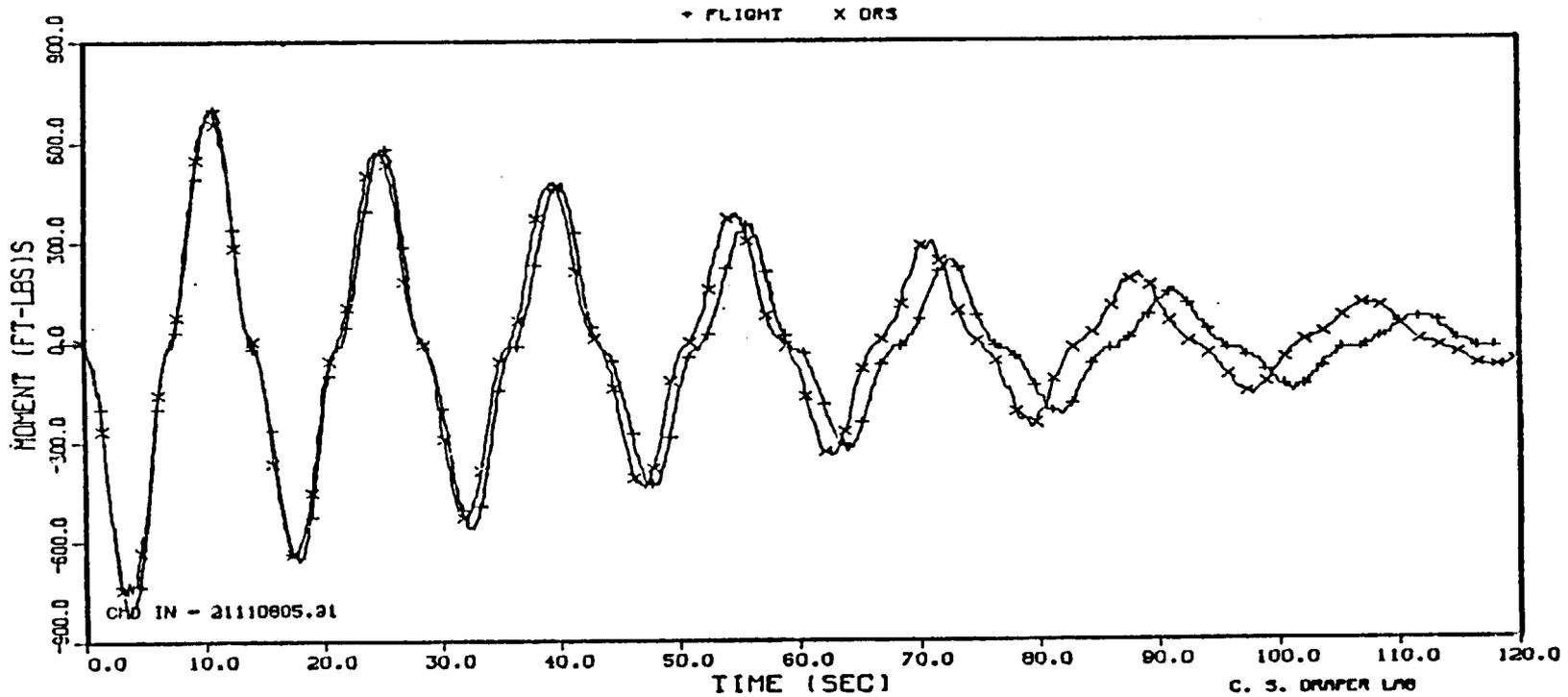
Simulation to Flight Overplot  
Cross-axis Bending Excited by PRCS Jet Firing  
8000 lb PFTA Grappled by RMS

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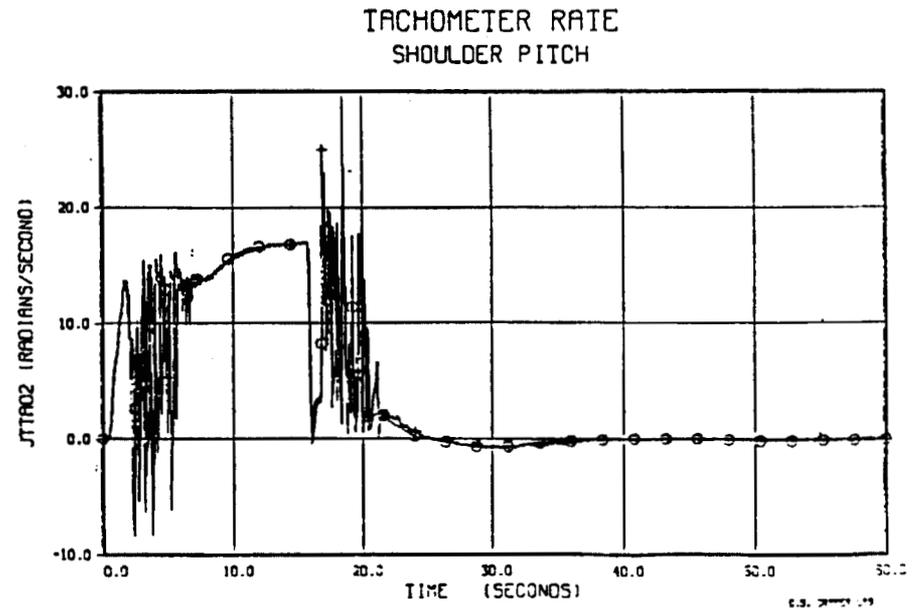
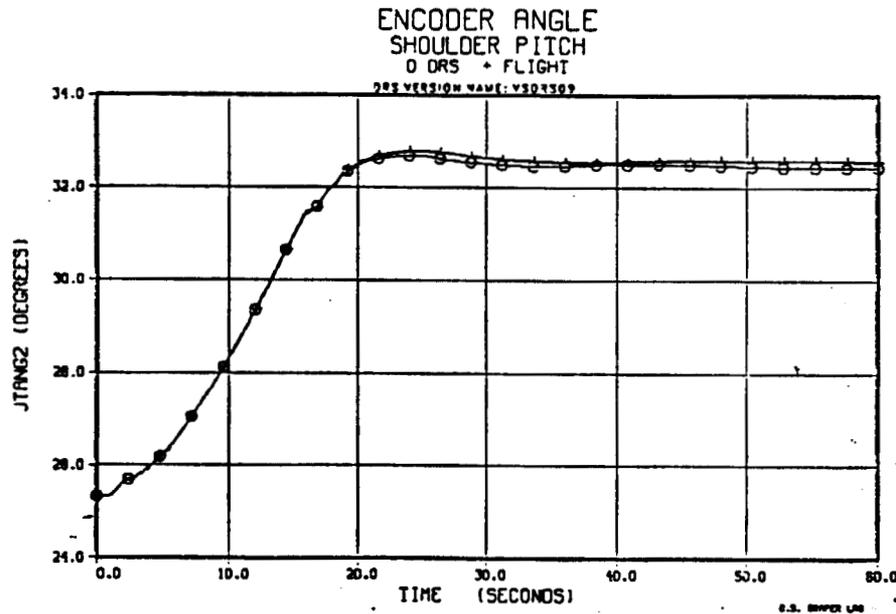
Simulation to Flight Overplot  
In-plane Bending Excited by PRCS Jet Firing  
8000 lb PFTA Grappled by RMS

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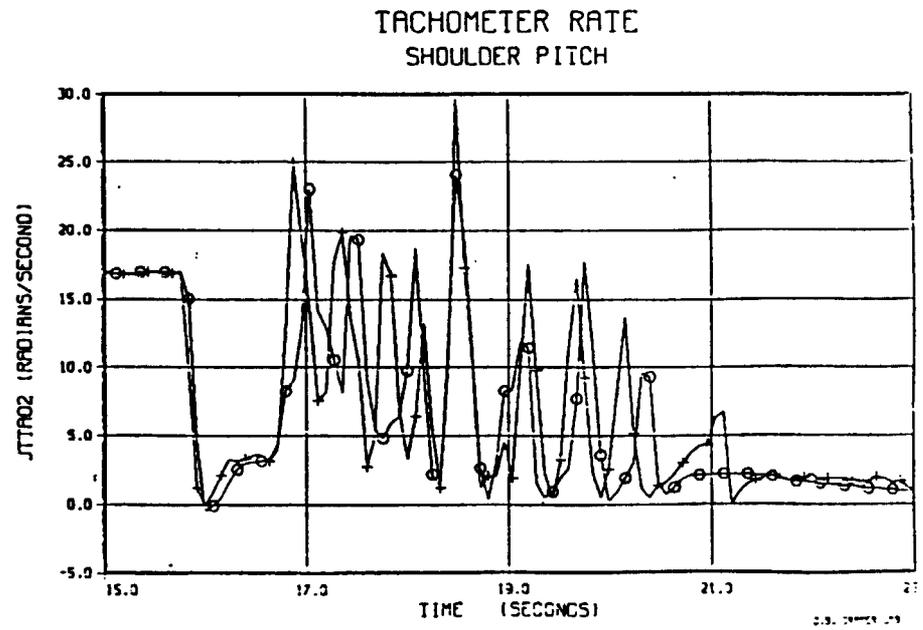
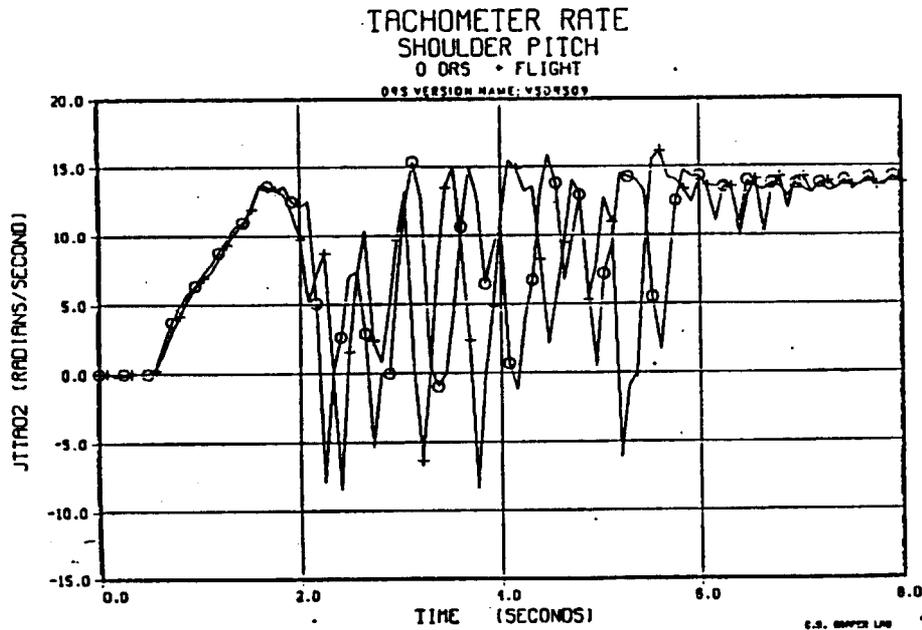
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# Simulation to Flight Overplot Joint Angle & Motor Rate for Shoulder Pitch Single Mode Drive 8000 lb PFTA Grappled by RMS



# Simulation to Flight Overplot

## Motor Rate Start and Stop Transients for Shoulder Pitch Single Mode Drive 8000 lb PFTA Grappled by RMS



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## Simulation Applications of the DRS

- Arm dynamics and performance analyses
  - Arm motion and loads during arm maneuvers
  - Payload tip off rates at payload release
  - Payload and arm motion during capture
  - Interaxis coupling during six joint coordinated motion
  - Post-flight estimation of joint brake effectiveness
  - Evaluation of on-board algorithm for detecting and arresting joint runaway malfunctions

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## Simulation Applications of the DRS (cont)

- FCS interaction
  - Stability analyses – Self sustaining limit cycles are possible because of the relative values of FCS bandwidth, phase lag and fundamental bending frequency
  - Estimation of accelerations at the arm to payload interface due to PRCS jet activity



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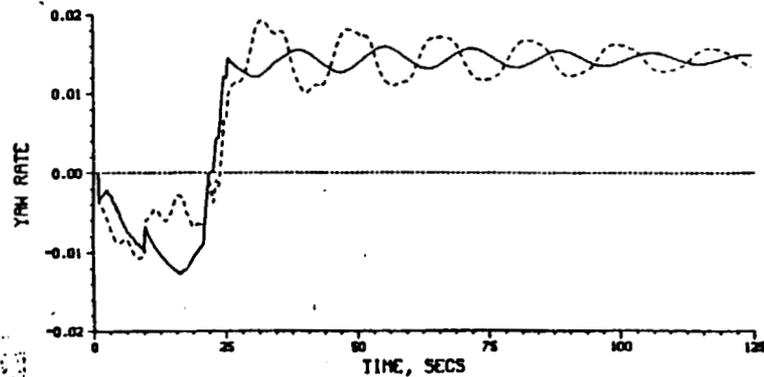
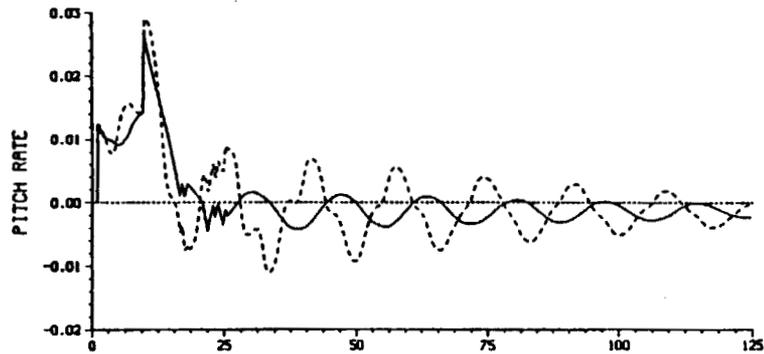
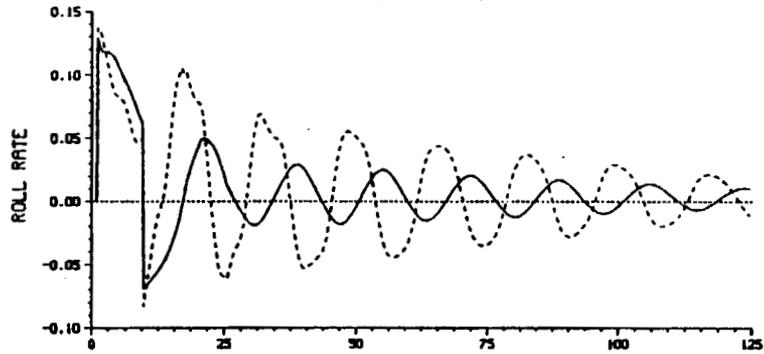
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## FCS Stability Analyses

- Stability is dependent on payload position and attitude relative to the Orbiter
- For a candidate position and attitude:
  - Apply open-loop PRCS pulses to excite the fundamental flex modes of the system and then activate the FCS closed loop attitude hold, or
  - Simulate three axis attitude maneuvers,
  - Observe whether a self-sustained limit cycle results.

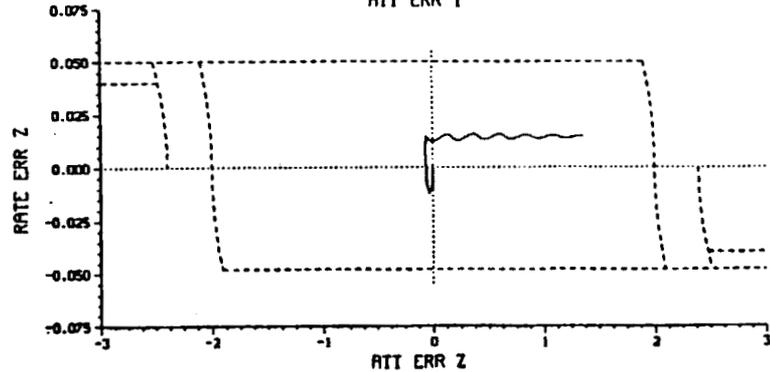
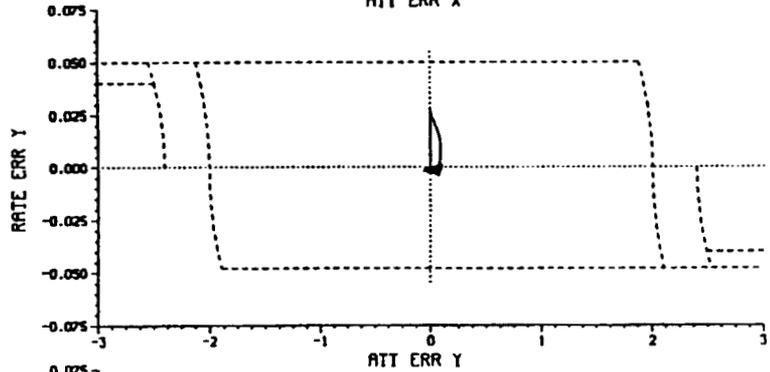
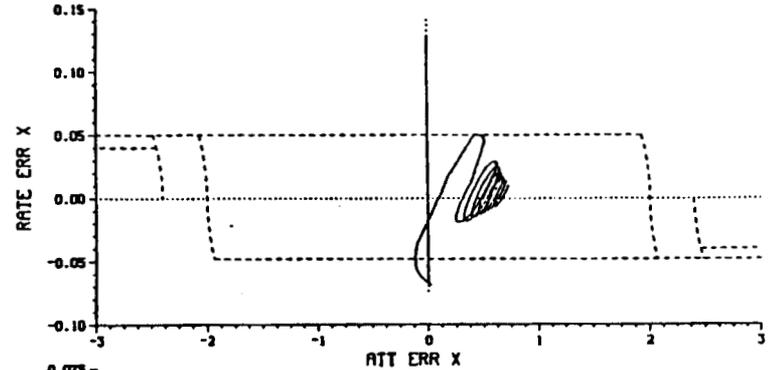
# Example of a FCS Interaction Test with No Self-sustained Limit Cycle

SJB24333 ESTIMATED, ACTUAL AND DESIRED ORBITER ATT. RATE  
IN DEG/SEC (solid-est, dash=true, dot-desired)



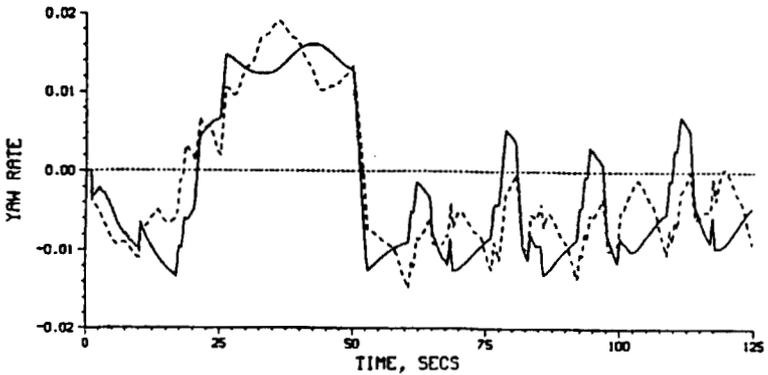
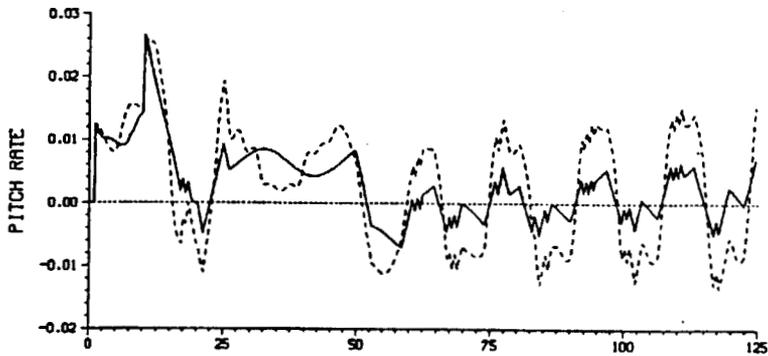
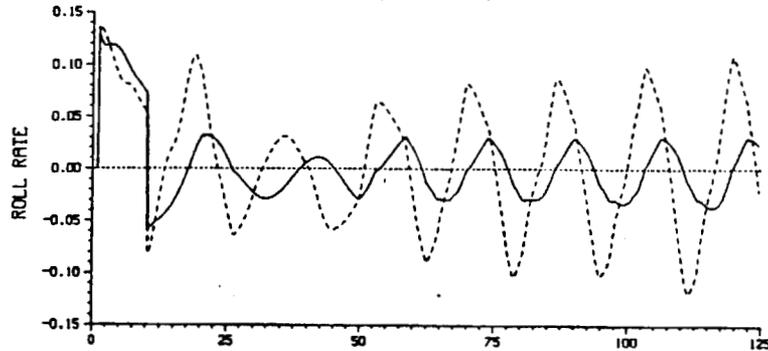
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SJB24333 PHASE PLANE PLOTS  
DAP VARIABLES IN DEG AND DEG/SEC

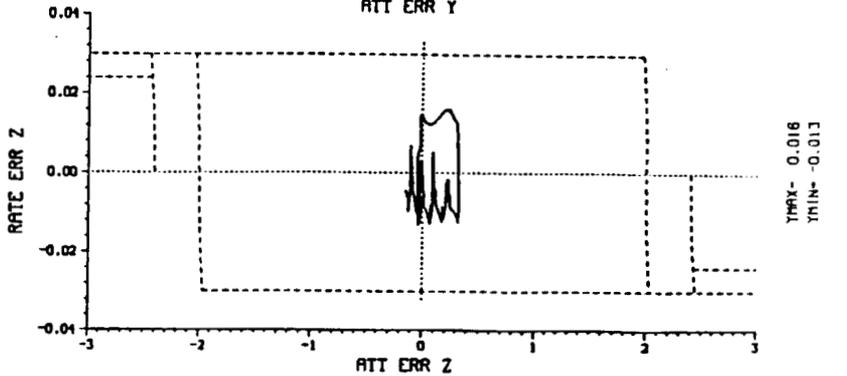
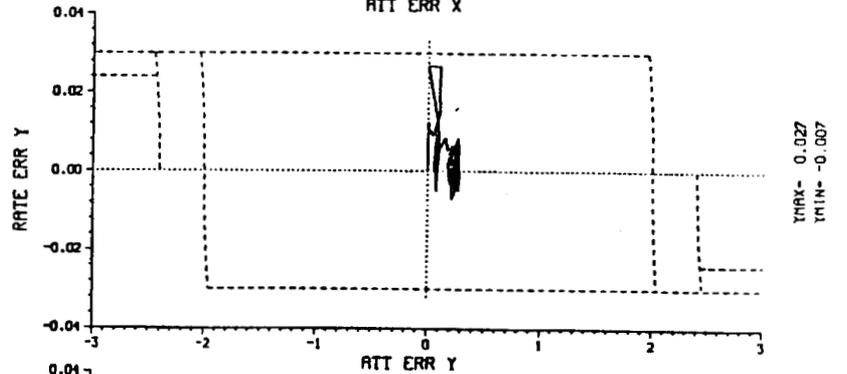
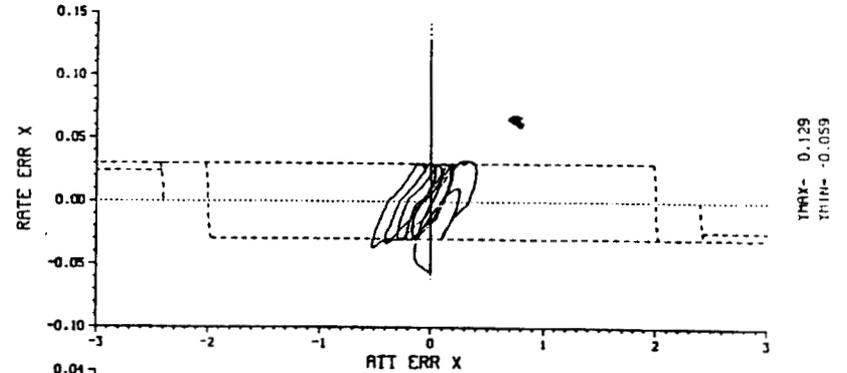


# Example of a FCS Interaction Test with A Self-sustained Limit Cycle

LOW50\$30 ESTIMATED, ACTUAL AND DESIRED ORBITER ATT. RATE  
IN DEG/SEC (solid=est,dash=true,dot=desired)



LOW50\$30 PHASE PLANE PLOTS  
DAP VARIABLES IN DEG AND DEG/SEC



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## Cost of Running the DRS

- Simulation of one minute of FCS/RMS interaction costs about \$15 when run at low priority over night.
- Normal priority for faster turn around is a factor of six more costly.
- For all its fidelity and capability, the DRS is reasonably efficient and economical.
- Nevertheless, the possible need to run hundreds of FCS interaction cases to map out stability regions was the motivation to develop an even more efficient simulation tailored to the needs of FCS interaction studies.

## Draper's LSAD Simulation

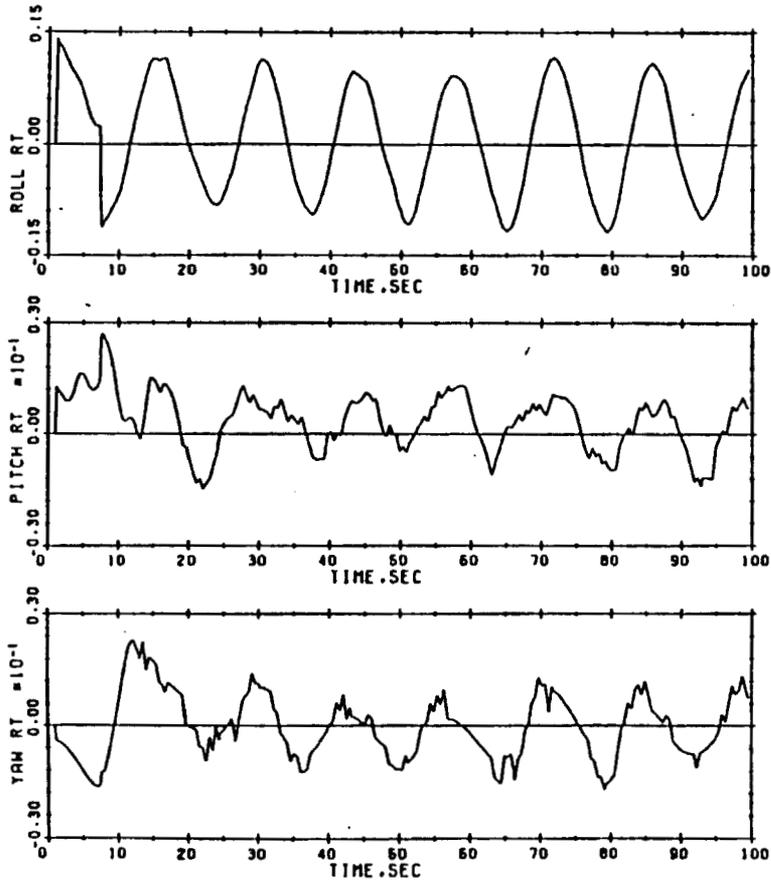
- All flexibility between the orbiter and the payload is lumped into six relative degrees of freedom. The arm is assumed to be massless. LSAD state vector has dimension six.
- A simplified algorithm is used to model the response of the joint servos. This algorithm can operate at an 80 ms time step as opposed to the 1ms DRS time step.
- Fidelity in the low frequency modes has been retained.
- Features have been added:
  - Ability to submit sets of position and attitude variation cases in a single batch,
  - Ability to start a simulation with the arm in an excited state.
- Cost of an LSAD run is about a factor of 10 less than the cost of a DRS run.

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# DRS to LSAD Comparison of Orbiter Rates During an FCS Stability Test

## DRS STABILITY TEST

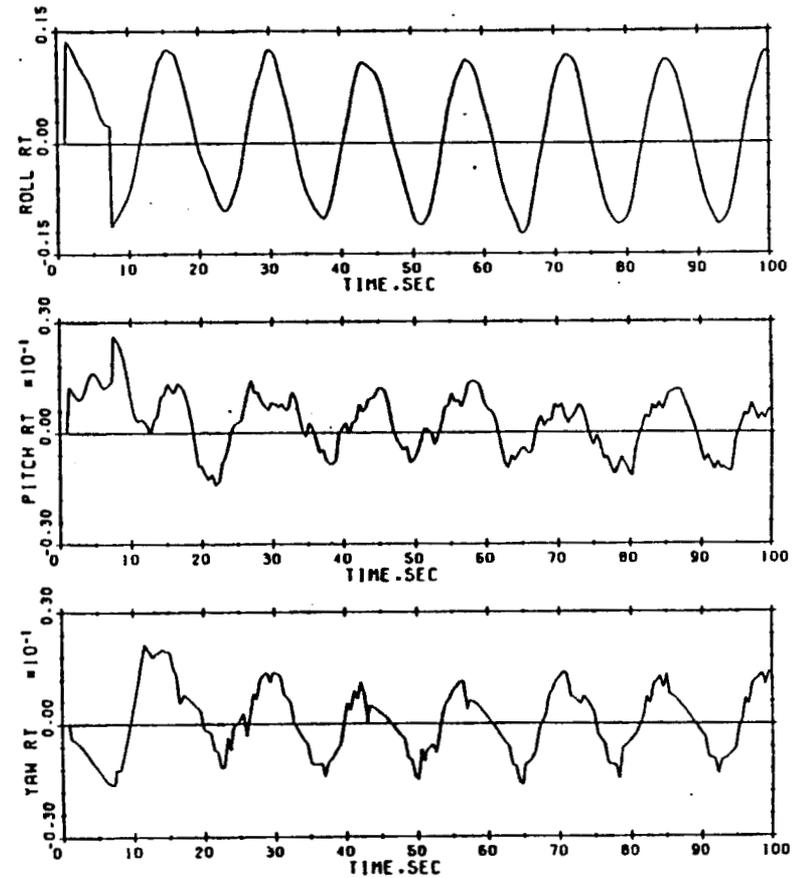
32K PL. B20501. BRAKES ON



## LSAD STABILITY TEST. RCS EXCITATION

32K PL. BRKS. B20501

DB= 2.000. RL= 0.010. DRB/JET/ARM PLOT



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N90-10101

**A PERFORMANCE COMPARISON OF INTEGRATION ALGORITHMS IN SIMULATING  
FLEXIBLE STRUCTURES**

By

R. M. Howe  
The University of Michigan  
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Ann Arbor, Michigan

**ABSTRACT**

Modeling of the dynamic vibration modes of a flexible structure can be achieved either by using a generalized coordinate for each mode considered in the simulation, or by discretizing the structure into a sufficiently large number of segments to provide the necessary modal accuracy. The accuracy and stability considerations in choosing appropriate numerical integration algorithms are different, depending on which modeling approach is utilized. In the generalized coordinate approach the frequency and shape of each mode is assumed to be known. The integration method should provide an accurate match to the modal frequency and damping, and should also exhibit sinusoidal transfer function errors which are acceptably small, especially for frequencies in the vicinity of the modal resonance. Since only those modes considered necessary for the required simulation fidelity are included as generalized coordinates, integrator stability for modes of higher frequency does not become an issue.

On the other hand, when the discretized structure approach is used, high frequency modes not of interest to the simulation will nevertheless be present. In this case it is important that the integration method not only provide satisfactory characteristic root and transfer function accuracy for the lower modes of interest, but also provide stable solutions with satisfactory damping for the higher modes which are not of interest.

In this paper asymptotic formulas for the characteristic root errors as well as transfer function gain and phase errors are presented for a number of traditional integration methods and for several new integration methods. Normalized stability regions in the  $\lambda h$  plane are compared for the various methods. In particular, it is shown that a modified form of Euler integration with root matching is an especially efficient method for simulating lightly-damped structural modes. The method has been used successfully for structural bending modes in the real-time simulation of missiles. Performance of this algorithm is compared with other special algorithms, including the state-transition method. A predictor-corrector version of the modified Euler algorithm permits it to be extended to the simulation of nonlinear models of the type likely to be obtained when using the discretized structure approach.

Performance of the different integration methods is also compared for integration step sizes larger than those for which the asymptotic formulas are valid. It is concluded that many traditional integration methods, such as RD-4, are not competitive in the simulation of lightly damped structures.

# A Performance Comparison of Integration Algorithms in Simulating Flexible Structures

R. M. Howe

The University of Michigan, Ann Arbor, Michigan  
and

Applied Dynamics International, Ann Arbor, Michigan

## ABSTRACT

In this paper a number of integration algorithms, including several new methods, are considered for the simulation of flexible structures. The effectiveness of the different algorithms is assessed by considering the characteristic root errors which they produce, the sinusoidal transfer function gain and phase errors, the stability regions, and the execution times. The suitability of the various algorithms for simulations with real-time inputs is also noted. When the structural modes in a simulation are represented by generalized (normal) coordinates, the selection criteria for integration methods are somewhat different than the criteria when the structure is discretized into a sufficiently large number of segments to provide the necessary modal accuracy. In this paper asymptotic formulas for the characteristic root errors as well as transfer function gain and phase errors are presented for a number of traditional integration methods and for several new integration methods. Normalized stability regions in the  $\lambda h$  plane are compared for the various methods. In particular, it is shown that a modified form of Euler integration with root matching is an especially efficient method for simulating structural modes. The method has been used successfully for structural bending modes in the real-time simulation of missiles. A predictor version of the modified Euler algorithm permits it to be extended to the simulation of nonlinear models of the type likely to be obtained when using the discretized structure approach.

## 1. Introduction

Modeling of the dynamic vibration modes of a flexible structure can be achieved either by using a generalized coordinate for each mode considered in the simulation, or by discretizing the structure into a sufficiently large number of segments to provide the necessary modal accuracy. In this latter case the mathematical model for a flexible structure with  $N$  degrees of freedom has the following general form:

$$M(q)\ddot{q} + C(q, \dot{q}) + K(q) = F(t) \quad (1)$$

where  $q$  is an  $N$ -component position state vector,  $M(q)$  is the mass matrix,  $C(q, \dot{q})$  is the coriolis and centrifugal acceleration vector,  $K(q)$  is the elastic and gravity force vector, and  $F(t)$  is the external force vector. When the vibration modes of the structure are represented by normal

(generalized) coordinates, a coordinate  $x$  representing the time-varying amplitude of a given mode with undamped natural frequency  $\omega_n$  and damping ratio  $\zeta$  obeys the equation

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \omega_n^2\phi(t) \quad (2)$$

Here  $\phi(t)$  is the generalized force associated with the coordinate  $x$ . When a number of modes are present, there will in general also be terms in Eq. (2) which couple the mode of amplitude  $x$  with other structural modes.

The accuracy and stability considerations in choosing appropriate numerical integration algorithms for solving differential equations of the type shown in (1) or (2) will be different. In the generalized coordinate approach of Eq. (2) the frequency and shape of each mode is assumed to be known. The integration method should provide an accurate match to the modal frequency and damping, and should also exhibit sinusoidal transfer function errors which are acceptably small, especially for frequencies in the vicinity of the modal resonance. Since only those modes considered necessary for the required simulation fidelity are included as generalized coordinates, integrator stability for higher frequency modes which are not of interest does not become an issue.

On the other hand, when the discretized structure approach represented by Eq. (1) is used, high frequency modes which are unimportant in the simulation will nevertheless be present. In this case it is important that the integration method not only provide satisfactory characteristic root and transfer function accuracy for the lower modes of interest, but also provide stable solutions with satisfactory damping for the higher modes which are not of interest.

In this paper asymptotic formulas for the characteristic root errors as well as transfer function gain and phase errors are presented for a number of traditional integration methods and for several new integration methods. Normalized stability regions in the  $\lambda h$  plane are compared for the various methods, where  $\lambda$  is an eigenvalue associated with the linearized perturbation equations of the structure and  $h$  is the integration step size. . In particular, it is shown that a modified form of Euler integration with root matching is an especially efficient method for simulating lightly-damped structural modes. The method has been used successfully for structural bending modes in the real-time simulation of missiles. Predictor versions of the modified Euler algorithm permit it to be extended to the simulation of nonlinear models of the type likely to be obtained when structures are represented by means of discretization. The stability regions in the  $\lambda h$  plane for the modified Euler methods are especially well suited to the requirements when using the discretized structure approach.

## 2. Dynamic Error Measures for Integration Algorithms

In comparing different integration methods for the simulation of flexible structures it is important to utilize meaningful performance measures which permit general conclusions to be drawn regarding the expected dynamic errors associated with each method. Our dynamic error analysis will be based on linearized perturbation equations derived from the original nonlinear differential equations used to model the structure. Thus we will assume that the system

eigenvalues are known, as well as the transfer functions relating specific input-output pairs. We will further assume that the simulation uses a fixed integration step size  $h$ . This is necessary in the case of a real-time simulation. It is likely to be true over a large number of steps even when a variable-step integration method is used in simulating a flexible structure. For linearized equations and a fixed integration step size we can apply the method of  $z$  transforms to analyze the dynamic errors resulting from specific integration algorithms [1,2]. There are two error measures which quite useful in predicting overall dynamic accuracy in the simulation. The first is the fractional error in each characteristic root (eigenvalue) of the digital simulation, defined as

$$\text{Fractional error in characteristic root} = e_\lambda = \frac{\lambda^* - \lambda}{\lambda} \quad (3)$$

where  $\lambda$  is the characteristic root of the continuous system being simulated and  $\lambda^*$  is the equivalent characteristic root for the digital simulation. For the case of complex roots (of which there will be many conjugate pairs in the simulation of a flexible structure) it is more appropriate to determine the fractional error,  $e_\omega$ , in root frequency and the damping ratio error,  $e_\zeta$ . Thus we define

$$e_\omega = \frac{\omega_d^* - \omega_d}{\omega_d}, \quad e_\zeta = \zeta^* - \zeta \quad (4)$$

Here  $\omega_d^*$  and  $\omega_d$  represent the frequencies of the digital and continuous system roots, respectively, while  $\zeta^*$  and  $\zeta$  represent the damping ratios for the digital and continuous system roots, respectively.

The second dynamic error measure of significance is the fractional error in digital system transfer function for sinusoidal inputs of frequency  $\omega$ . For any input-output pair let  $H(s)$  be the transfer function of the continuous system and  $H^*(z)$  be the  $z$  transform of the digital system that results when a particular integration algorithm is used. Then the fractional error in sinusoidal transfer function is given by [3]

$$\frac{H^*(e^{j\omega h})}{H(j\omega)} - 1 = e_M + je_A \quad (5)$$

For simulations of any reasonable accuracy the magnitude of this fractional error will be small compared with unity, in which case it is easily shown that the real part,  $e_M$ , is equal approximately to the fractional error in gain and the imaginary part,  $e_A$ , is equal to the phase error of the transfer function [3].

For any numerical integration algorithm the integrator transfer function for sinusoidal inputs of frequency  $\omega$  can be written approximately as [3]

$$H_I^*(e^{j\omega h}) \cong \frac{1}{j\omega h [1 + e_I(j\omega h)^k]}, \quad \omega h \ll 1 \quad (6)$$

where  $h$  is the integration step size. Since  $1/(j\omega h)$  is the ideal integrator transfer function, it is apparent that the term  $e_I(j\omega h)^k$  represents the integrator error. For Adams-Bashforth predictor and Adams-Moulton two-pass predictor-corrector algorithms of order 2, 3, and 4, integration methods that are candidates for simulation of flexible structures, the error coefficient  $e_I$  and algorithm order  $k$  are listed in Table 1.

**Table 1. Integrator Transfer Function Error Parameters for AB Predictor and AM Predictor Corrector Algorithms**

<u>Method</u>	<u>Error Coefficient, <math>e_I</math></u>	<u>Algorithm Order, <math>k</math></u>
AB-2	$\frac{5}{12}$	2
AB-3	$\frac{3}{8}$	3
AB-4	$\frac{251}{720}$	4
AM-2	$\frac{1}{12}$	2
AM-3	$\frac{1}{24}$	3
AM-4	$\frac{19}{720}$	4

In terms of  $e_I$  and  $k$  the following formula for  $e_\lambda$ , the fractional error in characteristic root as defined earlier in Eq. (3), can be derived [3]:

$$e_\lambda = \frac{\lambda^* - \lambda}{\lambda} \cong -e_I(\lambda h)^k, \quad |\lambda h| \ll 1 \quad (7)$$

It is apparent that  $e_\lambda$  is directly proportional to the integrator error coefficient,  $e_I$ . For complex characteristic roots equivalent asymptotic formulas for the root frequency and damping errors,  $e_\omega$  and  $e_\zeta$ , as defined in Eq. (4), can be derived [3]. As in Eq. (7), the errors are proportional to  $e_I|\lambda h|^k$ .

For digital simulation of a first order system with transfer function  $H(s) = 1/(s-\lambda)$  the fractional error in the transfer function for sinusoidal inputs, as defined in Eq. (5), can also be derived in terms of the integrator error parameters  $e_I$  and  $k$  [3]. From this result the following asymptotic formulas are obtained for  $e_M$ , the fractional error in transfer function gain, and  $e_A$ , the transfer function phase error:

$$\text{For } k \text{ odd, } e_M \equiv (-1)^{\frac{k+1}{2}} \frac{\omega \lambda e_I}{\omega^2 + \lambda^2} (\omega h)^k, \quad e_A \equiv (-1)^{\frac{k+1}{2}} \frac{\omega^2 e_I}{\omega^2 + \lambda^2} (\omega h)^k, \quad \omega h \ll 1 \quad (8)$$

$$\text{For } k \text{ even, } e_M \equiv -(-1)^{\frac{k}{2}} \frac{\omega^2 e_I}{\omega^2 + \lambda^2} (\omega h)^k, \quad e_A \equiv (-1)^{\frac{k}{2}} \frac{\omega \lambda e_I}{\omega^2 + \lambda^2} (\omega h)^k, \quad \omega h \ll 1 \quad (9)$$

Here the errors are proportional to  $e_I(\omega h)^k$ . Comparable asymptotic formulas for  $e_M$  and  $e_A$  can be derived for digital simulation of a second-order system with transfer function  $H(s) = 1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$  [3]. Again, the gain and phase errors are proportional to  $e_I(\omega h)^k$ .

The transfer function  $H(s)$  for any order linear system with both real and complex roots can be represented as the product of first and second-order transfer functions. In this case it can be shown that the asymptotic formulas for the digital system transfer function gain and phase errors is simply the sum of the individual first and second-order subsystem gain and phase errors, respectively, for predictor and predictor-corrector methods of the type shown in Table 1. If we simulate a flexible structure with a given integration method, this permits us to compute the linearized system gain and phase errors at the frequency  $\omega$  for any input-output pair as a function of integration step size  $h$ . In view of the reemerging popularity of frequency-domain methods for designing multiple input/multiple output control systems, this is a quite useful result. It permits us to estimate ahead of time for a given step size and integration method whether the simulation errors will be satisfactorily small. Conversely, for a given transfer function accuracy requirement, it allows us to compute the maximum allowable step size  $h$  for the simulation.

It should be noted that the methodology outlined above for determining characteristic root and transfer function errors for any order of linearized system from the simple integrator model given by Eq. (6) does not work in the case of multiple-pass, single step methods such as Runge-Kutta. This is because the results of individual passes within a single step in such methods depend on the particular form of the system transfer function. Asymptotic formulas for the root error parameters  $e_\lambda$ ,  $e_\omega$ , and  $e_\zeta$  can, of course, be derived separately for RK-2, RK-3, RK-4, and variations of these methods [3].

### 3. Modified Euler Integration Algorithms

In this section we describe some modifications of simple Euler integration which have potential advantages over conventional integration methods such as those listed in Table 1. First we introduce the concept of state variables defined at both integer and half integer sample times. Assume that the simulation of a mechanical degree of freedom with position state  $x$ , velocity state  $y$ , and acceleration  $a$  involves integrating the following simple state equations:

$$\dot{y} = a, \quad \dot{x} = y \quad (10)$$

Next assume that successive data points are defined at integer time samples in representing the acceleration  $a$  and position  $x$ , and at half-integer sample times in representing the velocity  $y$ . The following modified Euler algorithms can then be used for integration:

$$y_{n+1/2} = y_{n-1/2} + ha_n, \quad x_{n+1} = x_n + hy_{n+1/2} \quad (11)$$

The basic concept behind this modification of standard Euler integration is very simple; instead of using the state variable derivative defined at the beginning of the integration step, the method uses a state variable derivative defined halfway through the step. For this algorithm it is easy to show that the integrator error coefficient defined in Eq. (6) is given by  $e_I = 1/24$  and the order of the method is  $k = 2$ . Thus the accuracy of this single-pass algorithm is twice that of the two-pass AM-2 predictor-corrector. However, the algorithm does require that the velocity states be defined at half-integer sample times.

Let us apply this modified Euler method to the second-order system represented by Eq. (2) for the generalized coordinate  $x$ . We can replace Eq. (2) by the following two state equations:

$$\dot{y} = \omega_n[\phi(t) - x - 2\zeta y], \quad \dot{x} = \omega_n y \quad (12)$$

By analogy with Eq. (11) the modified Euler difference equations become:

$$y_{n+1/2} = y_{n-1/2} + \omega_n h(\phi_n - x_n - 2\zeta y'_n), \quad x_{n+1} = x_n + \omega_n h y_{n+1/2} \quad (13)$$

Since  $y_n$  is not explicitly computed, it is necessary to substitute an estimate  $y'_n$  in the damping term on the right side of the  $y_{n+1/2}$  equation. There are many ways in which the  $y'_n$  estimate can be computed. In Table 2 we list four candidate methods.

**Table 2. Methods for Estimating the Velocity  $y_n$  in Modified Euler Integration**

<u>Method</u>	<u>Formula for the Estimate, <math>y'_n</math></u>
1. Average of $y_{n+1/2}$ and $y_{n-1/2}$	$y'_n = \frac{y_{n+1/2} + y_{n-1/2}}{2}$
2. Extrapolation using $y_{n-1/2}$ and $y_{n-3/2}$	$y'_n = \frac{3}{2}y_{n-1/2} - \frac{1}{2}y_{n-3/2}$
3. Integration using $\dot{y}_{n-1}$ and $\dot{y}_{n-2}$	$y'_n = y_{n-1/2} + \frac{7}{8}\dot{y}_{n-1} - \frac{3}{8}\dot{y}_{n-2}$
4. Estimate based on $y_{n-1/2}$	$y'_n = y_{n-1/2}$

The estimate for  $y_n$  in the first method is simply based on averaging  $y_{n+1/2}$  and  $y_{n-1/2}$ . This is equivalent to utilizing trapezoidal integration for the damping term. Although this means that  $y_{n+1/2}$  now appears on both sides of the difference equation in (13), for the linear system considered here it is possible to solve explicitly for  $y_{n+1/2}$ , as we will see in the next section. In the second method the estimate for  $y_n$  is based on a linear extrapolation from  $y_{n-1/2}$  and  $y_{n-3/2}$ . This is equivalent to using trapezoidal integration for the damping term. Since  $y_{n+1/2}$  now appears only on the left side of the difference equation in (13), this method can be used in the simulation of equations where  $dy/dt$  is a nonlinear function of  $y$ . This is also true for the third and fourth methods. The third is based on a second-order predictor integration over the interval  $h/2$ , starting with  $y_{n-1/2}$  and using  $dy/dt$  at the  $n-1$  and  $n-2$  intervals. This is equivalent to estimating  $y_n$  with quadratic extrapolation based on  $y_{n-1/2}$ ,  $y_{n-3/2}$  and  $y_{n-5/2}$ . In the fourth method we simply use  $y_{n-1/2}$  as our estimate for  $y_n$ . This is equivalent to Euler integration for the damping term.

#### 4. Modified Euler Integration with Trapezoidal Damping

We have seen in Table 2 that the velocity estimate  $y'_n$  for the modified Euler difference equations in (13) can be based on the average of  $y_{n+1/2}$  and  $y_{n-1/2}$ . Thus

$$y'_n = \frac{y_{n+1/2} + y_{n-1/2}}{2} \quad (14)$$

As noted earlier, this is equivalent to utilizing trapezoidal integration for the damping term. Although this means that  $y_{n+1/2}$  now appears on both sides of the difference equation, for the linear system considered here it is possible to solve explicitly for  $y_{n+1/2}$ . In this way we obtain the following equations:

$$y_{n+1/2} = C_1 y_{n-1/2} + C_2 (\phi_n - x_n), \quad x_{n+1} = x_n + \omega_n h y_{n-1/2} \quad (15)$$

where

$$C_1 = \frac{1 - \zeta \omega_n h}{1 + \zeta \omega_n h}, \quad C_2 = \frac{\omega_n h}{1 + \zeta \omega_n h} \quad (16)$$

From the method of z transforms applied to Eqs. (15) and (16) we obtain the following asymptotic formulas for the frequency and damping ratio errors of the digital simulation [4]:

$$e_\omega = \frac{\omega_d^* - \omega_d}{\omega_d} \cong \frac{1 + 4\zeta^2 - 8\zeta^4}{24(1 - \zeta^2)} \omega_n^2 h^2, \quad e_\zeta = \zeta^* - \zeta \cong \frac{\zeta}{24} (4\zeta^2 - 1), \quad \omega_n h \ll 1 \quad (17)$$

The transfer function gain and phase errors are given approximately by

$$\text{Fractional gain error} = \frac{|H^*|}{|H|} - 1 = e_M \cong \frac{-\frac{\zeta\omega}{\omega_n} \left[ 1 - \frac{\omega^2}{\omega_n^2} \right]}{\left[ 1 - \frac{\omega^2}{\omega_n^2} \right]^2 + \left[ \frac{2\zeta\omega}{\omega_n} \right]^2} (\omega h), \quad \omega h \ll 1 \quad (18)$$

$$\text{Phase error} = e_A \cong \frac{\frac{2\zeta^2\omega^2}{\omega_n^2}}{\left[ 1 - \frac{\omega^2}{\omega_n^2} \right]^2 + \left[ \frac{2\zeta\omega}{\omega_n} \right]^2} (\omega h), \quad \omega h \ll 1 \quad (19)$$

The characteristic root errors in Eq. (17) and the transfer function gain and phase errors in Eqs. (18) and (19) are comparable with those for AM-2 integration for the same step size  $h$  [3]. Yet AM-2 is a two-pass method whereas the modified Euler with trapezoidal damping, as used here, is a single-pass method. Thus it will take only half as long to execute as AM-2 while producing comparable accuracy. Its accuracy is approximately 5 times better than the accuracy of AB-2 integration when applied to the same second-order system.

The accuracy of modified Euler integration when applied to a linear second-order system can be further improved by the technique of root matching, which was originally employed by Fowler to improve the performance of conventional Euler integration [5]. By taking the  $z$  transform of Eqs. (15) and (16) we can obtain exact analytic formulas for the undamped natural frequency  $\omega_n^*$  and damping ratio  $\zeta^*$  in terms of  $\omega_n$  and  $\zeta$ . From these formulas we can solve for  $\omega_n$  and  $\zeta$  in terms of  $\omega_n^*$  and  $\zeta^*$ . If in these formulas we then replace  $\omega_n$  and  $\zeta$  by  $\omega_n'$  and  $\zeta'$ , respectively, and  $\omega_n^*$  and  $\zeta^*$  by  $\omega_n$  and  $\zeta$ , respectively, we obtain the following [4]:

$$\omega_n' = \frac{1}{h} \sqrt{2 - \frac{2\cos(\omega_n h \sqrt{1 - \zeta^2})}{\cosh(\zeta \omega_n h)}} \quad (20)$$

$$\zeta' = \frac{\tanh(\zeta \omega_n h)}{\omega_n' h} \quad (21)$$

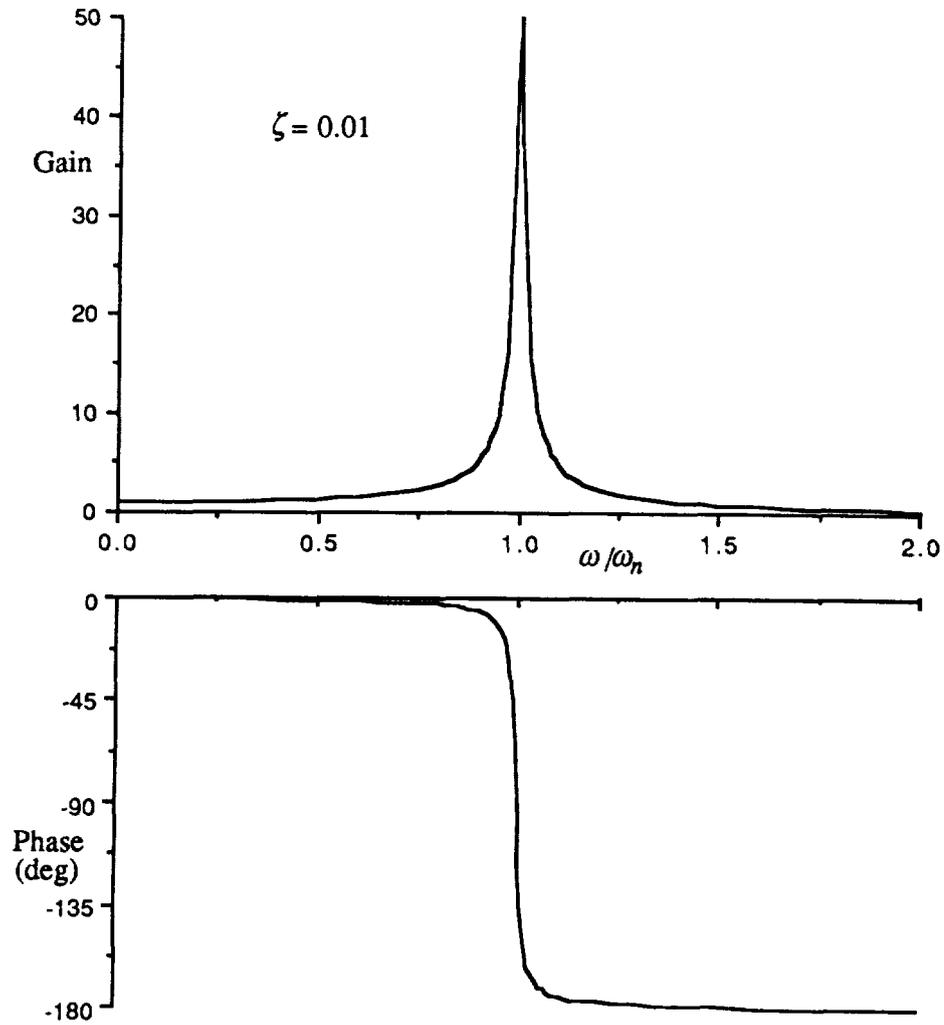
If  $\omega_n'$  and  $\zeta'$  from these formulas are used instead of  $\omega_n$  and  $\zeta$  in Eqs. (15) and (16), the resulting digital simulation will exhibit  $\omega_n^*$  and  $\zeta^*$  values which exactly match the  $\omega_n$  and  $\zeta$  of the continuous system being simulated. For a given step size  $h$  the  $\omega_n'$ ,  $\zeta'$ ,  $C_1$  and  $C_2$  can be precomputed, so that each integration step in simulating the second-order system only requires 3 multiplies and 2 adds, as before. Now the characteristic roots of the digital simulation will be exactly equal to those of the continuous system, regardless of the integration step size  $h$ . The approximate formulas for the transfer function gain and phase errors are given by [4]:

$$e_M \cong \frac{1}{24} (\omega h)^2, \quad e_A \cong -\frac{\zeta \omega_n}{6\omega} (\omega_n h)^2, \quad \omega h \ll 1 \quad (22)$$

Note that the fractional error in gain,  $e_M$ , is completely independent of the damping ratio  $\zeta$ , and the phase error  $e_A$  approaches zero as  $\zeta$  approaches zero. Thus our modified Euler algorithm with root matching will be especially effective in simulating lightly-damped second-order systems, as will be the case in structural modes. This is illustrated in Figure 1, where gain and phase versus frequency for a second-order system with  $\zeta = 0.01$  are plotted. Because of the sharp resonant peak in gain and the extremely rapid change in phase as  $\omega$  passes through  $\omega_n$ , it is very critical that both the natural frequency and damping ratio of the digital simulation match that of the continuous system. The table at the bottom of the figure shows the transfer function errors for input frequencies in the vicinity of  $\omega_n$  for the specific case of  $\omega_n h = 0.5$ , which corresponds to only 2 integration steps per radian or 12.57 steps per cycle. Shown in the table are the gain and phase errors based on both an exact calculation from the system  $z$  transform,  $H^*(e^{j\omega h})$ , as well as the approximate formulas of Eq. (31). Note how closely the approximate calculations agree with the exact, even for the example here for which  $\omega h = 0.5$ .

Until now we have only analyzed the dynamic performance of the modified Euler method in the frequency domain. This has been accomplished by examining the gain and phase errors of the transfer function for sinusoidal inputs. We now consider the errors in computed response of the second-order system to a unit-step input. Figure 2 shows the errors which result when using RK-2 integration (Heun's method); modified Euler with trapezoidal integration for the damping term, i.e., Eqs. (15) and (16); and modified Euler with root matching, i.e.,  $\omega_n'$  and  $\zeta'$  from Eqs. (20) and (21) substituted for  $\omega_n$  and  $\zeta$  in Eqs. (15) and (16). For the example in the figure the damping ratio  $\zeta = 0.707$  and the integration step size is given by  $\omega_n h = 0.5$ . The results show that the RK-2 errors are 4 to 10 times larger than the modified Euler errors. It should also be noted that RK-2 is a two-pass method, that is, it requires two evaluations of the state-variable derivatives per integration step. It follows that RK-2 will take approximately twice as long to execute per integration step as the single-pass modified Euler methods. To provide the same output integration frame rate in real time the RK-2 method will therefore require twice the mathematical step size  $h$  in comparison with the modified Euler methods considered here. This will further increase by a factor of 4 the RK-2 errors relative to the modified Euler errors in Figure 2.

The modified Euler results shown in Figure 2 were obtained using an initial step of  $h/2$  in integrating  $dy/dt$  to obtain  $y$ . After one integration step this provides the calculation of  $y_{1/2}$  starting with the initial condition  $y_0$ . The step size is taken as  $h$  for all subsequent  $dy/dt$  integration steps. This results in successive velocity values representing  $y$  at half-integer step times, consistent with the concept introduced in the beginning of this section.



		Fractional Gain Error		Phase Error (radians)	
		Exact	Eq. (31)	Exact	Eq. (31)
$\omega_n h = 0.5$	0.7	0.01040	0.01021	-0.000296	-0.000292
	0.9	0.01727	0.01888	-0.000381	-0.000375
12.57 steps per cycle)	1.0	0.02137	0.02083	-0.000424	-0.000317
	1.1	0.02592	0.02521	-0.000467	-0.000458
	1.4	0.04240	0.04083	-0.000595	-0.000583

**Figure 1.** Frequency response of lightly-damped second-order system using modified Euler integration with root matching,  $\omega_n h = 0.5$ .

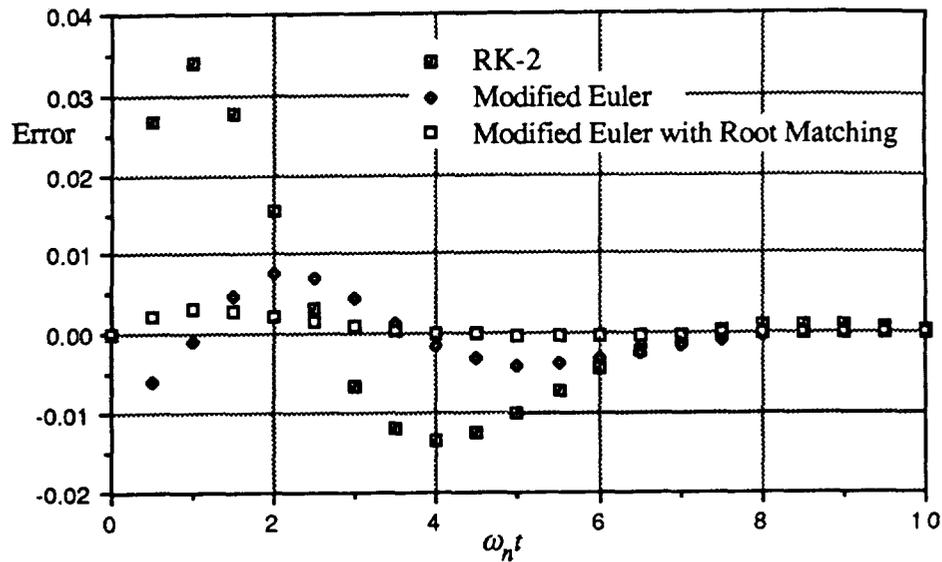


Figure 2. Unit step response errors in simulating a second-order system with damping ratio  $\zeta = 0.707$ , integration step size given by  $\omega_n h = 0.5$ .

### 5. Performance of Other Versions of Modified Euler Integration

In this section we present the asymptotic formulas for characteristic root and transfer function errors when modified Euler integration is used to simulate a second-order system with methods 2, 3, or 4 in Table 2 utilized to calculate the velocity estimate  $y'_n$  in Eq. (13). For method 2, which is equivalent to AB-2 integration for the damping term, the following results are obtained for  $e_\omega$ , the fractional error in root frequency, and  $e_\zeta$ , the damping ratio error [6]:

$$e_\omega \cong \frac{1 - 32\zeta^2 + 40\zeta^4}{24(1 - \zeta^2)} (\omega_n h)^2, \quad e_\zeta \cong \frac{11\zeta - 20\zeta^3}{24} (\omega_n h)^2, \quad \omega_n h \ll 1 \quad (23)$$

These errors are significantly less than the errors when AB-2 is used for all integrations. For method 3 in Table 2, which uses a second-order predictor integration algorithm to compute  $y'_n$ , the following asymptotic formulas are obtained for the root frequency and damping errors:

$$e_\omega \cong \frac{(1 - 4\zeta^2)}{24} (\omega_n h)^2, \quad e_\zeta \cong \frac{\zeta - \zeta^3}{12} (\omega_n h)^2, \quad \omega_n h \ll 1 \quad (24)$$

The transfer function gain and phase errors are given by

$$e_M \equiv -\frac{2\frac{\omega^2}{\omega_n^2}\left[1 - 2\zeta^2 - \frac{\omega^2}{\omega_n^2}\right]}{\left[1 - \frac{\omega^2}{\omega_n^2}\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} \frac{(\omega_n h)^2}{24}, \quad e_A \equiv \frac{2\zeta\frac{\omega}{\omega_n}\left[1 + \frac{\omega^2}{\omega_n^2}\right]}{\left[1 - \frac{\omega^2}{\omega_n^2}\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} \frac{(\omega_n h)^2}{24}, \quad \omega h \gg 1 \quad (25)$$

In both Eqs. (24) and (25) the errors are a factor of two smaller than the corresponding errors when AM-2 is used for all integrations. In addition, the AM-2 algorithm is a two-pass method which will therefore take twice as long to execute on a given computer. For method 4 in Table 2, which is equivalent to using Euler integration for the damping term, the following formulas are obtained for the characteristic root and transfer function errors [4]:

$$e_\omega \equiv \frac{\frac{1}{2}\zeta - \zeta^3}{1 - \zeta^2} \omega_n h, \quad e_\zeta \equiv \frac{1}{2}\zeta^2 \omega_n h, \quad \omega_n h \ll 1 \quad (26)$$

$$e_M \equiv \frac{-\zeta\frac{\omega}{\omega_n}\left[1 - \frac{\omega^2}{\omega_n^2}\right]}{\left[1 - \frac{\omega^2}{\omega_n^2}\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} (\omega h), \quad e_A \equiv \frac{2\zeta^2\frac{\omega^2}{\omega_n^2}}{\left[1 - \frac{\omega^2}{\omega_n^2}\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} (\omega h), \quad \omega h \gg 1 \quad (26)$$

Note that the errors are all proportional to the first power of the step size  $h$ . This is because of the first-order Euler algorithm used for integration of the damping term. For  $\zeta = 0$ , however, the first-order errors in Eqs. 25) and (26) vanish, meaning that the errors become second-order in  $h$ . This is to be expected, since the conventional Euler integration plays no role when  $\zeta = 0$ . In fact it can be shown that when  $\zeta = 0$ , the digital solution will have zero damping regardless of the step size  $h$ .

When method 2,3, or 4 in Table 1 (or any other explicit method) is used to provide the estimate  $y'_n$  for the velocity state, the modified Euler method can be used as the algorithm for integrating the nonlinear state equations represented by (1). The vector difference equations become the following:

$$\dot{q}_{n+1/2} = \dot{q}_{n-1/2} + h\{F_n - [M(q_n)]^{-1}C(q_n, \dot{q}'_n) - K(q_n)\}, \quad q_{n+1} = q_n + h\dot{q}_{n+1/2} \quad (28)$$

We now turn to a consideration of integration algorithm stability.

## 6. Stability of Integration Methods

It has already been pointed out that the stability of numerical integration algorithms becomes an important consideration when the flexible structure is modeled by discretization. This is because the discretized model will contain high frequency modes which are unimportant in the simulation but can cause numerical instabilities for reasonable integration step sizes. For a given integration method the stability boundary in the  $\lambda h$  plane can be obtained by considering a simulation of the linear system with transfer function  $H(s) = 1/(s-\lambda)$ . From the difference equation the  $z$  transform,  $H^*(z)$ , is obtained. The stability boundary is defined by the  $\lambda h$  values for which the denominator of  $H^*(z)$  vanishes when  $|z| = 1$ . These  $\lambda h$  values can be obtained by letting  $z = e^{j\theta}$  in the denominator of  $H^*(z)$  and solving for  $\lambda h$  for  $\theta$  values ranging between 0 and  $\pi$ . When this is done for the AB predictor methods, the stability regions plotted in Figure 3 are obtained. The regions are symmetric with respect to the real axis so that only the upper half plane is shown. For any values of  $\lambda h$  lying outside the boundaries the digital simulation will be unstable. In the case of both AB-3 and AB-4 the boundary crosses over into the right half plane. This means that a continuous system with roots on the imaginary axis which correspond to undamped transients can exhibit stable transients in the digital solution. Put another way, it means that AB-3 and AB-4 solutions will exhibit more damping than the continuous system being simulated. This is actually desirable in the case of the high frequency modes which are not of interest in a given simulation. On the other hand the AB predictor methods do not have particularly large stability regions and therefore do not permit very large integration step sizes  $h$  compared with the reciprocal magnitude,  $1/|\lambda|$ , of the largest eigenvalues in the simulation.

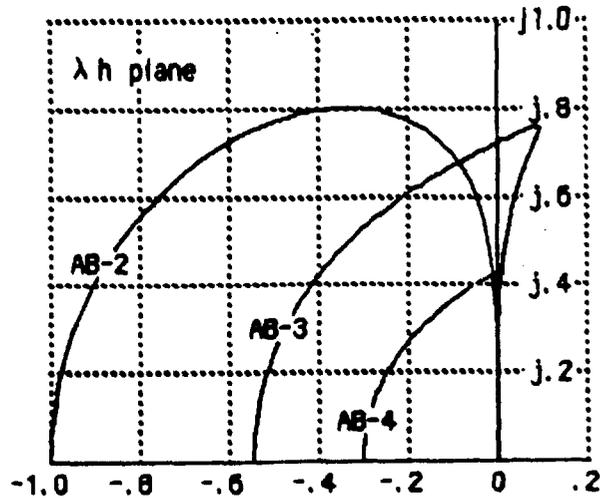


Figure 3. Stability boundaries for AB predictor integration.

In Figure 4 the stability boundaries are shown for the two-pass AM predictor-corrector methods. Although the boundaries are considerably larger than those for the AB methods, it must be remembered that the AM algorithms will take twice as long to execute. Thus the boundaries should be reduced by a factor of two for a valid comparison with AB-2. When this

is done, the AM-2 and 3 boundaries actually fall inside the AB-2 and 3 boundaries, although the AM-4 boundary still lies outside the AB-4 boundary. In all cases the higher-order algorithms exhibit less stability and are therefore unlikely to be candidates for simulating flexible structures.

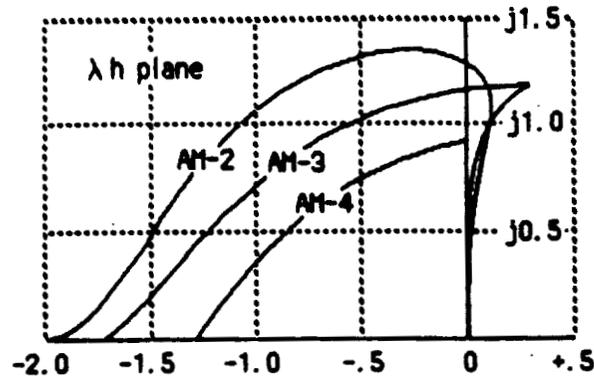


Figure 4. Stability boundaries for two-pass AM predictor-corrector integration.

For comparison purposes the stability boundaries for RK-2, 3 and 4 are shown in Figure 5. We recall that these algorithms require 2, 3 and 4 passes, respectively, through the state equations per integration step. Thus for proper comparison with single-pass methods the boundaries shown should be reduced by factors of 2, 3 and 4, respectively. When this is done, the RK-2 boundary roughly matches the AB-2 boundary, while the RK-3 and RK-4 boundaries still fall outside the AB-3 and 4 boundaries, respectively.

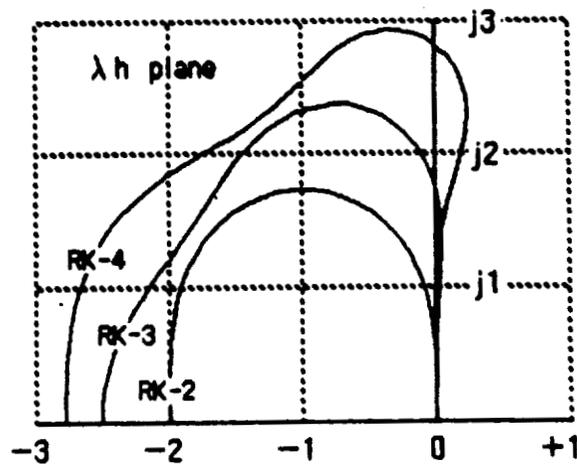


Figure 5. Stability boundaries for Runge-Kutta integration methods.

Finally, in Figure 6 are shown the stability boundaries for various modified Euler methods, as described in Sections 4 and 5. The trapezoidal damping case corresponds to method 1 in Table 2, the Euler damping case to method 4, the AB-2 damping case to method 2, and the predictor damping case to method 3. Also shown for comparison purposes in Figure 6 are the stability boundaries for AB-2, AM-2 and RK-2, as presented earlier in Figures 3, 4 and 5, respectively. The AM-2 and RK-2 stability boundaries have been reduced by a factor of two to reflect the two passes per integration step required in the implementation of these methods. Note that all four of the Modified Euler methods in Figure 6 have stability regions which permit values of  $|\lambda h|$  up to 2 for lightly damped transients, e.g., eigenvalues near the imaginary axis. In this regard the methods are considerably superior to the AB-2, AM-2 and RK-2 algorithms and should perform especially well in the simulation of flexible structures.

It should also be noted that the modified Euler methods are ideally suited for real-time simulation in that they do not require inputs prior to their occurrence in real time. For example, if  $F(t)$  in Eq. (1) is a real time input, then the single-pass modified Euler algorithm of Eq. (28) only requires  $F_n$  at the beginning of the  $n$ th integration step. On the other hand, the AM predictor-corrector algorithms require  $F_{n+1}$  at the start of the second pass for the  $n$ th integration step, and  $F_{n+1}$  is not yet available in real time. There is, however, a modified version of the AM-2 predictor method which is compatible with real-time inputs [6]. The AB predictor methods are also compatible with real time inputs, and there are versions of RK-2 and RK-3 which permit real-time inputs [3]. RK-4 is not compatible with real-time inputs, since it requires  $F_{n+1/2}$  at the beginning of the second pass and  $F_{n+1}$  at the start of the fourth pass, in both cases prior to their availability in real time.

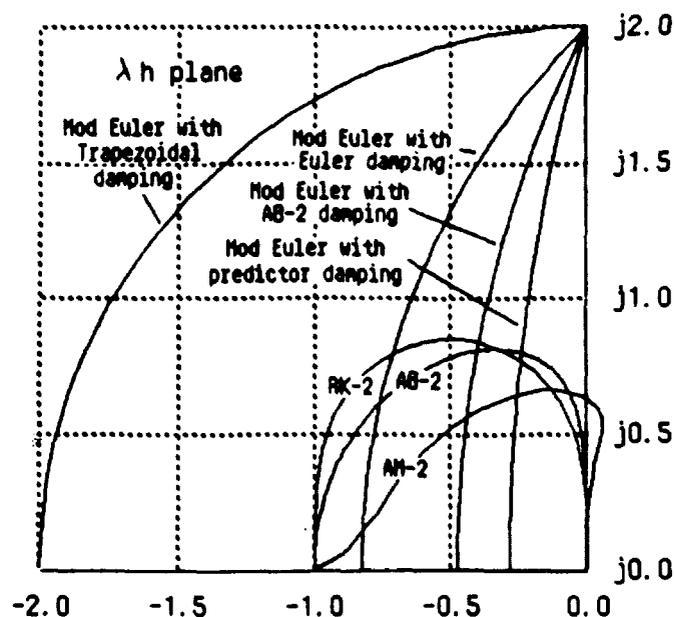


Figure 6. Stability boundaries for modified Euler integration methods.

## 7. Conclusions

In this paper we have considered the dynamic performance of integration methods in the context of simulating flexible structures. In terms of both characteristic root errors and transfer function errors, both important in such simulations, we have compared the performance of traditional integration methods with various versions of modified Euler integration. We have shown that modified Euler integration is especially effective in simulating lightly-damped structural modes. We have also shown that the modified Euler methods have very favorable stability boundaries in the  $\lambda h$  plane with respect to requirements in the simulation of lightly-damped modes. This is especially significant when a flexible structure is modeled by discretization as opposed to normal coordinates, since it will allow larger integration step sizes before the solution goes unstable due to the presence of higher modes which are unimportant to the simulation.

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# **OPTICAL PROCESSING FOR DISTRIBUTED SENSORS IN CONTROL OF FLEXIBLE SPACECRAFT**

**BY**

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**PRESENTATION FOR THE  
WORKSHOP ON COMPUTATIONAL ASPECTS IN THE  
CONTROL OF FLEXIBLE SYSTEMS  
WILLIAMSBURG, VA**

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OPTICAL PROCESSING FOR DISTRIBUTED SENSORS IN  
CONTROL OF FLEXIBLE SPACECRAFT

by

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ABSTRACT

~~This presentation will deal with a recent potential of distributed image processing.~~ Applications in the control of flexible spacecraft will be emphasized. Devices are currently being developed at NASA and in universities and industries that allow the real-time processing of holographic images. Within 5 years, it is expected that, in real-time, one may add or subtract holographic images at optical accuracy. Images are stored and processed in crystal mediums. The accuracy of their storage and processing is dictated by the grating level of laser holograms. It is far greater than that achievable using current analog-to-digital, pixel oriented, image digitizing and computing techniques.

Processors using image processing algebra can conceptually be designed to mechanize Fourier transforms, least square lattice filters, and other complex control system operations. Thus, actuator command inputs derived from complex control laws involving distributed holographic images can be generated by such an image processor. ~~The presentation will reveal plans for the development of a Conjugate Optics Processor for control of a flexible object.~~

## **PRESENTATION OUTLINE**

- **OBJECTIVES OF RESEARCH**
- **OPTICAL COMPUTING**
- **HOLOGRAPHIC INFORMATION STORAGE**
- **A PROPOSED EXPERIMENT IN THE CONTROL OF FLEXIBLE STRUCTURES**
- **CONCLUDING REMARKS**

#### PRESENTATION OUTLINE

This talk deals with plans to develop optical distributed sensors and computation techniques for the control of flexible structures. The plan is to develop the technology relative to active vibration damping of structural dynamics systems and, then, to demonstrate it with a closed-loop control system experiment in the laboratory. First, objectives of the research will be presented. Next, fundamentals of optical computing will be briefly overviewed and new capabilities in real-time holography and holographic information storage will be discussed. An experiment being developed at Langley will be presented with emphasis on the sensor concept and the operations unique to optical distributed processing. The talk concludes with a status summary of both the analytical and laboratory work in this area.

# **OBJECTIVES OF RESEARCH**

## **CONTROL OF FLEXIBLE STRUCTURES**

- DEVELOP DISTRIBUTED SENSORS**
- DEVELOP DISTRIBUTED COMPUTATION TECHNIQUES**
- USE NEW TECHNOLOGY FOR OPTICAL PROCESSING**

#### OBJECTIVES OF RESEARCH

Objectives of this research center around the interest in active control of flexible spacecraft structures. The intention is to develop distributed sensors for this application and complimentary distributed computing techniques. This has been enabled by recent advances in real-time holography using photorefractive crystals. Hence, some research objectives will be directed to laboratory development of optical processing using the new real-time holography techniques.

## **ADVANTAGES OF OPTICAL COMPUTING**

- **PARALLEL MULTIPLICATION**
- **PARALLEL ADDITION**
- **INTEGRATED DISTRIBUTED SENSING AND PROCESSING**

#### ADVANTAGES OF OPTICAL COMPUTING

Here we list some advantages of optical computing. Parallel multiplication and addition are possible via optical computing but, more importantly, the potential of distributed sensing and signal processing exists.

# OPTICAL COMPUTING TECHNIQUES

- ANALOGUE

COHERENT  
INCOHERENT

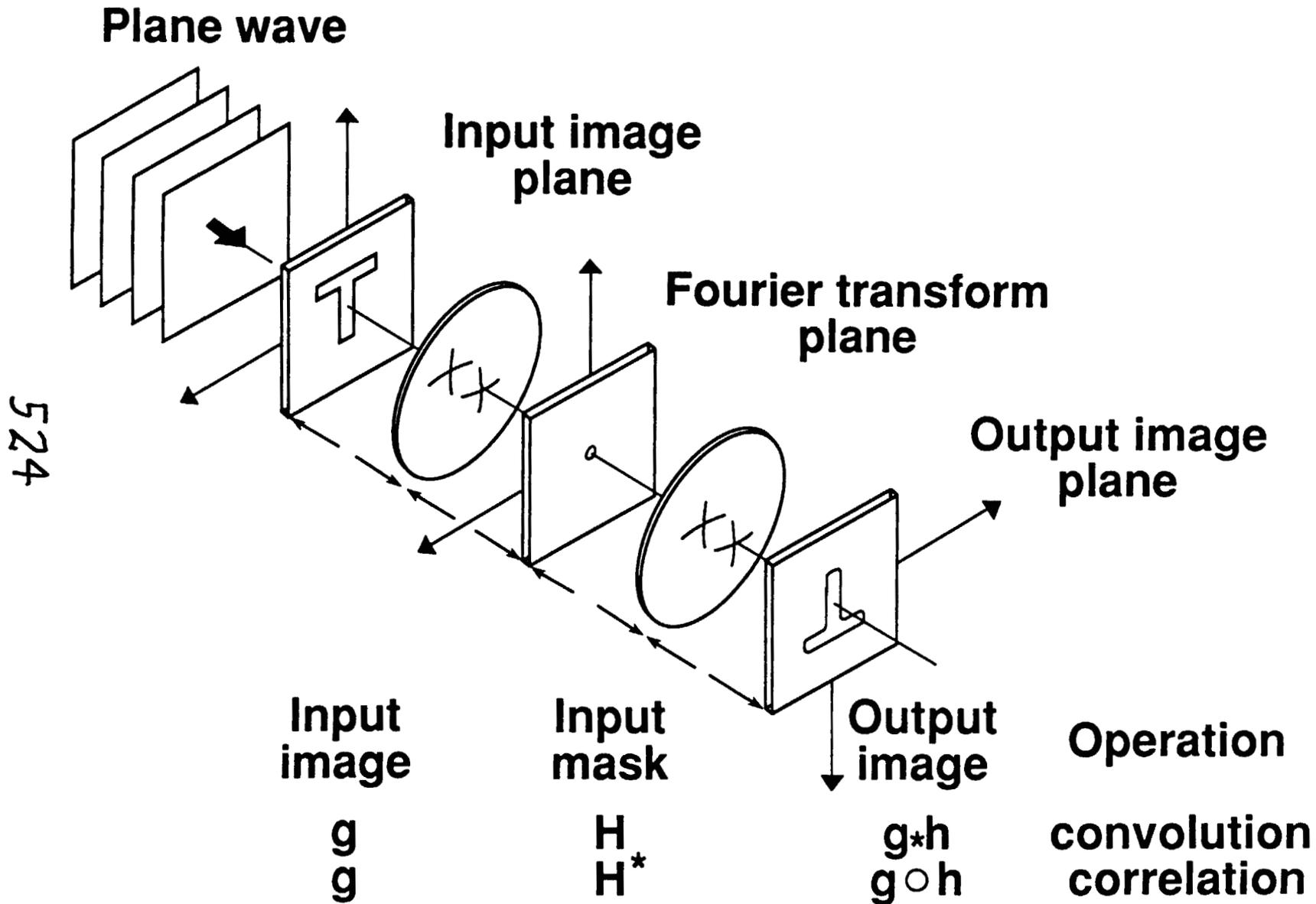
- DIGITAL

COHERENT  
INCOHERENT

#### OPTICAL COMPUTING TECHNIQUES

Both analogue and digital computing are possible via optics. These can be accomplished via both coherent and incoherent light. Our work will, however, be directed at analogue and coherent processing.

# COHERENT ANALOGUE OPTICAL COMPUTING



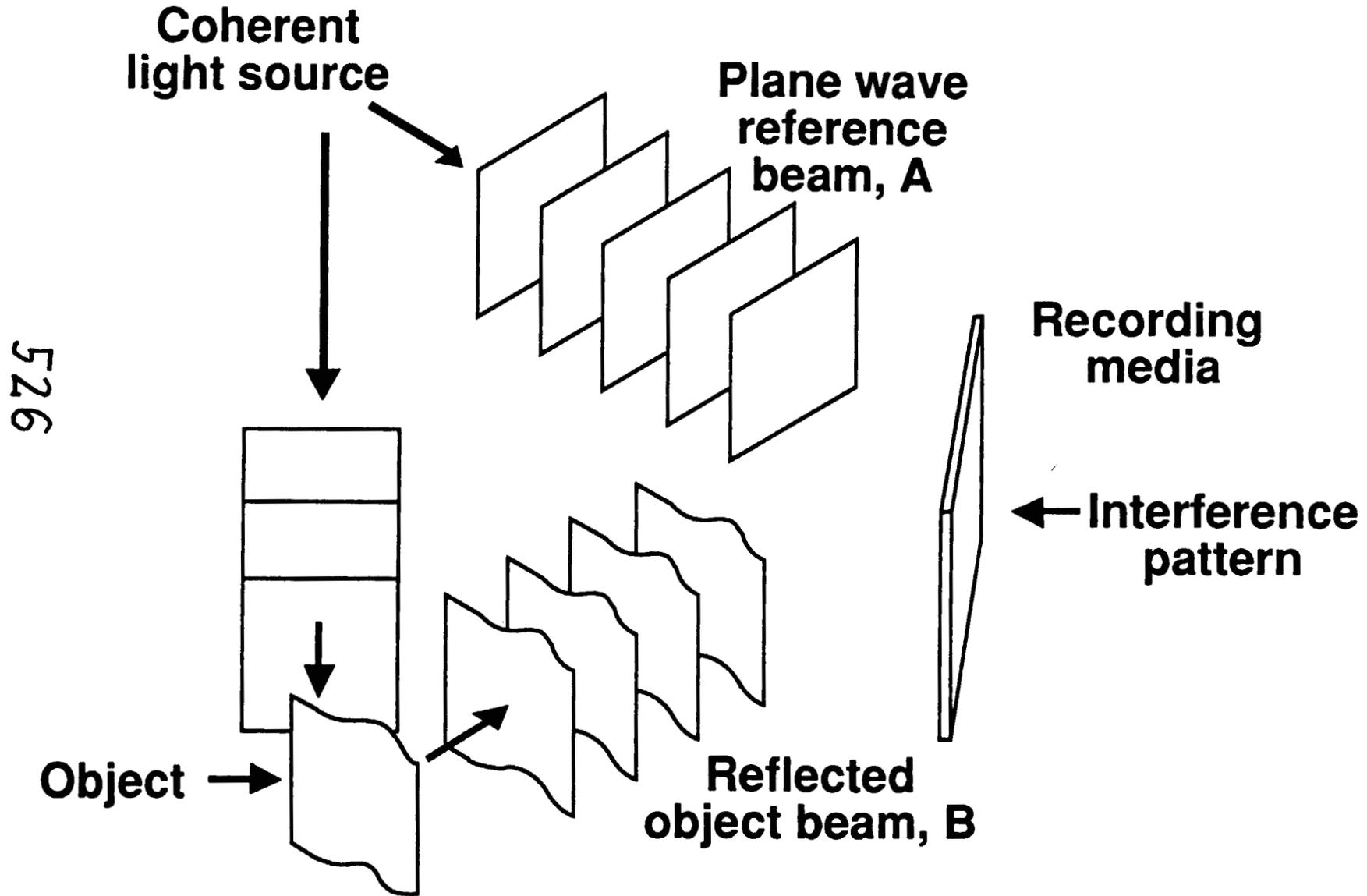
## COHERENT ANALOGUE OPTICAL COMPUTING

An example of a coherent analogue optical processor, a plane wave correlator, is illustrated in this slide. The plane wave correlator performs two-dimensional correlation or convolution operations on two input functions  $g(x,y)$ , and  $h(x,y)$ . The operations of correlation and convolution are performed in the following way. An image transparency (mask) with transmittance  $g(x,y)$ , shown here as an image of the letter "T", is placed in the front focal plane of lens L1 (Input Image Plane). An undistorted plane wave of coherent light is passed through the image mask which introduces phase and amplitude changes in the light. In the back focal plane of lens L1 the light amplitude distribution is proportional to the Fourier Transform of the transmittance of the input mask,  $g(x,y)$ . Thus, the back focal plane of L1, the Fourier Transform Plane, has coordinates which correspond to spatial frequencies, here denoted by  $k_x$  and  $k_y$ . If a second mask is inserted in the Fourier Transform Plane with transmittance  $H(k_x, k_y)$ , then the light propagated through this mask is  $GH$ , where  $G$  is the Fourier transform of  $g$ . The second lens L2 performs a Fourier transform of the product  $GH$ , and thus, the image which is formed in the Output Image Plane is the intensity of the convolution of  $g$  with  $h$ , where  $h$  is the inverse Fourier transform of the second mask  $H$ . Similarly, if the mask inserted in the Fourier transform plane is  $H^*$ , the image formed in the Output Image Plane is the correlation of  $g$  with  $h$ .

In the example shown in this slide, the input function is an image of the letter "T", and  $H(k_x, k_y)$  is a circular aperture. The image formed in the Output Image Plane is the convolution of the image "T" with the inverse Fourier transform of the circular aperture function  $H(k_x, k_y)$ . The image which appears in the Output Image Plane is an image of the letter "T" with the high spatial frequencies removed.

Although a simple function was used in this example for  $H$ , in general the mask or filter may be complex and contain both amplitude and phase variations. To construct a mask (filter) which contains both amplitude and phase information, interferometric techniques are used. The mask thus formed is a holographic filter.

# CONSTRUCTION OF A HOLOGRAPHIC FILTER



### CONSTRUCTION OF A HOLOGRAPHIC FILTER

Now consider the construction of a holographic filter (mask). A holographic filter (or hologram) is simply the recorded intensity distribution corresponding to the interference of two (or more) light waves. The basic geometry for constructing a hologram of a three dimensional diffusely reflecting object is shown in this slide. An incident coherent plane wave is divided into two plane waves. One, the reference beam, travels directly toward the recording media (A). The second, travels toward the object. The portion of the wave reflected from the object (B) is changed in both amplitude and phase. This reflected object beam interferes with the reference beam at the plane of the recording media. The recording media (usually film) records the light intensity distribution of the interference of A with B.

The interference pattern need not be constructed as described above, but may be generated artificially using a computer. Computer generated holograms can be recorded on film or on a spatial light modulator. The recorded hologram provides a means of modulating (i.e., filtering) a light beam.

# HOLOGRAM FILTER TRANSMITTANCE

Light intensity at the filter plane (film)

$$I(x,y) = |A(x,y)|^2 + |B(x,y)|^2 + 2A(x,y)B(x,y) \cos (P(x,y) - Q(x,y))$$

Filter transmittance

$$T_f(x,y) = T_b + C(|B|^2 + A^*B + AB^*)$$

### HOLOGRAPHIC FILTER TRANSMITTANCE

In this slide is the mathematical expression which describes the light intensity distribution at the recording media (previous slide). The expression for the intensity  $I(x,y)$  is composed of three terms. The first two terms are proportional to the intensities of the reference wave and object wave respectively. The third term is the interference term and depends upon the relative phases of the reference and object beams.

The second expression is the transmittance of the developed film. The first term in this expression is a constant which results from uniform exposure over the recording media, that is, the  $A^2$  term in the expression for the intensity. The remaining three terms are, respectively, the intensity of the object beam, the product of the complex conjugate of the field amplitude of the reference and the field amplitude of the object and the product of the field amplitude of the reference and the complex conjugate of the object.

When the recorded hologram is illuminated with a beam which is the exact duplicate of the reference wave, the filter reconstructs the original object wave,  $B$ . If the hologram is illuminated with a beam which is the conjugate of the reference wave, the filter reconstructs the conjugate of the object beam, or  $B^*$ .

## REAL-TIME HOLOGRAPHY WITH A VARIABLE MASK

- REQUIRED FOR MANY OPTICAL COMPUTING APPLICATIONS
- TWO TECHNIQUES AVAILABLE
  - SPATIAL LIGHT MODULATORS
  - DFWM IN PHOTOREFRACTIVE CRYSTALS

#### REAL-TIME HOLOGRAPHY WITH A VARIABLE MASK

For many optical processing applications it is necessary to change or update the holographic filter real-time. Two methods exist for recording a holographic filter real-time, using devices known as spatial light modulators, or using a nonlinear optical technique known as degenerate four wave mixing (DFWM) in a photorefractive crystal.

A spatial light modulator is a device composed of a matrix of individual pixels of variable optical transmittance. The transmittance of an individual pixel is adjusted by varying a voltage to the pixel. The pixel voltages are addressable in x and y. One example of a spatial light modulator is a liquid crystal display.

# SPATIAL LIGHT MODULATORS

## ADVANTAGES

- PROVEN TECHNOLOGY
- INTERFACE WITH DIGITAL SYSTEMS

## DISADVANTAGES

- LOW RESOLUTION

## SPATIAL LIGHT MODULATORS

Advantages of spatial light modulators is that they are a proven technology. They can also be driven conveniently with signals from digital computers. Also, their outputs can be conveniently interfaced via photodiodes to digital computers.

The disadvantages of these devices is that they have relatively low spatial resolution.

# DFWM IN PHOTOREFRACTIVE CRYSTALS

## ADVANTAGES

- HIGH RESOLUTION
- ALL OPTICAL
- SIGNAL AMPLIFICATION POSSIBLE

## DISADVANTAGES

- DIFFICULT TO WORK WITH (MANY VARIABLES)

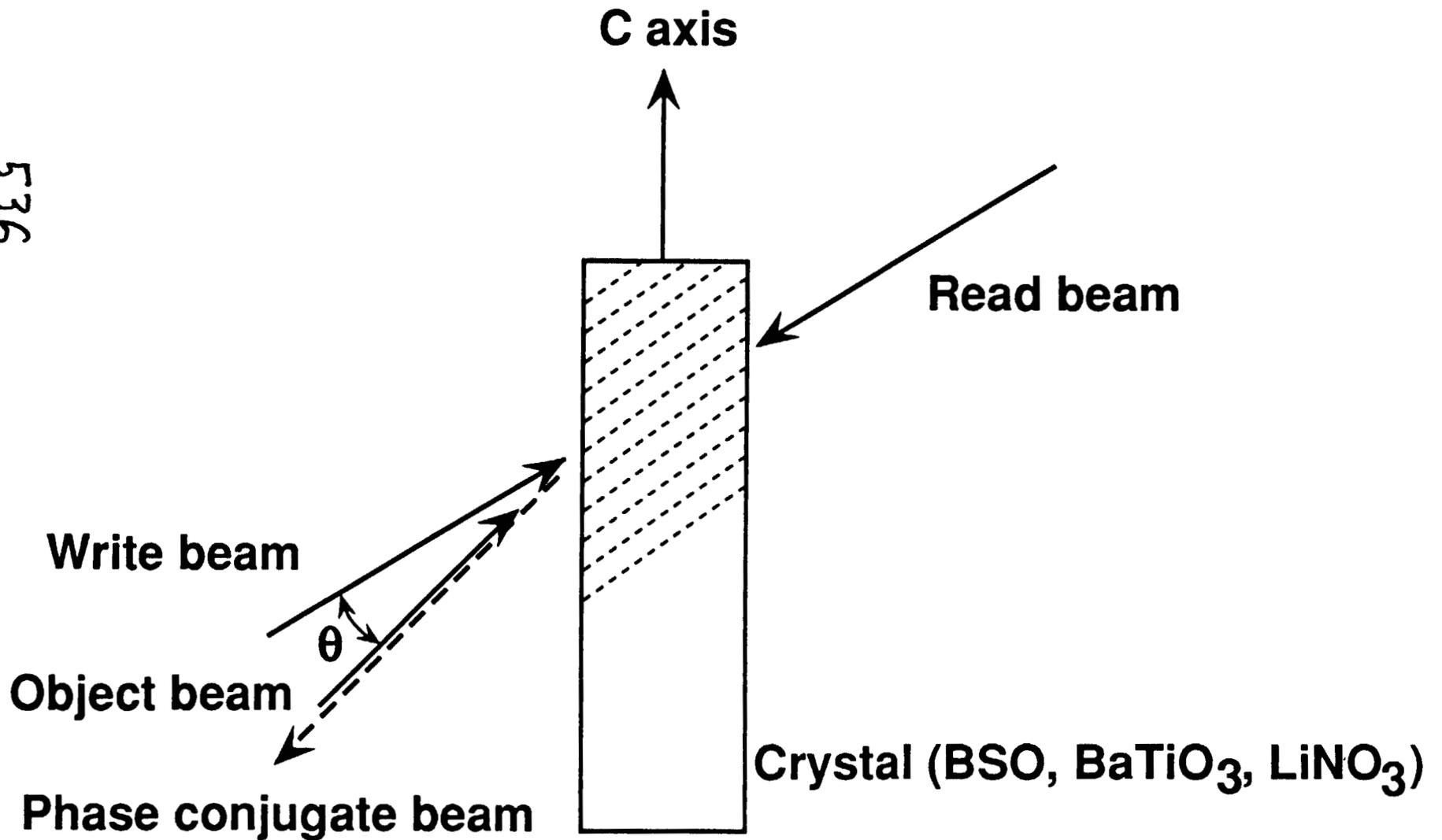
#### DFWM IN PHOTOREFRACTIVE CRYSTALS

Advantages of DFWM are: extremely high spatial resolution (at the level of atomic particles), all optical systems, and the potential for optical signal amplification. Signal amplification is perhaps one of the most important advantages. Typically, in an optical system about 4% of the light is lost as the beam passes through each optical element due to scattering, internal reflections, and so on. This can be reduced somewhat with optical coatings, but even with optical coatings some light is lost. The result is that the light beam will be significantly attenuated after passing through a relatively few number of components (i.e., lenses, masks, etc.). With DFWM, signal amplification on the order of 100 is possible, and thus, the problem of signal attenuation can be overcome.

The greatest disadvantage of DFWM is that the devices are still in the development stage. There are many variables such as beam alignment, temperature, light intensity, etc. that must be carefully adjusted, and as a result, this technique of doing real-time holography is difficult to implement.

# DFWM IN PHOTOREFRACTIVE MEDIA

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## DFWM IN PHOTOREFRACTIVE MEDIA

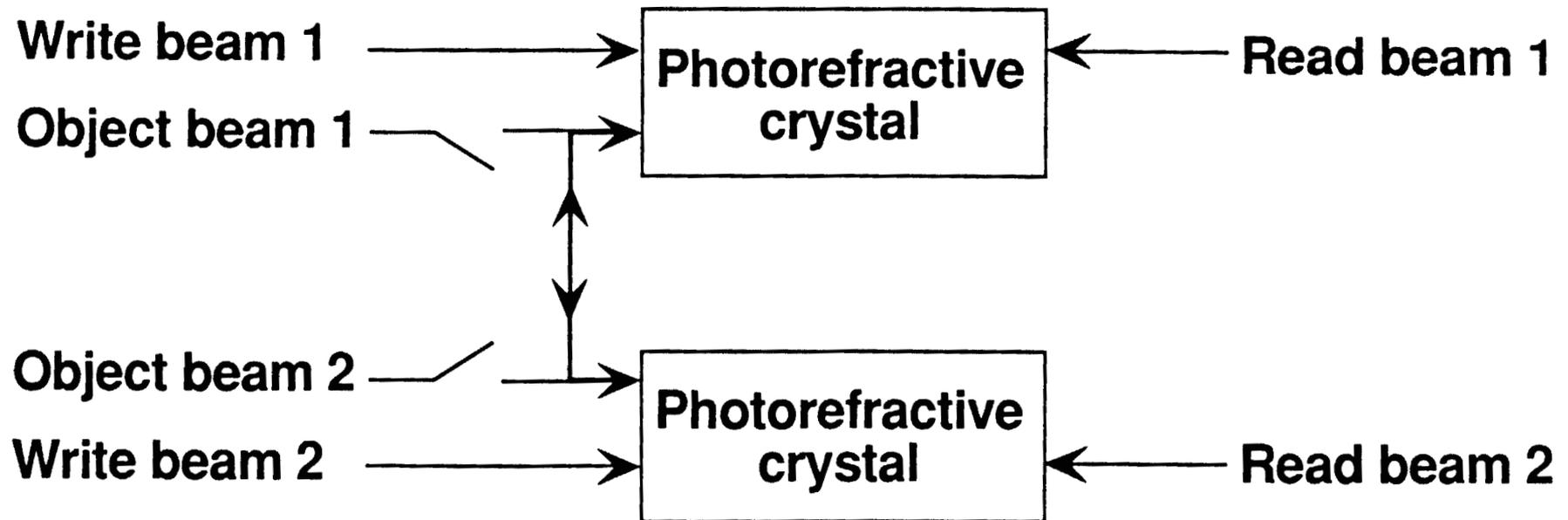
In this slide is a diagram illustrating the typical geometry for performing degenerate four wave mixing (DFWM) in a photorefractive crystal. Some examples of photorefractive crystals are Bismuth Silicon Oxide (BSO), Barium Titanate ( $\text{BaTiO}_3$ ), and Lithium Niobate ( $\text{LiNO}_3$ ). DFWM is a nonlinear optical process whereby a phase conjugate beam is produced by mixing (or causing to interfere) three coherent beams of light of the same wavelength within a medium.

The medium in this case is a photorefractive crystal. The photorefractive crystal provides a unique way of recording light intensity which allows multiple beams to be mixed. Light incident upon a photorefractive crystal causes trapped charges within the crystal to migrate. The charges migrate to regions of low light intensity and become retrapped. If the light intensity distribution over the crystal is not uniform, then a non-uniform distribution of charge will be established within the crystal. This distribution of charge will in turn give rise to a spatially varying electric field. The induced electric field causes a change in the index of refraction through the electro-optic effect. In this way, the distribution of light intensity over the volume of the crystal is recorded as a change in index of refraction.

In the diagram, three coherent beams of light, the write beam, the object beam, and the read beam are incident upon a photorefractive crystal simultaneously. The interference pattern generated by the interference of these three beams, produces (as described above) a spatially varying index of refraction within the crystal. The variation in index of refraction within the crystal is similar to a recorded hologram, and causes diffraction of the beams. If the read beam is identical to the write beam and counterpropagating with respect to the write beam, the diffracted beam is the phase conjugate of the object beam.

DFWM in a photorefractive crystal is similar to conventional holography where the hologram is illuminated with the phase conjugate of the reference, producing a phase conjugate of the object. The difference between DFWM and conventional holography is that in the DFWM process, the hologram is written and read simultaneously. Because of the fast response times of many photorefractive crystals, holography may be performed real-time using DFWM in a photorefractive crystal.

# TWO-CRYSTAL OSCILLATOR FOR STORAGE AND COMPUTING

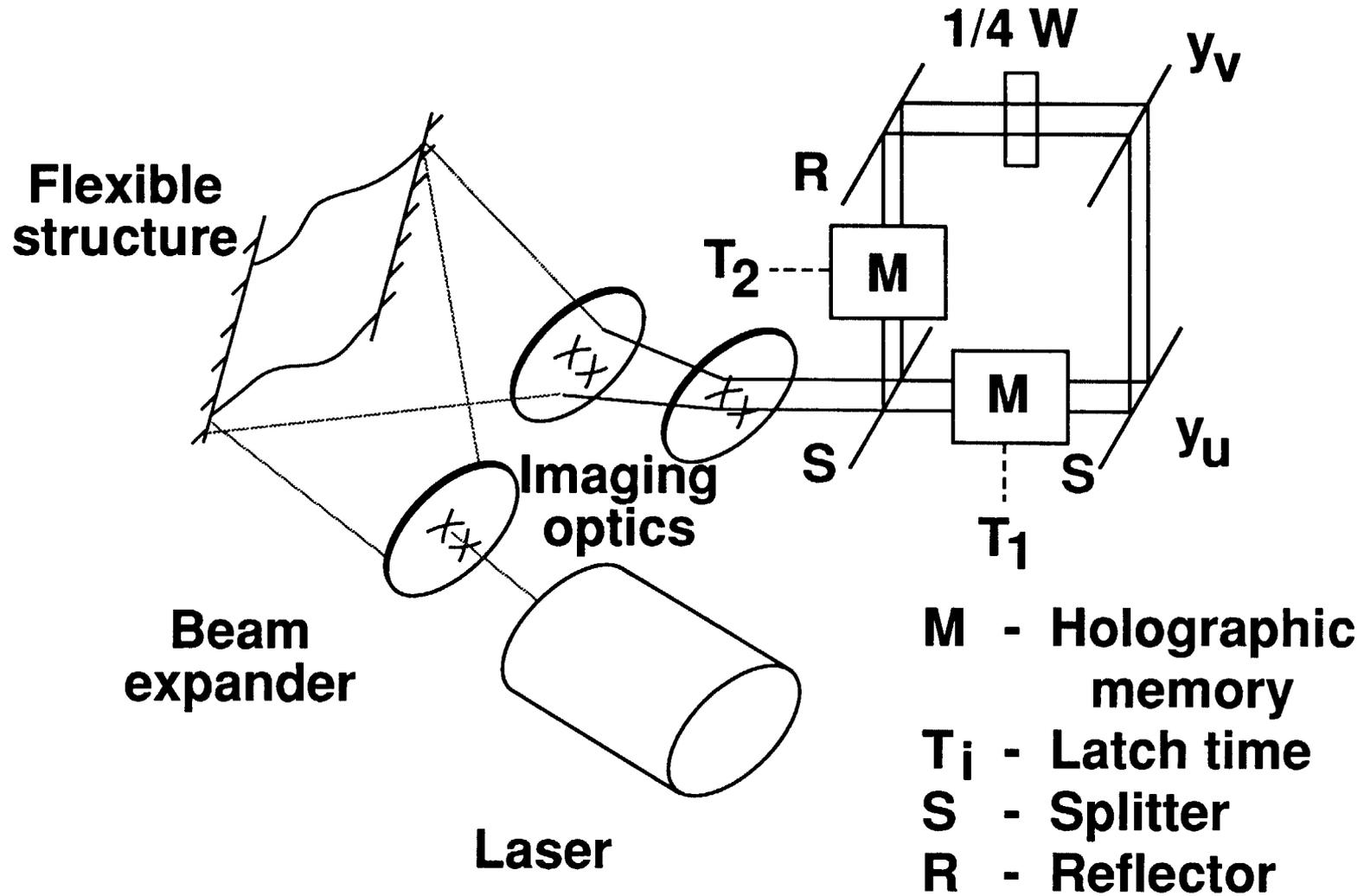


## TWO-CRYSTAL OSCILLATOR FOR STORAGE AND COMPUTING

Illustrated in this slide is a schematic diagram of a new optical memory being developed at NASA Langley. The memory will allow more efficient computational use of the crystals used in real-time holography. The memory consists of a two crystal oscillator. Each crystal has independent write, object, and read beams. Here, the top crystal may be used to write a hologram and (simultaneously) produce a phase conjugate beam. The phase conjugate output of this crystal is routed as the object beam to the bottom crystal. The phase conjugate output of the lower beam is thus the original object beam. The input to the top crystal may be switched to the phase conjugate output of the bottom crystal. The images will then oscillate in the optical path between the crystals and, with proper gain stabilization, retain the original object image operating like a conventional MOS dynamic RAM.

# SENSOR SYSTEM

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## SENSOR SYSTEM

Our intention is to develop a optical control system, based on the concepts presented, and to demonstrate it in a closed-loop laboratory test. The test structure will most probably be a beam because of the simplicity in representing the dynamics as a partial differential equation. This slide illustrates the sensor concept for the experiment. A Q Switched Laser source through a beam expander is used to illuminate the Flexible Structure. The object beam from the structure is focused to a small beam and is then defocused to a straight beam. This beam contains in phase information the image of the object. It is passed through a beam splitter to two holographic memory devices where the images are retained. These are latched at different times and the inteference between these images is used to obtain a rate image. One image is phase shifted by one quarter wave length to produce the correct intensity variation in the interference beam.

## CONTROLLER STRUCTURE

- PDE MODEL -  $U_{tt} + U_{xxxx} = 0$
- A PRIORI MODEL -  $U(X, K) = U(X, K-1) + V(X, K-1)T$   
 $\quad \quad \quad -F^{-1}\{k^4 F[U(X, K-1)]\}T / 2$   
 $V(X, K) = V(X, K-1) -F^{-1}\{k^4 F[U(X, K-1)]\}T$
- UPDATE EQUATION -  $U(X, K) = U(X, K) + \int_B G_Y(X, Z) E(Z, K) DZ$   
 $E(Z, K) = Y_M(Z, K) - Y(Z, K)$   
 $Y(Z, K) = H_u(Z)U(Z, K) + H_v(Z)V(Z, K)$
- CONTROL LAW -  $F(K) = \int_B G_U(Z)U(Z, K)DZ$

NOTE: IF  $G_Y(X, Z) = G_Y(X-Z)$  AND  $G_U(X, Z) = G_U(X-Z)$  THEN  
 THE INTEGRAL CAN BE COMPUTED VIA FOURIER OPTICS.

## CONTROLLER STRUCTURE

The controller will process state type stored images. In this case the state images are position and velocity images. A conventional estimator structure will be first attempted. In this case the a priori model is obtained from the partial differential equation model of the system by an Euler integration scheme. Hence, the a priori estimate of the state at sample  $k$  is obtained from the position and velocity information at sample  $k-1$ . The true value of the scheme is the fact that the Fourier transform operation shown on the slide can be accomplished using Fourier optics as previously described. This type of distributed model of the system completely eliminated spillover caused by modal representations of the system dynamics.

The estimator update equation would usually appear as shown on the slide. It involves an integral of estimation errors taken over the space of the state image. If the gain operator is shift invariant, depending only on  $x-z$ , then the integral in the update equation can be accomplished also via Fourier optics. Finally, the control law takes the form of the integral shown. Again, if the gain operator is shift invariant, the integral can be accomplished via Fourier optics. If the gain operator is not shift invariant a more general holographic technique would have to be developed. The output of the control law to drive an actuator will probably be accomplished using a photodiode. The signal would be amplified appropriately and used to drive an actuator such as a torque wheel attached to the beam.

## CONCLUDING REMARKS

- EXPERIMENT PROPOSED FOR DEVELOPMENT AND DEMONSTRATION OF DISTRIBUTED SENSING
- POTENTIAL TO ELIMINATE SPILLOVER CAUSED BY MODAL MODELLING
- STATUS OF HARDWARE DEVELOPMENT
  - HOLOGRAPHY WITH PHOTOGRAPHIC PLATES DONE
  - CRYSTALS HAVE BEEN DELIVERED TO LARC
  - REAL-TIME HOLOGRAPHY DEMONSTRATION IS IN PROGRESS
- STATUS OF DISTRIBUTED SENSING RESEARCH
  - SYSTEM HAS BEEN CONCEPTUALLY DESIGNED
  - SIMULATION OF OPTICAL COMPONENTS IN PROGRESS

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#### CONCLUDING REMARKS

An experiment has been proposed to facilitate development of an optical processor that processes distributed signals. The input to the processor is a distributed image of a structure and the output of the optical processor will probably be used to drive a torque wheel. Processing using distributed images will eliminate spillover caused by modal representations of controllers.

To this time we have accomplished conventional holography via photographic plates in our laboratory. This was done to create holograms of structures which are digitized and input to a Sun workstation computer system thus enabling simulation development to proceed in pace with simultaneous laboratory development. Photorefractive crystals have been delivered to Langley and we are now in the process of developing an in-house real-time holography capability.

At this time the control system has been conceptually designed. Detail design awaits the real-time holographic and dynamic memory developments and demonstrations. Analytical simulations of the optical components involved are currently being programmed for the Sun workstation.



# Report Documentation Page

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